Since the estimates $\hat{\boldsymbol{c}}$ for regression (13.81) are zero when the regressors are $\hat{\boldsymbol{G}}$, those regressors have no explanatory power for $\boldsymbol{\iota}$, and the sum of squared residuals is therefore equal to the total sum of squares. Because the latter is

$$\boldsymbol{\iota}^\top\boldsymbol{\iota} = \sum_{t=1}^{n} 1 = n,$$

the ML estimate of the residual variance in (13.81) is just unity:

$$\frac{1}{n}\text{SSR} = \frac{1}{n}\boldsymbol{\iota}^\top\boldsymbol{\iota} = \frac{1}{n}n = 1.$$

The OLS variance estimate, which is $\text{SSR}/(n-k) = n/(n-k)$, is asymptotically equivalent to this, but it will simplify the exposition if we suppose that the ML estimate is used. The covariance matrix estimate for the vector $\hat{\boldsymbol{c}}$ from (13.81) is then

$$\big(\hat{\boldsymbol{G}}^\top\hat{\boldsymbol{G}}\big)^{-1}.$$

It is this expression that gives the OPG regression its name, for its inverse is precisely the OPG estimator of the information matrix; see (8.48) and (8.50).[7] It follows that, as with the GNR, n^{-1} times the covariance matrix estimator from the OPG regression is asymptotically equal to the covariance matrix of $n^{1/2}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0)$.

The property just established is not the only one shared by the Gauss-Newton and OPG regressions. We will now establish two further properties of the OPG regression that are in fact shared by all regressions to which we give the name "artificial." The first of these properties is what allows one to use artificial regressions to perform one-step efficient estimation. According to this property, if the OPG regression (13.81) is evaluated at some parameter vector $\acute{\boldsymbol{\theta}}$ that is root-n consistent for $\boldsymbol{\theta}_0$, so that $\acute{\boldsymbol{\theta}} - \boldsymbol{\theta}_0 = O(n^{-1/2})$, then the artificial parameter estimates $\acute{\boldsymbol{c}}$ are such that

$$n^{1/2}\acute{\boldsymbol{c}} \overset{a}{=} n^{1/2}(\hat{\boldsymbol{\theta}} - \acute{\boldsymbol{\theta}}), \tag{13.82}$$

where $\hat{\boldsymbol{\theta}}$ is the ML estimator of $\boldsymbol{\theta}$. This result is essentially the same as the one proved for the Gauss-Newton regression in Section 6.6.

The result (13.82) is important. Because of it, we can proceed in one step from any root-n consistent estimator $\acute{\boldsymbol{\theta}}$ to an estimator asymptotically equivalent to the asymptotically efficient estimator $\hat{\boldsymbol{\theta}}$. The one-step estimator $\grave{\boldsymbol{\theta}}$ defined by $\grave{\boldsymbol{\theta}} \equiv \acute{\boldsymbol{\theta}} + \acute{\boldsymbol{c}}$ has the property that

$$n^{1/2}(\grave{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) = n^{1/2}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) + o(1), \tag{13.83}$$

[7] As we noted in Section 8.6, some authors refer to the OPG estimator of the information matrix as the BHHH estimator, after Berndt, Hall, Hall, and Hausman (1974), who advocated its use, although they did not explicitly make use of the OPG regression itself.