We leave it as an exercise to prove that a test of whether the vector (7.69) has mean zero asymptotically may be accomplished by testing whether the q–vector $\boldsymbol{\delta}$ is equal to zero in the regression

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{P}_2\boldsymbol{X}^*\boldsymbol{\delta} + \text{residuals}. \tag{7.70}$$

Here $\boldsymbol{P}_2\boldsymbol{X}^*$ consists of the q columns of $\boldsymbol{P}_2\boldsymbol{X}$ that are not annihilated by $\boldsymbol{M}_{P_1X}$. Regression (7.70) must be estimated by IV using $\boldsymbol{W}_1$ as the matrix of instruments, and any of the tests discussed in Section 7.7 may then be used to test whether $\boldsymbol{\delta} = \boldsymbol{0}$.

7.10 Conclusion

This chapter has introduced all of the important concepts associated with the technique of instrumental variables estimation. For a more detailed treatment, see Bowden and Turkington (1984). Another useful reference is Godfrey (1988, Chapter 5), which discusses a large number of specification tests for both linear and nonlinear models that have been estimated by IV.

In this chapter, we applied the method of instrumental variables only to univariate linear and nonlinear regression models with i.i.d. errors. We will encounter numerous other applications later in the book, notably in Chapters 17 and 18, in which we discuss GMM estimation and simultaneous equations models, respectively. In many other cases, we will state a result in the context of OLS or NLS estimation and point out that it goes through with minor modification in the context of IV estimation as well.

Terms and Concepts

criterion function
Durbin-Wu-Hausman (DWH) tests
errors in variables
exactly identified (just identified) model
Gauss-Newton regression (GNR)
generalized IV estimator
identification: local, global, and asymptotic
instrumental variables (IV) estimator
instruments (instrumental variables)
nonlinear IV estimator
nonlinear two-stage least squares (NL2SLS) estimator
normalization (of a simultaneous equations model)
overidentified model
overidentifying restrictions
predetermined variable
reduced form (of a simultaneous equations model)
restricted reduced form (RRF)
simple IV estimator
simultaneous equations bias
simultaneous equations model
structural form (of a simultaneous equations model)
two-stage least squares (2SLS) estimator
unrestricted reduced form (URF)
vector of contrasts