since the distribution of the u_t's has not been specified. Thus, for a sample of size n, the model $\mathbb{M}$ described by (5.08) is the set of all DGPs generating samples $\boldsymbol{y}$ of size n such that the expectation of y_t conditional on some information set Ω_t that includes $\boldsymbol{Z}_t$ is $x_t(\boldsymbol{\beta})$ for some parameter vector $\boldsymbol{\beta} \in \mathbb{R}^k$, and such that the differences $y_t - x_t(\boldsymbol{\beta})$ are independently distributed error terms with common variance σ^2, usually unknown.

It will be convenient to generalize this specification of the DGPs in $\mathbb{M}$ a little, in order to be able to treat **dynamic models**, that is, models in which there are **lagged dependent variables**. Therefore, we explicitly recognize the possibility that the regression function $x_t(\boldsymbol{\beta})$ may include among its (until now implicit) dependences an arbitrary but bounded number of lags of the dependent variable itself. Thus x_t may depend on $y_{t-1}, y_{t-2}, \ldots, y_{t-l}$, where l is a fixed positive integer that does not depend on the sample size. When the model uses time-series data, we will therefore take $x_t(\boldsymbol{\beta})$ to mean the expectation of y_t conditional on an information set that includes the entire past of the dependent variable, which we can denote by $\{y_s\}_{s=1}^{t-1}$, and also the entire history of the exogenous variables up to and including the period t, that is, $\{\boldsymbol{Z}_s\}_{s=1}^{t}$. The requirements on the disturbance vector $\boldsymbol{u}$ are unchanged.

For asymptotic theory to be applicable, we must next provide a rule for extending (5.08) to samples of arbitrarily large size. For models which are not dynamic (including models estimated with cross-section data, of course), so that there are no time trends or lagged dependent variables in the regression functions x_t, there is nothing to prevent the simple use of the fixed-in-repeated-samples notion that we discussed in Section 4.4. Specifically, we consider only sample sizes that are integer multiples of the actual sample size m and then assume that $x_{Nm+t}(\boldsymbol{\beta}) = x_t(\boldsymbol{\beta})$ for $N > 1$. This assumption makes the asymptotics of nondynamic models very simple compared with those for dynamic models.[3]

Some econometricians would argue that the above solution is too simple-minded when one is working with time-series data and would prefer a rule like the following. The variables $\boldsymbol{Z}_t$ appearing in the regression functions will usually themselves display regularities as time series and may be susceptible to modeling as one of the standard stochastic processes used in time-series analysis; we will discuss these standard processes at somewhat greater length in Chapter 10. In order to extend the DGP (5.08), the out-of-sample values for the $\boldsymbol{Z}_t$'s should themselves be regarded as random, being generated by appropriate processes. The introduction of this additional randomness complicates the asymptotic analysis a little, but not really a lot, since one would always assume that the stochastic processes generating the $\boldsymbol{Z}_t$'s were independent of the stochastic process generating the disturbance vector $\boldsymbol{u}$.

[3] Indeed, even for *linear* dynamic models it is by no means trivial to show that least squares yields consistent, asymptotically normal estimates. The classic reference on this subject is Mann and Wald (1943).