order to provide consistent estimates of the parameters $\boldsymbol{\theta}$; see Section 18.4. If (18.04) is thought of as providing conditional moment conditions, then either predeterminedness or strict exogeneity allows us to use the columns of the matrix $\boldsymbol{X}$ as instruments for the estimation of $\boldsymbol{\theta}$ by some sort of IV procedure, such as 2SLS, 3SLS, or GMM. In claiming this, we assume of course that there are enough instruments in $\boldsymbol{X}$ to *identify* all of the parameters in $\boldsymbol{\theta}$.

Unfortunately, the concept of strict exogeneity is much too restrictive, at least for time-series applications. In this context, very few variables are strictly exogenous, although many are predetermined. However, as we now show, a variable can be predetermined or not in one and the same model depending on how the model is parametrized. Furthermore, predeterminedness is not always necessary for consistent estimation. Thus predeterminedness is not a very satisfactory concept.

Consider the following simultaneous model, taken from Engle, Hendry, and Richard (1983):

$$y_t = \beta x_t + \varepsilon_{1t} \tag{18.05}$$

$$x_t = \delta_1 x_{t-1} + \delta_2 y_{t-1} + \varepsilon_{2t}, \tag{18.06}$$

where the error terms are normally, independently, and identically distributed for each t, with covariance matrix

$$\boldsymbol{\Sigma} \equiv \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}.$$

If $\sigma_{12} \neq 0$, x_t is correlated with ε_{1t} and estimation of (18.05) by OLS will not be consistent because x_t is not predetermined in (18.05).

Now let us consider the expectation of y_t conditional on x_t and all lagged y_t's and x_t's. We have

$$E(y_t \,|\, x_t, y_{t-1}, x_{t-1} \cdots) = \beta x_t + E(\varepsilon_{1t} \,|\, x_t, y_{t-1}, x_{t-1} \cdots). \tag{18.07}$$

Notice that ε_{2t} is defined by (18.06) as a linear combination of the conditioning variables. Thus the conditional expectation of ε_{1t} in (18.07) is

$$E(\varepsilon_{1t} \,|\, \varepsilon_{2t}) = \frac{\sigma_{12}}{\sigma_{22}}\,\varepsilon_{2t} = \frac{\sigma_{12}}{\sigma_{22}}\,(x_t - \delta_1 x_{t-1} - \delta_2 y_{t-1}).$$

We may therefore write

$$y_t = b x_t + c_1 x_{t-1} + c_2 y_{t-1} + v_t, \tag{18.08}$$

with

$$b = \beta + \frac{\sigma_{12}}{\sigma_{22}}, \quad c_1 = -\,\delta_1 \frac{\sigma_{12}}{\sigma_{22}}, \quad c_2 = -\,\delta_2 \frac{\sigma_{12}}{\sigma_{22}}, \tag{18.09}$$

and with v_t independent of x_t. Thus x_t is predetermined in (18.08), whatever the value of σ_{12}, even though it is not predetermined in (18.05) when $\sigma_{12} \neq 0$.