

Table 1: Estimation Results by Iteration of the Search Algorithm. DGP: Duration Dependence = Negative; Unobserved Heterogeneity = Discrete, Two Points of Support. QL: Duration Dependence = Cubic.

	β	α_1	α_2	α_3	θ_1	P_1	θ_2	P_2	θ_3	P_3	θ_4	P_4	$LogL$	$\bar{\theta}_i$	\bar{P}_i
Sample Size=500															
1	0.731	2.603	-1.197	0.181	1.565	1.000							-1418.22	-11.513	0.017
2	0.740	2.224	-0.965	0.145	1.706	0.961	-25.560	0.039					-1417.39	-0.588	0.034
3	1.079	1.282	-0.626	0.098	2.788	0.528	-25.560	0.032	0.944	0.439			-1415.61	-0.847	0.067
4*	1.615	-0.879	0.347	-0.045	4.692	0.338	-0.261	0.285	2.328	0.376			-1412.57	3.106	0.004
5	1.737	-1.179	0.471	-0.062	5.115	0.285	-0.294	0.287	2.263	0.309	3.506	0.118	-1412.52		
Sample Size=1000															
1	0.665	2.486	-1.049	0.150	1.570	1.000							-2811.36	-11.513	0.011
2	0.671	2.244	-0.900	0.125	1.658	0.975	-11.513	0.025					-2810.68	-0.987	0.036
3*	0.976	1.602	-0.752	0.113	2.785	0.464	0.980	0.536					-2808.03	-11.513	0.013
4	1.333	-0.228	0.132	-0.021	4.178	0.352	1.862	0.441	-0.442	0.207			-2804.76		
Sample Size=5000															
1	0.670	2.558	-1.040	0.139	1.549	1.000							-14067.84	-11.513	0.006
2	1.066	1.276	-0.563	0.077	2.935	0.493	0.852	0.507					-14042.82	-11.513	0.008
3	1.216	0.507	-0.190	0.020	3.543	0.418	1.364	0.434	-0.292	0.147			-14038.13	2.197	0.002
4	1.269	0.340	-0.124	0.011	3.767	0.380	0.980	0.288	-0.530	0.111	1.926	0.221	-14037.78		

Notes: DGP = data generating process, QL = quasi-likelihood. β is the parameter on the observed heterogeneity, the α_i are the parameters of the cubic in duration and the θ_i and P_i are the parameters of the distribution of unobserved heterogeneity. $LogL$ is the value of the loglikelihood. $\bar{\theta}_i$ is the position suggested by the search algorithm for the new point of support on the next iteration. \bar{P}_i is the suggested value for the probability associated with the new point of support. * indicates that a point of support was dropped in this iteration.

Table 2: A Summary of Estimates of β Across DGP's and Quasi Likelihoods

Sample Size		500				1000				5000			
DGP	Quasi Likelihood	Initial	True	MLE	HQIC	Initial	True	MLE	HQIC	Initial	True	MLE	HQIC
DD: none Het.: none	DD: none	1.000 (0.108)	1.000 (0.108)	1.004 (0.110)	1.000 (0.108)	0.992 (0.068)	0.992 (0.068)	0.996 (0.069)	0.993 (0.068)	0.998 (0.033)	0.998 (0.033)	0.999 (0.033)	0.998 (0.033)
	DD: cubic	1.001 (0.117)	1.001 (0.117)	1.394 (0.505)	1.011 (0.140)	0.992 (0.076)	0.992 (0.076)	1.177 (0.219)	1.001 (0.106)	0.997 (0.036)	0.997 (0.036)	1.075 (0.076)	0.997 (0.036)
	DD: step	1.004 (0.118)	1.004 (0.118)	4.023 (1.813)	1.012 (0.132)	0.993 (0.076)	0.993 (0.076)	3.289 (1.295)	0.996 (0.084)	0.997 (0.036)	0.997 (0.036)	2.499 (0.879)	0.997 (0.036)
DD: none Het.: discrete	DD: none	0.905 (0.118)	1.015 (0.132)	1.028 (0.132)	1.015 (0.132)	0.896 (0.083)	1.004 (0.091)	1.012 (0.091)	1.004 (0.091)	0.901 (0.039)	1.005 (0.042)	1.010 (0.043)	1.006 (0.042)
	DD: cubic	0.701 (0.086)	0.959 (0.226)	1.372 (0.594)	0.827 (0.234)	0.692 (0.060)	0.967 (0.173)	1.148 (0.225)	0.875 (0.228)	0.698 (0.029)	0.990 (0.077)	1.062 (0.065)	1.011 (0.053)
	DD: step	0.702 (0.087)	1.003 (0.206)	3.436 (1.559)	0.843 (0.247)	0.697 (0.060)	0.995 (0.161)	2.914 (0.875)	0.901 (0.238)	0.698 (0.029)	0.998 (0.068)	2.268 (0.845)	1.010 (0.051)
DD: none Het.: Gamma	DD: none	0.776 (0.150)		1.003 (0.133)	0.984 (0.132)	0.774 (0.115)		0.998 (0.100)	0.983 (0.098)	0.766 (0.051)		0.998 (0.044)	0.993 (0.045)
	DD: cubic	0.651 (0.113)		1.430 (0.644)	0.944 (0.198)	0.649 (0.087)		1.164 (0.225)	0.952 (0.128)	0.642 (0.036)		1.060 (0.084)	0.960 (0.066)
	DD: step	0.652 (0.113)		3.917 (1.461)	0.946 (0.181)	0.650 (0.087)		3.491 (1.310)	0.954 (0.119)	0.643 (0.036)		2.631 (0.896)	0.966 (0.073)
DD: negative Het.: none	DD: none	1.191 (0.120)	1.191 (0.120)	1.310 (0.142)	1.265 (0.137)	1.198 (0.081)	1.198 (0.081)	1.316 (0.099)	1.284 (0.101)	1.194 (0.040)	1.194 (0.040)	1.303 (0.047)	1.290 (0.051)
	DD: cubic	1.011 (0.101)	1.011 (0.101)	1.523 (0.483)	1.042 (0.209)	1.014 (0.067)	1.014 (0.067)	1.339 (0.241)	1.045 (0.123)	1.005 (0.035)	1.005 (0.035)	1.200 (0.094)	1.030 (0.080)
	DD: step	1.012 (0.102)	1.012 (0.102)	4.212 (1.473)	1.027 (0.131)	1.015 (0.068)	1.015 (0.068)	3.913 (1.203)	1.026 (0.087)	1.004 (0.034)	1.004 (0.034)	2.608 (1.031)	1.004 (0.034)

Notes: DGP = data generating process. DD = Duration Dependence; Het. = unobserved Heterogeneity. The reported statistics are the mean and standard deviation of β across 100 samples on the indicated iteration of the algorithm. Initial = on the first iteration; True = on the iteration on which the distribution of unobserved heterogeneity is correctly specified; MLE = the Maximum Likelihood estimate; HQIC = the estimate using the Hannon-Quinn Information Criterion. In the column titled True, not all means are calculated over 100 samples as in some cases the search algorithm stopped before reaching the true number of points.

Table 2 (cont.)

Sample Size		500				1000				5000			
DGP	Quasi Likelihood	Initial	True	MLE	HQIC	Initial	True	MLE	HQIC	Initial	True	MLE	HQIC
DD: negative Het.: discrete	DD: none	0.897 (0.152)	1.138 (0.232)	1.325 (0.240)	1.223 (0.269)	0.890 (0.105)	1.128 (0.162)	1.310 (0.175)	1.250 (0.181)	0.867 (0.048)	1.107 (0.070)	1.269 (0.077)	1.249 (0.077)
	DD: cubic	0.678 (0.110)	0.897* (0.246)	1.409 (0.523)	0.830 (0.337)	0.676 (0.077)	0.929 (0.184)	1.247 (0.264)	0.885 (0.283)	0.662 (0.036)	0.901 (0.163)	1.152 (0.109)	1.054 (0.108)
	DD: step	0.677 (0.110)	0.995 (0.223)	4.123 (1.744)	0.813 (0.340)	0.674 (0.077)	0.980 (0.162)	3.670 (1.247)	0.829 (0.264)	0.660 (0.036)	0.987 (0.084)	2.768 (0.823)	0.994 (0.076)
DD: negative Het.: Gamma	DD: none	0.912 (0.153)		1.318 (0.177)	1.278 (0.183)	0.925 (0.116)		1.323 (0.130)	1.296 (0.128)	0.920 (0.052)		1.308 (0.057)	1.293 (0.058)
	DD: cubic	0.721 (0.114)		1.411 (0.386)	0.958 (0.234)	0.727 (0.087)		1.258 (0.230)	1.011 (0.174)	0.723 (0.038)		1.166 (0.099)	1.028 (0.104)
	DD: step	0.721 (0.115)		4.253 (1.817)	0.951 (0.222)	0.727 (0.087)		3.992 (1.174)	0.983 (0.154)	0.722 (0.038)		2.898 (1.155)	0.975 (0.053)
DD: positive Het.: none	DD: none	0.803 (0.074)	0.803 (0.074)	0.803 (0.074)	0.803 (0.074)	0.795 (0.055)	0.795 (0.055)	0.795 (0.055)	0.795 (0.055)	0.795 (0.026)	0.795 (0.026)	0.795 (0.026)	0.795 (0.026)
	DD: cubic	1.008 (0.101)	1.008 (0.101)	1.954 (1.183)	1.024 (0.154)	0.995 (0.076)	0.995 (0.076)	1.407 (0.783)	1.003 (0.096)	0.994 (0.036)	0.994 (0.036)	1.061 (0.086)	0.994 (0.036)
	DD: step	1.012 (0.103)	1.012 (0.103)	4.003 (1.981)	1.039 (0.177)	0.998 (0.076)	0.998 (0.076)	3.529 (1.541)	1.009 (0.102)	0.994 (0.036)	0.994 (0.036)	2.851 (1.258)	0.994 (0.036)
DD: positive Het.: discrete	DD: none	0.796 (0.110)	0.806 (0.113)	0.808 (0.113)	0.806 (0.113)	0.795 (0.075)	0.807 (0.077)	0.808 (0.077)	0.808 (0.077)	0.794 (0.031)	0.806 (0.032)	0.807 (0.032)	0.806 (0.032)
	DD: cubic	0.712 (0.098)	0.946 (0.205)	1.299 (0.659)	0.800 (0.214)	0.711 (0.067)	0.953 (0.171)	1.104 (0.256)	0.863 (0.208)	0.707 (0.029)	0.969 (0.097)	1.008 (0.092)	0.978 (0.088)
	DD: step	0.715 (0.098)	0.974 (0.205)	2.910 (1.297)	0.835 (0.259)	0.712 (0.067)	0.958 (0.165)	2.607 (0.816)	0.893 (0.245)	0.708 (0.029)	0.966 (0.097)	2.317 (0.855)	0.988 (0.067)
DD: positive Het.: Gamma	DD: none	0.667 (0.132)		0.800 (0.107)	0.794 (0.105)	0.670 (0.103)		0.801 (0.077)	0.796 (0.076)	0.664 (0.043)		0.792 (0.034)	0.789 (0.033)
	DD: cubic	0.627 (0.101)		1.413 (0.562)	0.935 (0.173)	0.629 (0.077)		1.188 (0.263)	0.959 (0.123)	0.623 (0.031)		1.042 (0.070)	0.991 (0.056)
	DD: step	0.632 (0.102)		3.709 (1.550)	0.952 (0.240)	0.632 (0.077)		3.200 (1.025)	0.937 (0.125)	0.625 (0.031)		2.555 (0.927)	0.972 (0.074)

Table 3: A Summary of Estimates of β in Large Samples

Sample Size		100000			
DGP	Quasi Likelihood	Initial	True	MLE	HQIC
DD: none Het.: none	DD: none	0.997	0.997	0.998	0.997
		(0.008)	(0.008)	(0.008)	(0.008)
	DD: cubic	0.996	0.996	1.008	0.996
		(0.009)	(0.009)	(0.012)	(0.009)
	DD: step	0.996	0.996	1.079	0.996
		(0.009)	(0.009)	(0.098)	(0.009)
DD: none Het.: discrete	DD: none	0.901	1.005	1.006	1.005
		(0.008)	(0.008)	(0.008)	(0.008)
	DD: cubic	0.698	1.004	1.011	1.004
		(0.007)	(0.010)	(0.011)	(0.010)
	DD: step	0.697	1.005	1.128	1.005
		(0.007)	(0.010)	(0.110)	(0.010)
DD: none Het.: Gamma	DD: none	0.765		0.996	0.996
		(0.016)		(0.009)	(0.009)
	DD: cubic	0.642		1.000	0.989
		(0.012)		(0.013)	(0.012)
	DD: step	0.642		1.169	0.989
		(0.012)		(0.150)	(0.012)

Notes: DGP = data generating process. DD = Duration Dependence; Het. = unobserved Heterogeneity. The reported statistics are the mean and standard deviation of β across 5 samples on the indicated iteration of the algorithm. Initial = on the first iteration; True = on the iteration on which the distribution of unobserved heterogeneity is correctly specified; MLE = the Maximum Likelihood estimate; HQIC = the estimate using the Hannon-Quinn Information Criterion.

Table 4: Estimates of β When the Variance of Observable Heterogeneity is Increased

Sample Size		500				1000				5000			
DGP	Quasi Likelihood	Initial	True	MLE	HQIC	Initial	True	MLE	HQIC	Initial	True	MLE	HQIC
DD: negative Het.: discrete	DD: none	0.906 (0.071)	1.205 (0.098)	1.294 (0.109)	1.246 (0.110)	0.902 (0.054)	1.190 (0.071)	1.282 (0.073)	1.255 (0.077)	0.914 (0.027)	1.192 (0.037)	1.281 (0.037)	1.267 (0.038)
	DD: cubic	0.687 (0.055)	0.961 (0.170)	1.302 (0.284)	0.977 (0.223)	0.683 (0.039)	0.960 (0.147)	1.204 (0.146)	1.013 (0.137)	0.690 (0.019)	0.984 (0.109)	1.122 (0.068)	1.033 (0.057)
	DD: step	0.685 (0.056)	0.994 (0.146)	3.551 (1.376)	0.919 (0.212)	0.681 (0.039)	0.984 (0.108)	3.295 (1.341)	0.972 (0.178)	0.687 (0.019)	0.996 (0.059)	2.130 (1.015)	1.001 (0.050)

Notes: DGP = data generating process. DD = Duration Dependence; Het. = unobserved Heterogeneity. The reported statistics are the mean and standard deviation of β across 100 samples on the indicated iteration of the algorithm. Initial = on the first iteration; True = on the iteration on which the distribution of unobserved heterogeneity is correctly specified; MLE = the Maximum Likelihood estimate; HQIC = the estimate using the Hannon-Quinn Information Criterion. For these results, the variance of observable heterogeneity (X) is set to 1.00 (it is set to 0.25 for the results in table 1-3).

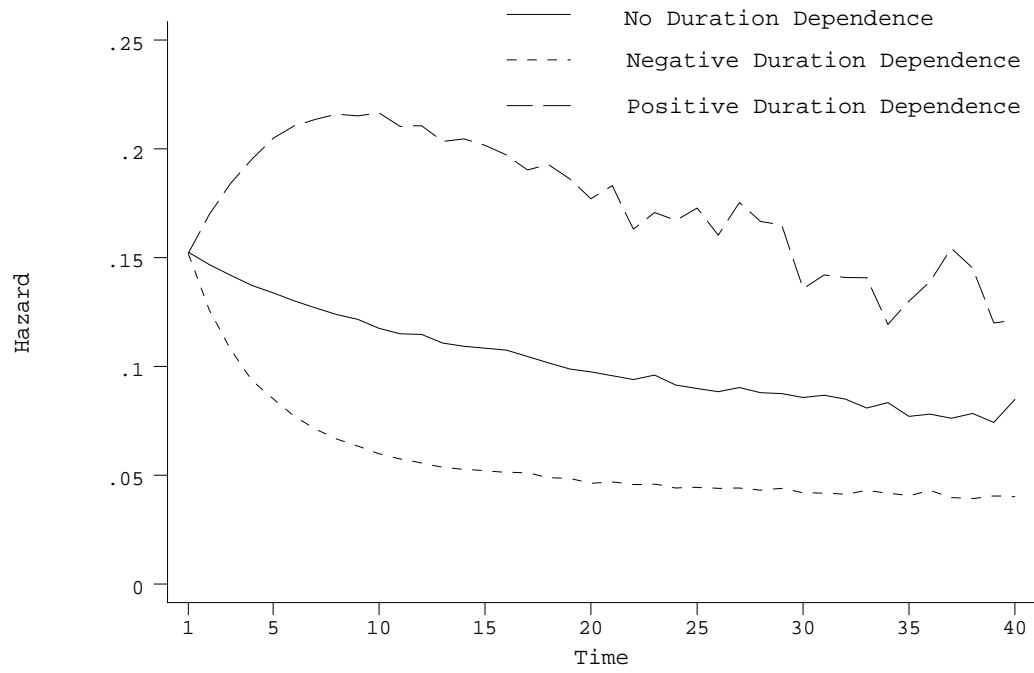


Figure 1: Empirical Hazard for DGP with No Heterogeneity

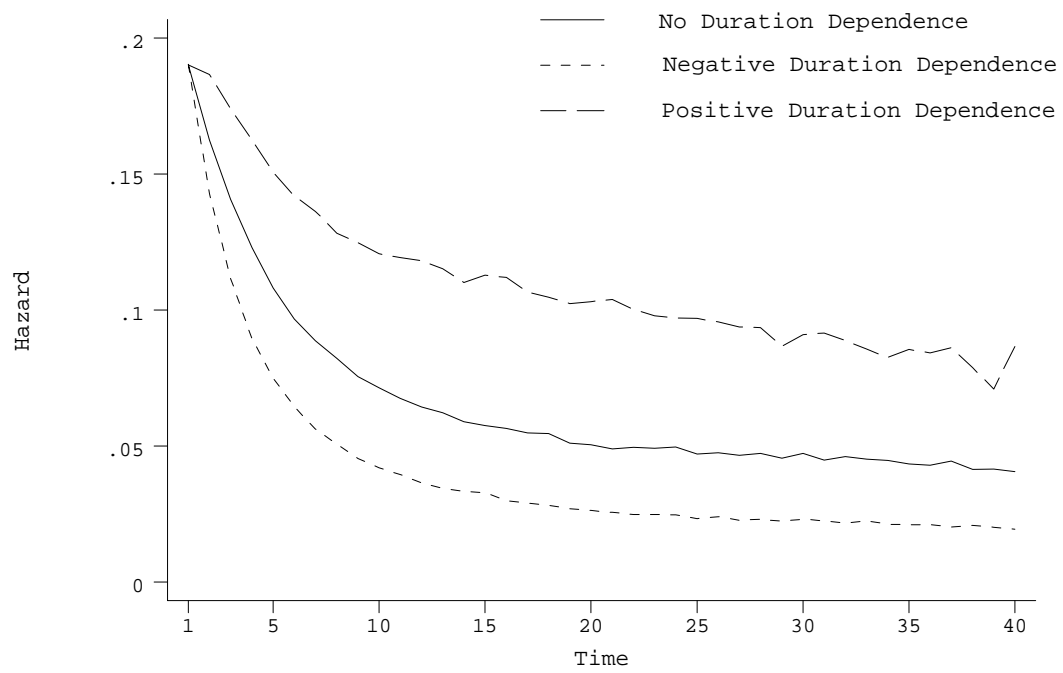


Figure 2: Empirical Hazard for DGP with Two Point Heterogeneity

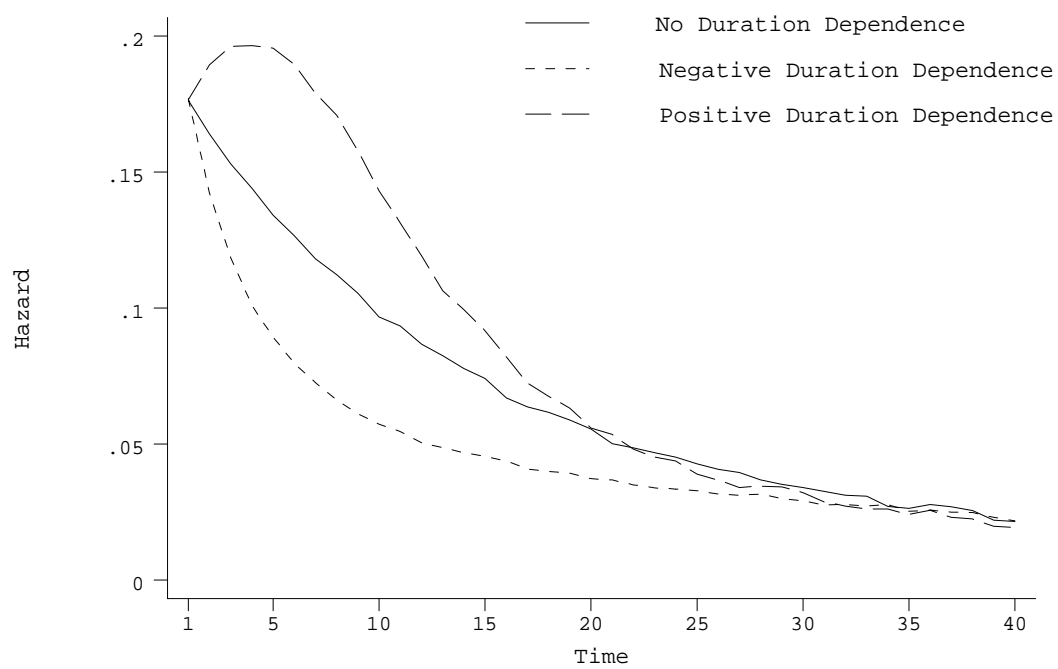


Figure 3: Empirical Hazard for DGP with Gamma Heterogeneity

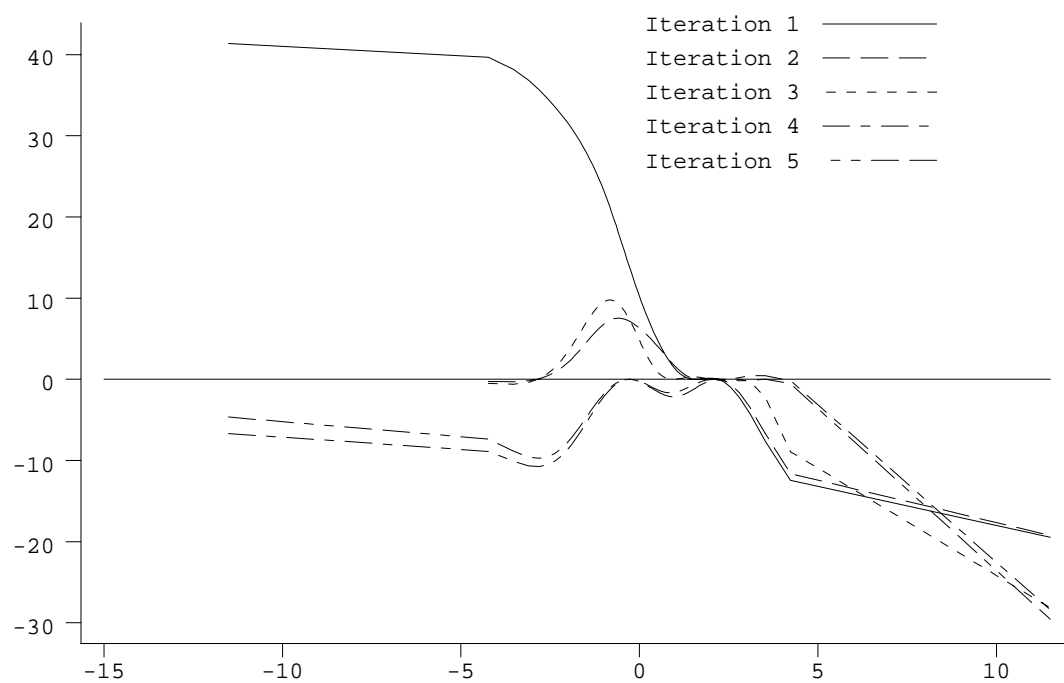


Figure 4: The Gateaux Derivative When Sample Size=500.

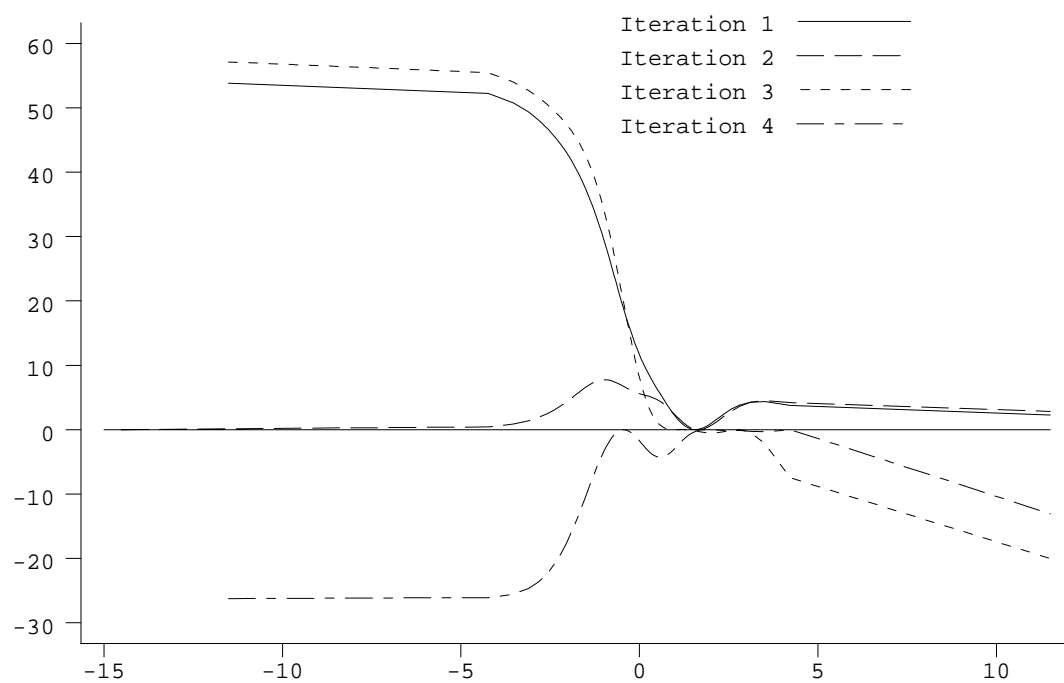


Figure 5: The Gateaux Derivative When Sample Size=1000.

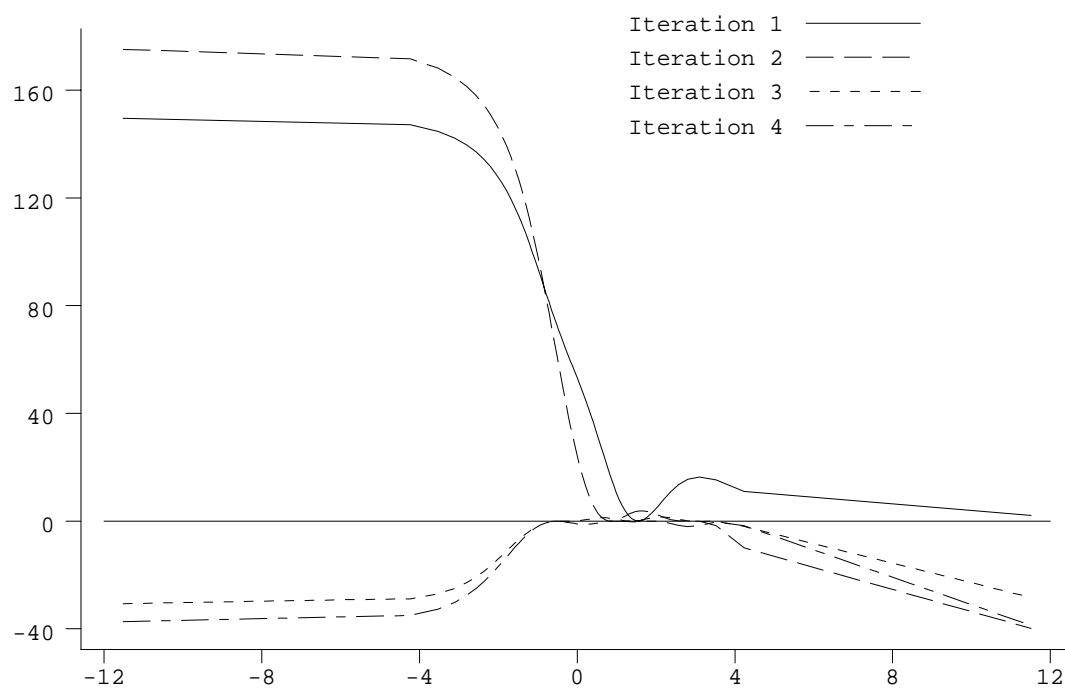


Figure 6: The Gateaux Derivative When Sample Size=5000.

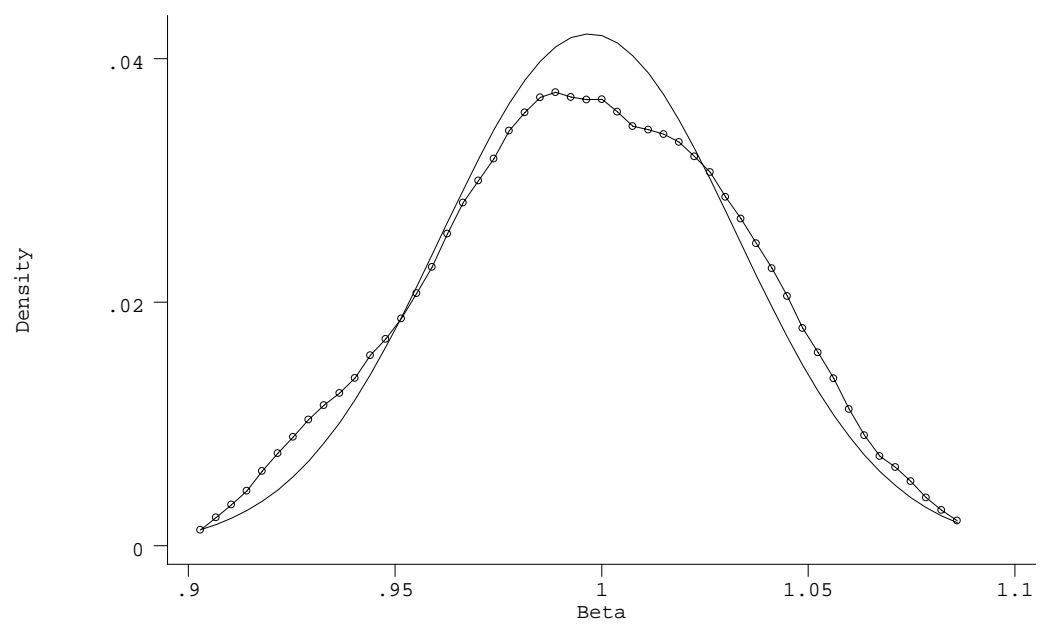


Figure 7: The Density of Beta (HQIC). DGP: No Heterogeneity,
No Duration Dependence. QL: Cubic.

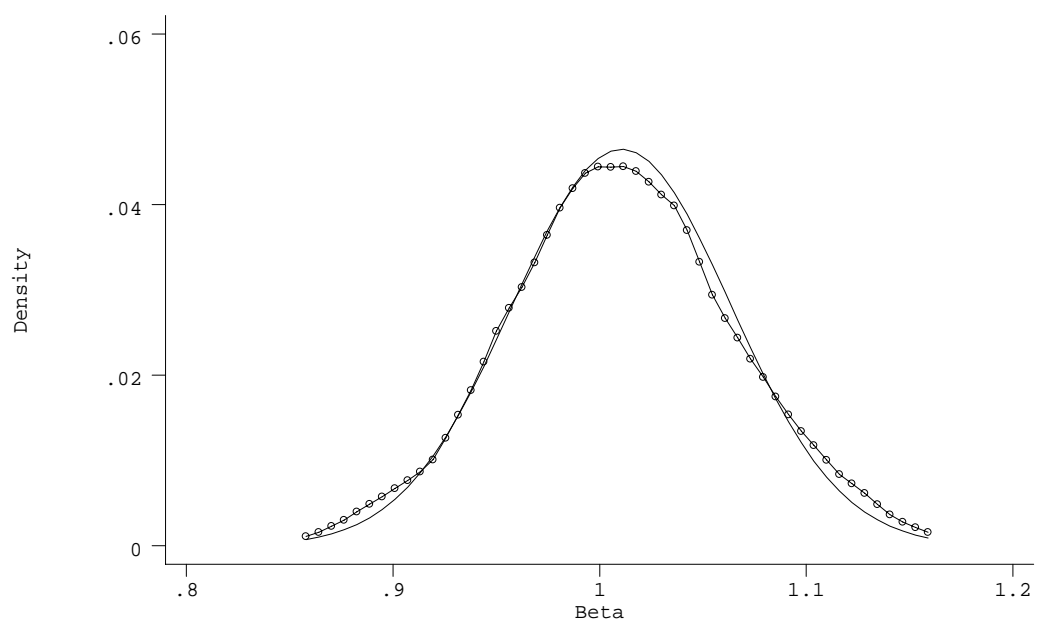


Figure 8: The Density of Beta (HQIC). DGP: Two Point Heterogeneity, No Duration Dependence. QL: Cubic.

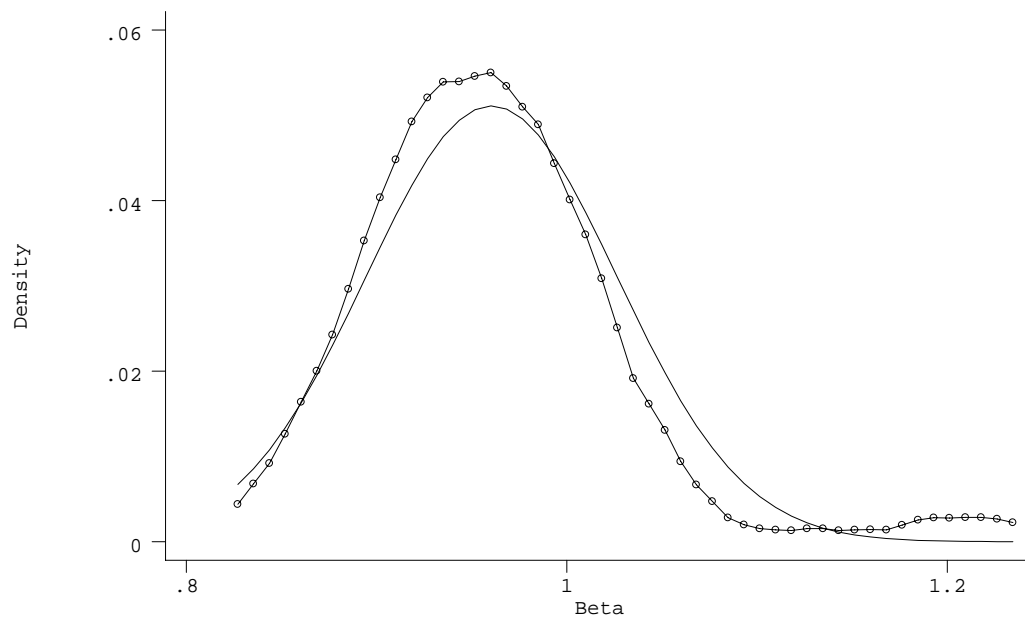
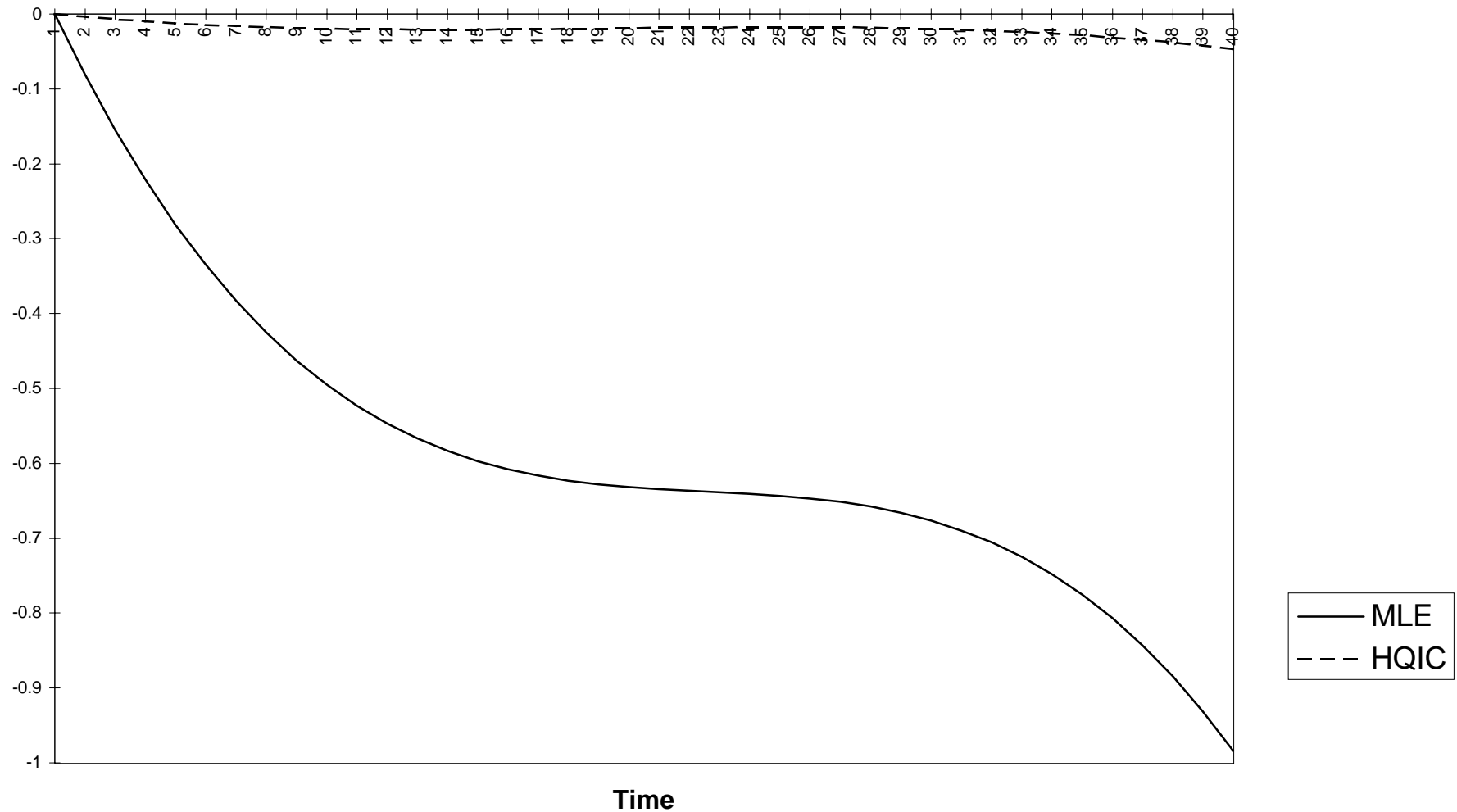


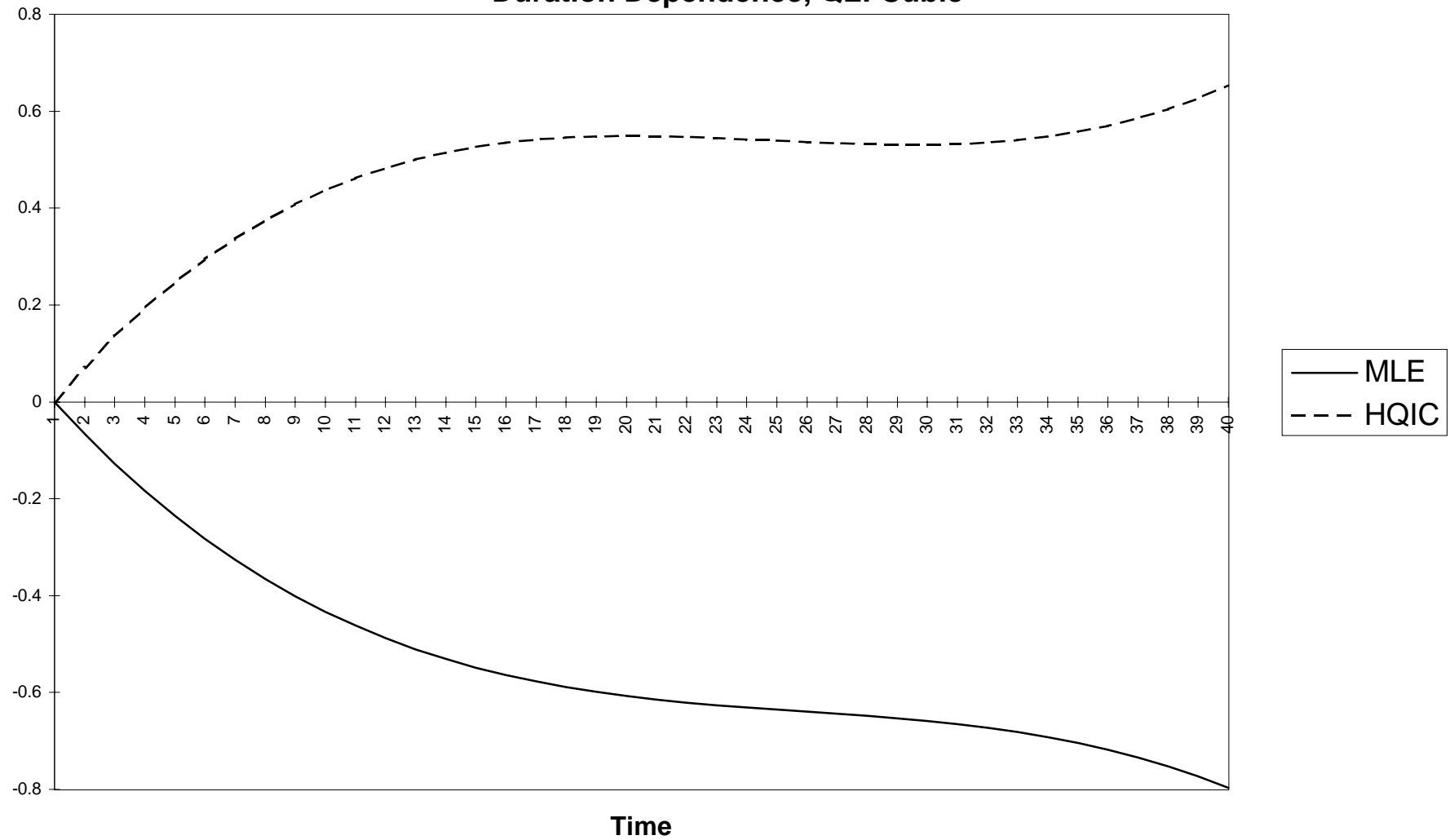
Figure 9: The Density of Beta (HQIC). DGP: Gamma Heterogeneity,
No Duration Dependence. QL: Cubic.

Figure 10: Duration Dependence when DGP: No Heterogeneity, No Duration Dependence, QL: Cubic



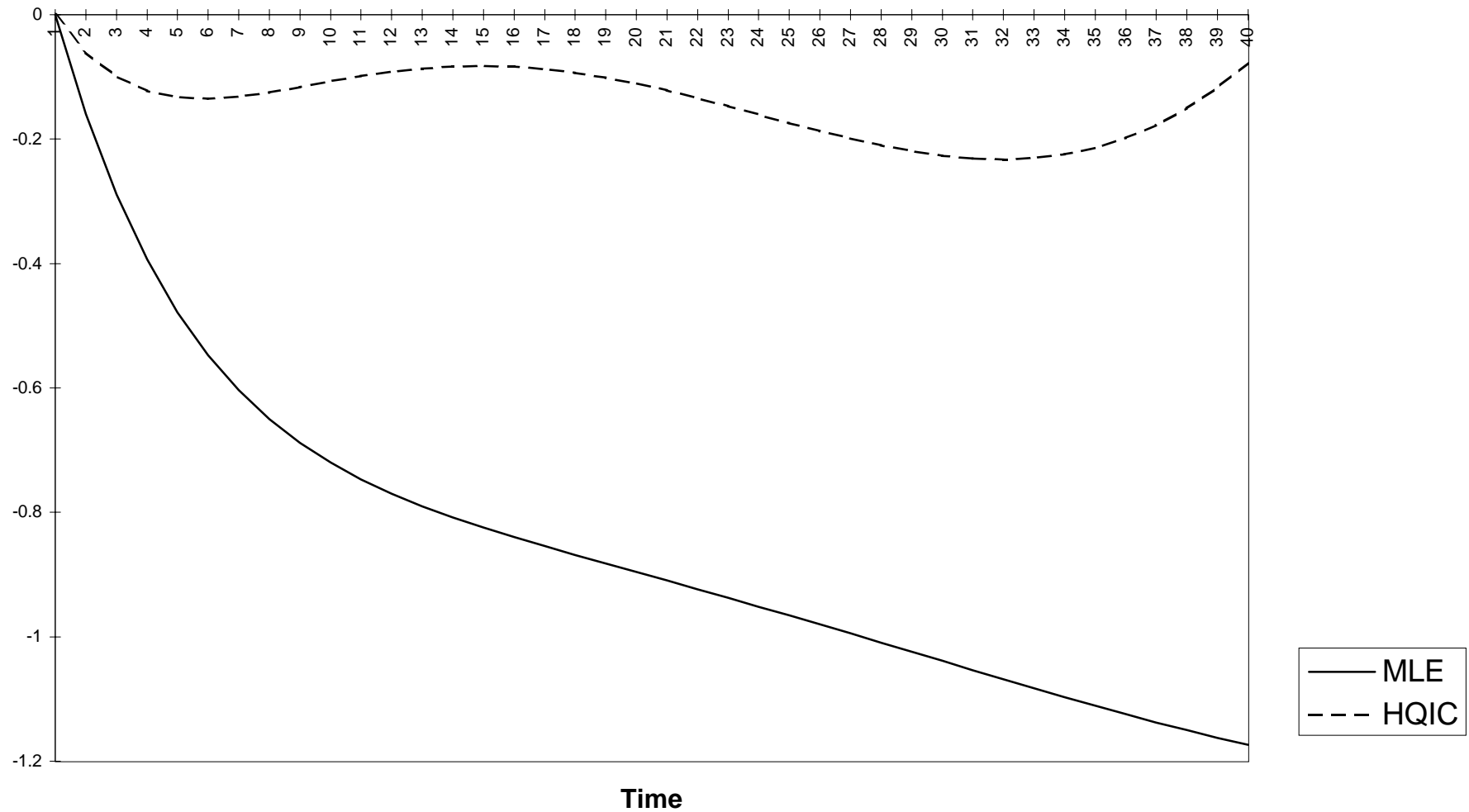
Note: Duration dependence is graphed in deviations (by period) from the true duration dependence. MLE=Maximum Likelihood estimate, HQIC=Hannan-Quinn Information Criterion estimate.

Figure 11: Duration Dependence when DGP: Two Point Heterogeneity, No Duration Dependence, QL: Cubic



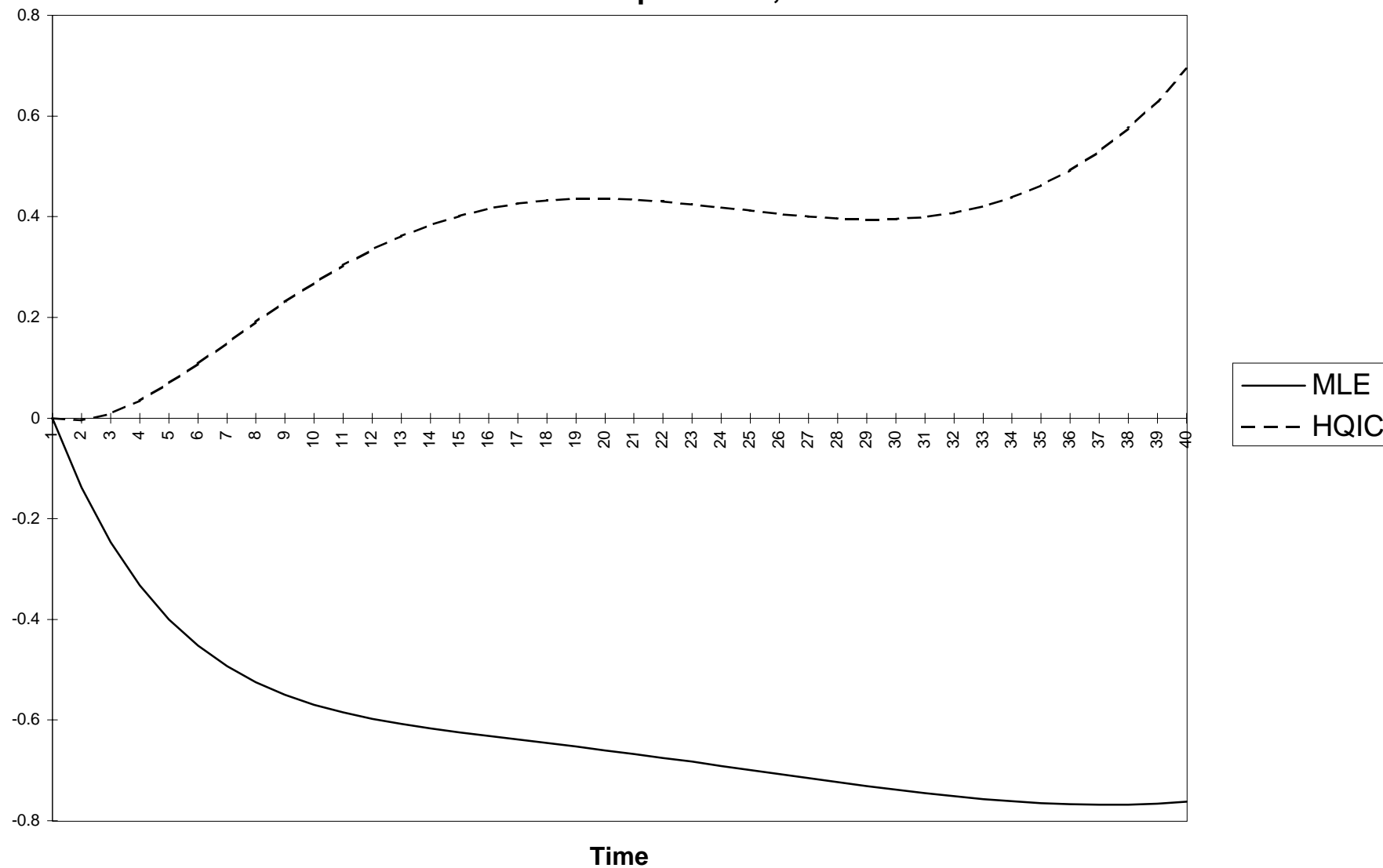
Note: Duration dependence is graphed in deviations (by period) from the true duration dependence. MLE=Maximum Likelihood estimate, HQIC=Hannan-Quinn Information Criterion estimate.

Figure 12: Duration Dependence when DGP: No Heterogeneity, Negative Duration Dependence, QL: Cubic



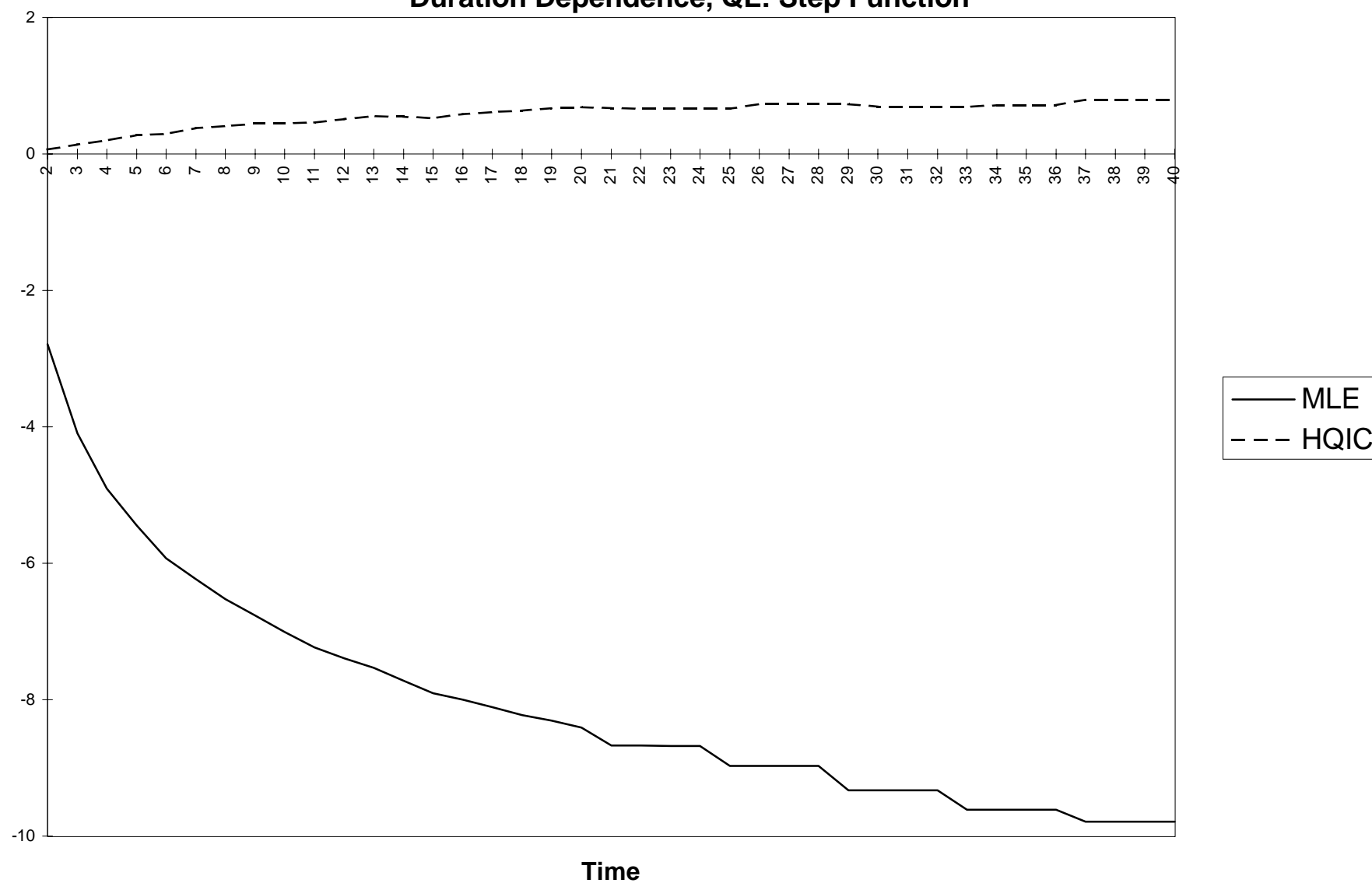
Note: Duration dependence is graphed in deviations (by period) from true duration dependence. MLE=Maximum Likelihood Estimate, HQIC=Hannan-Quinn Information Criterion estimate.

Figure 13: Duration Dependence when DGP: Two Point Heterogeneity, Negative Duration Dependence, QL: Cubic



Note: Duration dependence is graphed in deviations (by period) from the true duration dependence. MLE=Maximum Likelihood estimate, HQIC=Hannan-Quinn Information Criterion estimate.

Figure 14: Duration Dependence when DGP: Two Point Heterogeneity, Negative Duration Dependence, QL: Step Function



Note: Duration dependence is graphed in deviations (by period) from true duration dependence. MLE=Maximum Likelihood estimate, HQIC=Hannan-Quinn Information Criterion estimate.

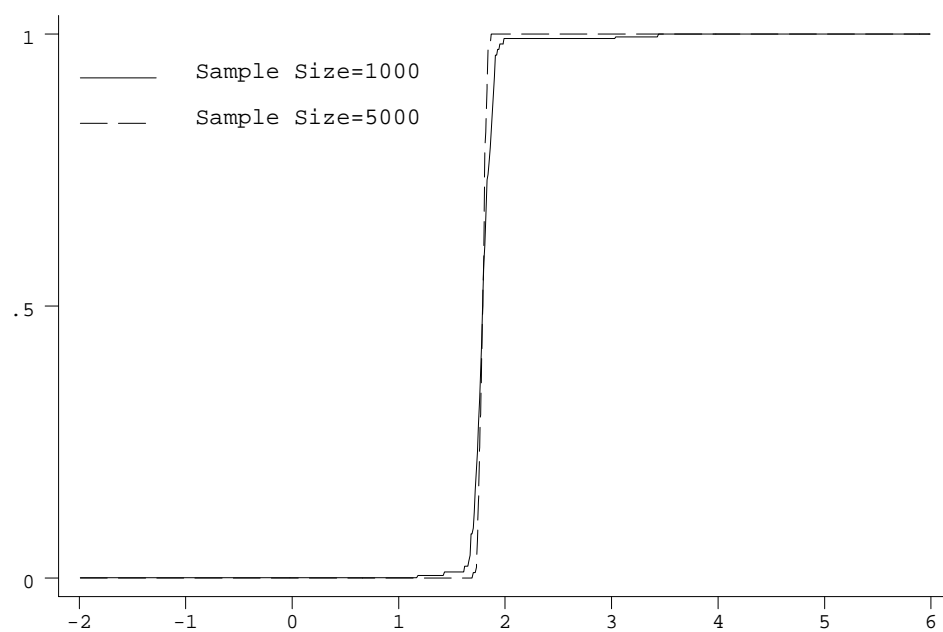


Figure 15: The CDF of Unobserved Heterogeneity (HQIC). DGP: No Heterogeneity, No Duration Dependence. QL: Cubic.

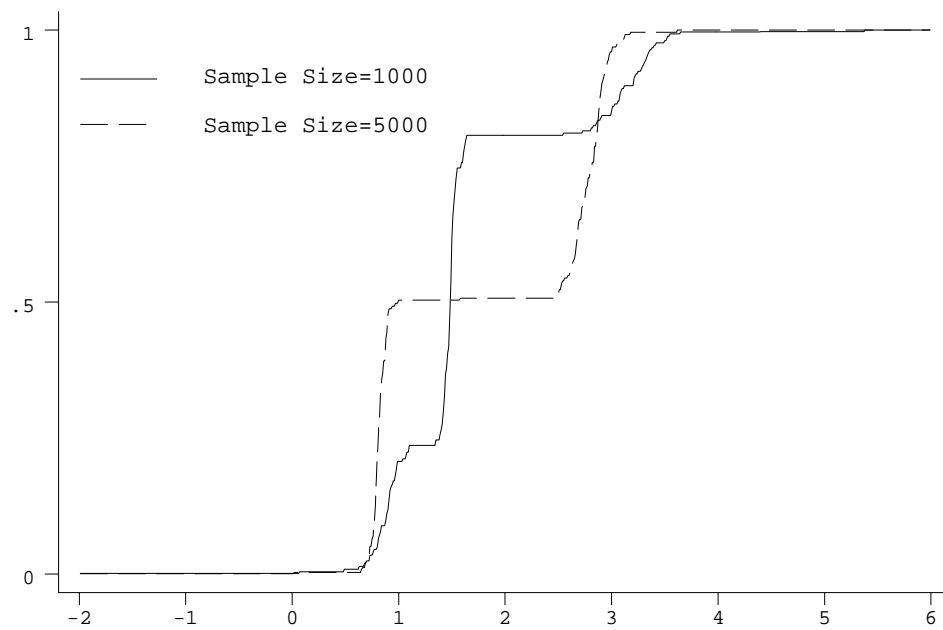


Figure 16: The CDF of Unobserved Heterogeneity (HQIC). DGP: Two Point Heterogeneity, No Duration Dependence. QL: Cubic.

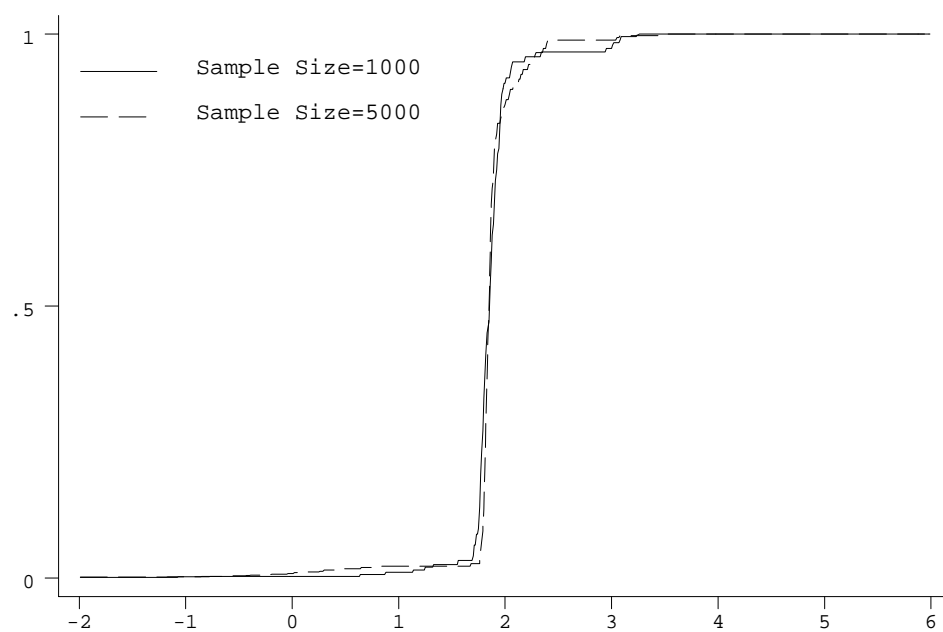


Figure 17: The CDF of Unobserved Heterogeneity (HQIC). DGP: No Heterogeneity, Negative Duration Dependence. QL: Cubic.

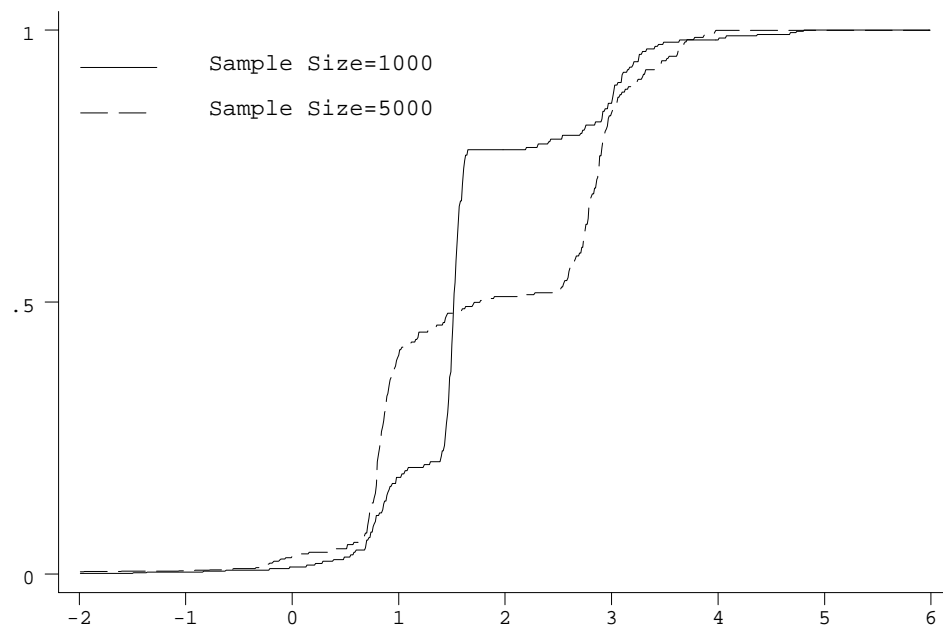


Figure 18: The CDF of Unobserved Heterogeneity (HQIC). DGP: Two Point Heterogeneity, Negative Duration Dependence. QL: Cubic.

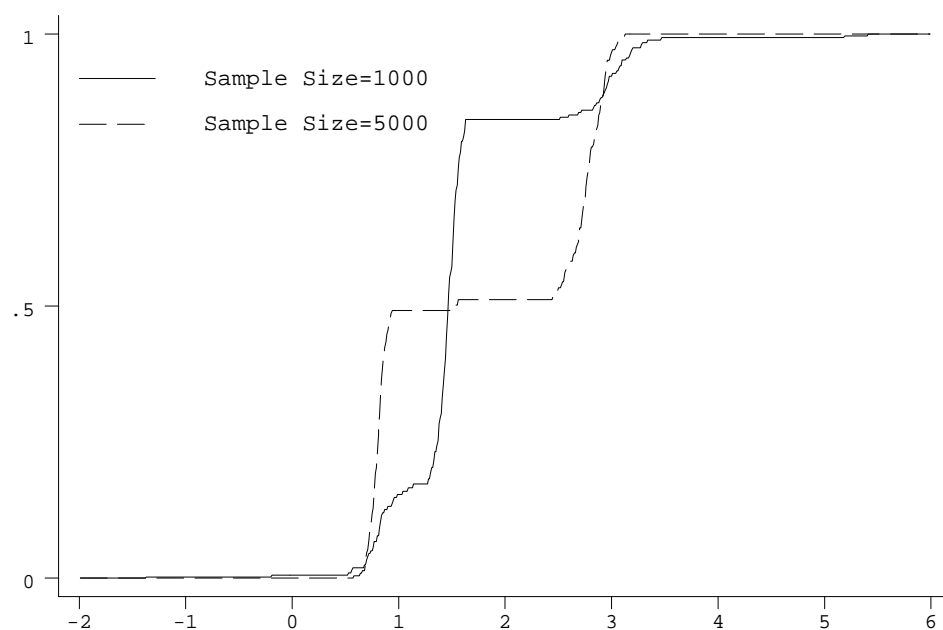
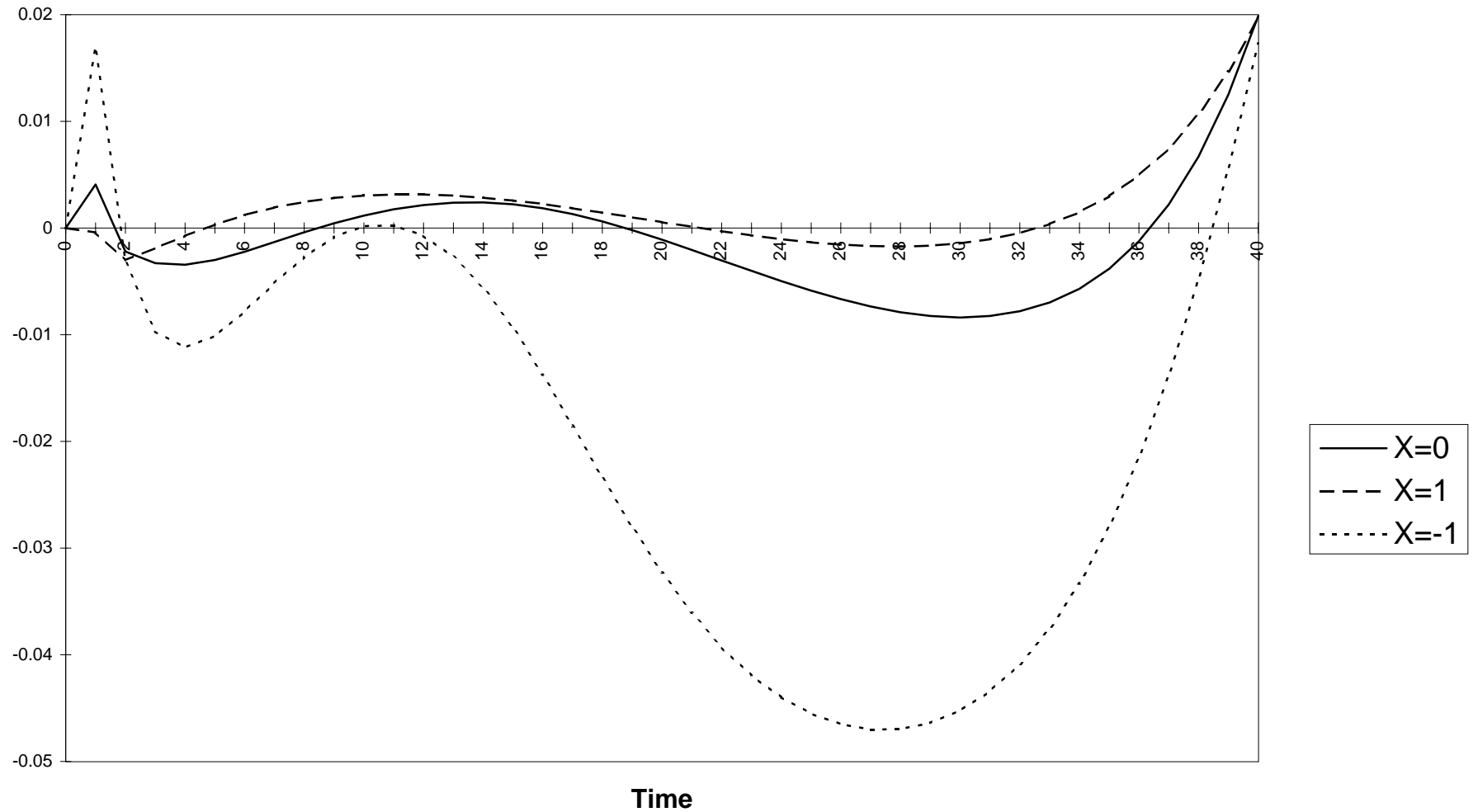


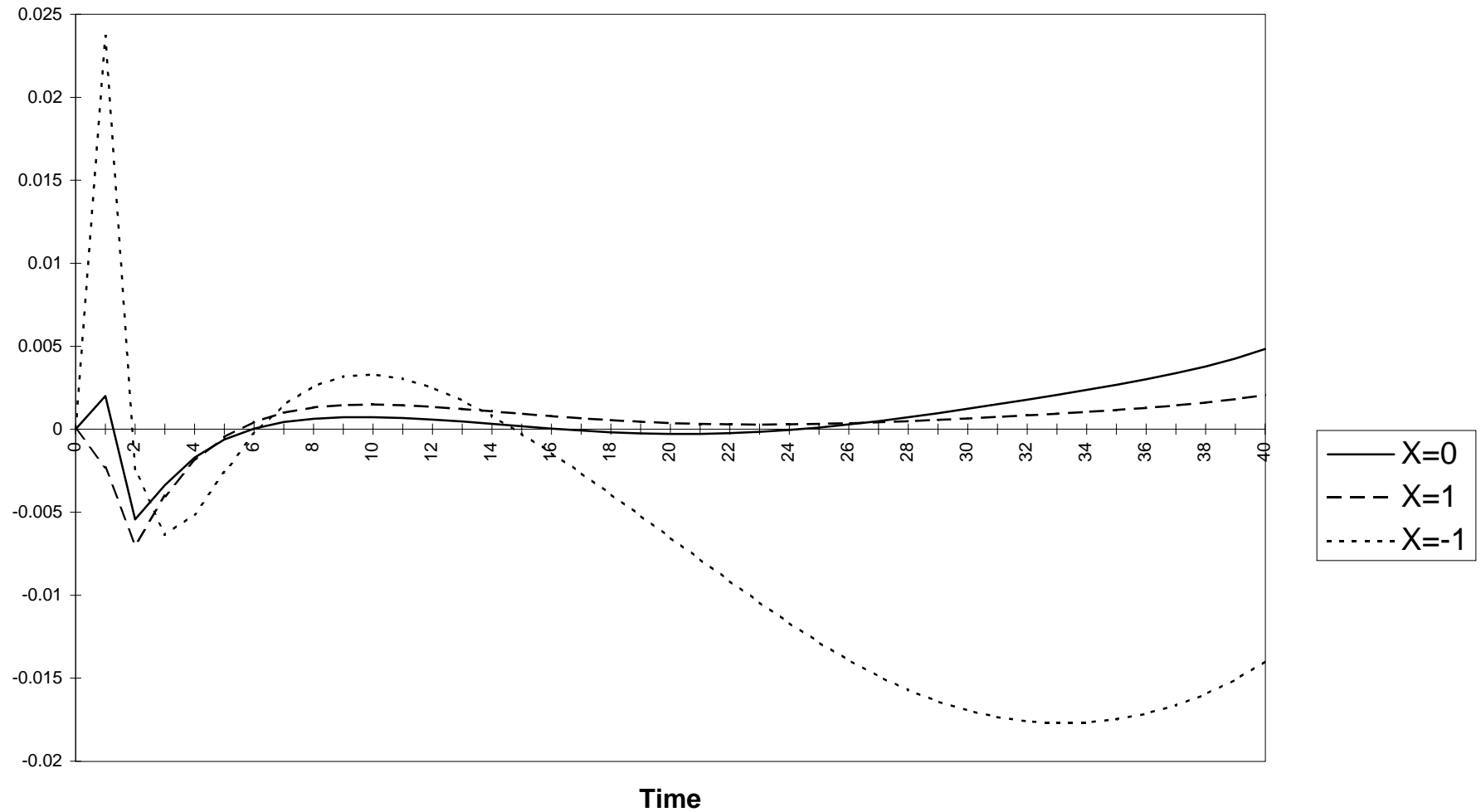
Figure 19: The CDF of Unobserved Heterogeneity (HQIC). DGP: Two Point Heterogeneity, Negative Duration Dependence. QL: Step Function.

Figure 20: MLE Predicted Hazards when DGP: No Heterogeneity, No Duration Dependence, QL: Cubic



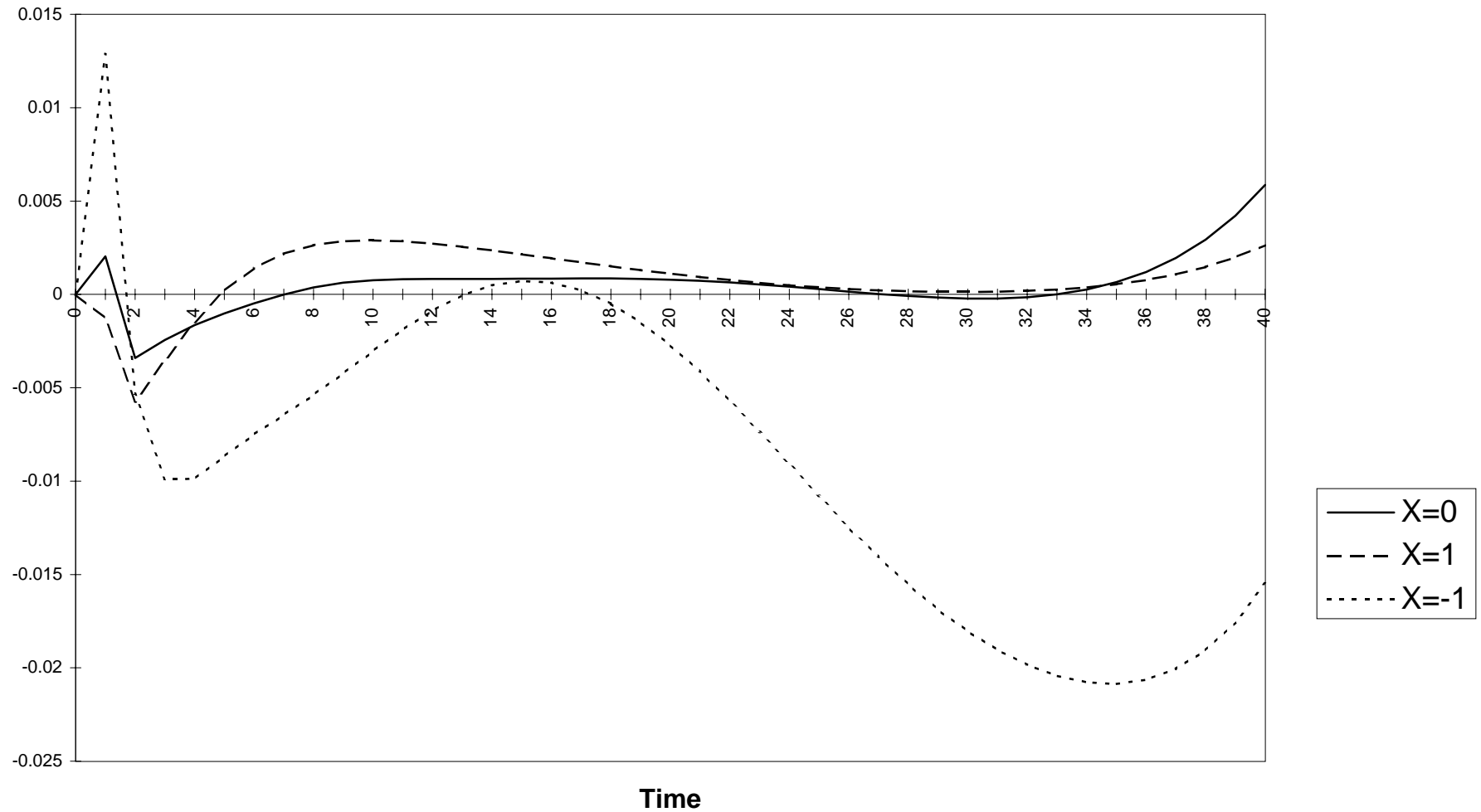
Note: Predicted hazard is graphed in deviations (by period) from the true hazard.

Figure 21: MLE Predicted Hazards when DGP:Two Point Heterogeneity, No Duration Dependence, QL: Cubic



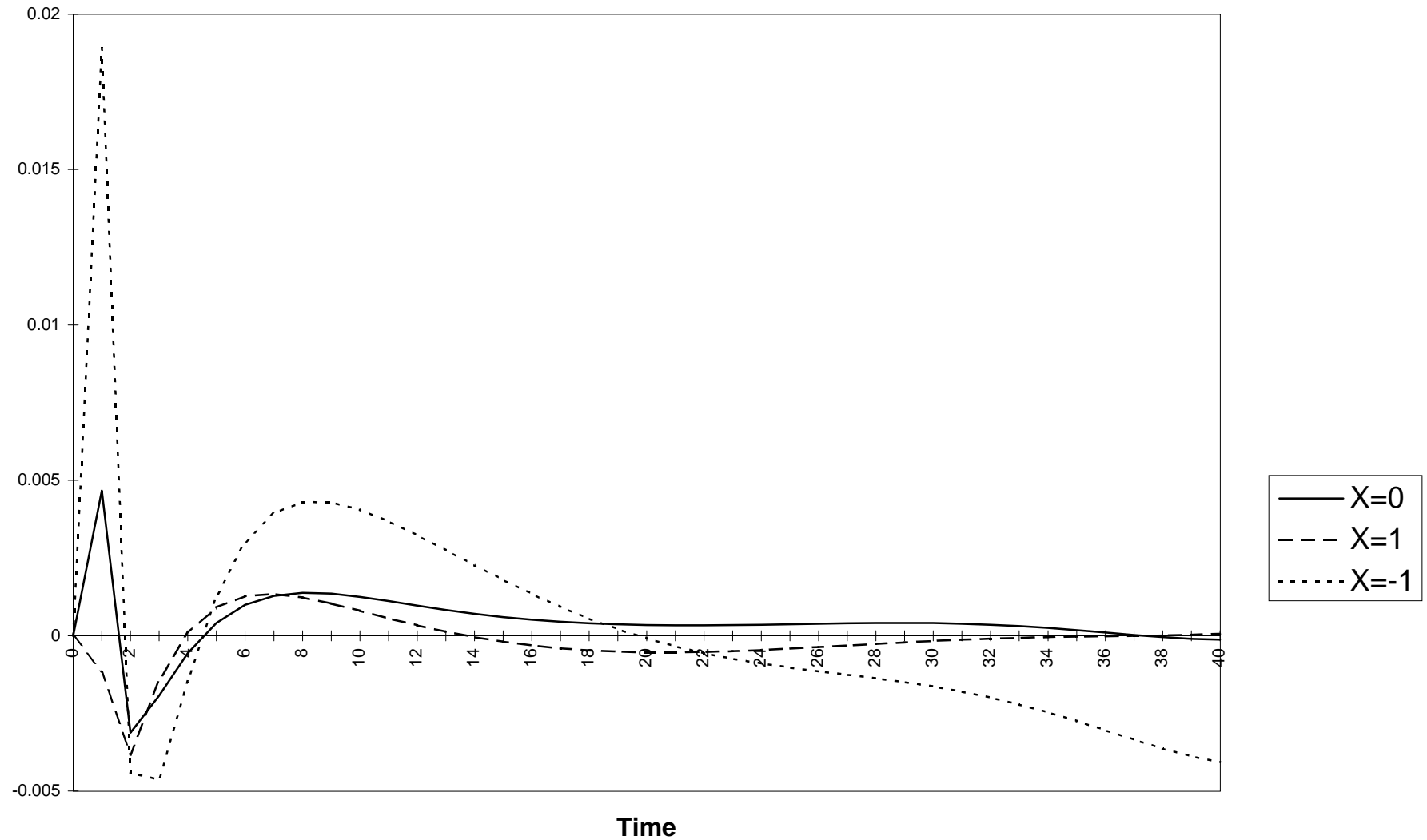
Note: Predicted hazard is graphed in deviations (by period) from true hazard.

Figure 22: MLE Predicted Hazards when DGP: No Heterogeneity, Negative Duration Dependence, QL: Cubic



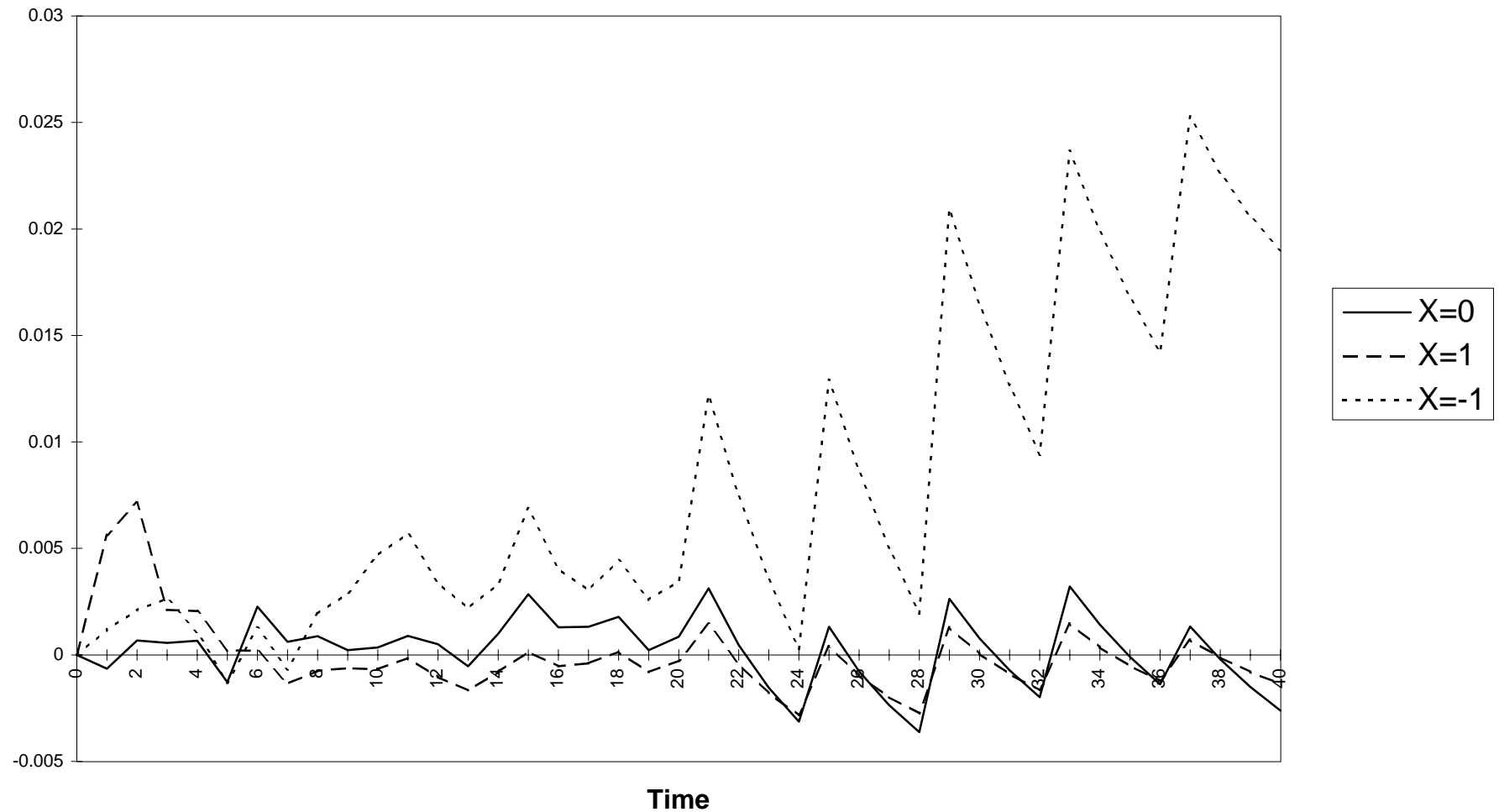
Note: Predicted hazard is graphed in deviations (by period) from the true hazard.

**Figure 23: MLE Predicted Hazards when DGP: Two Point Heterogeneity,
Negative Duration Dependence, QL: Cubic**



Note: Predicted hazard is graphed in deviations (by period) from true hazard.

**Figure 24: MLE Predicted Hazards when DGP: Two Point Heterogeneity,
Negative Duration Dependence, QL: Step Function**



Note: Predicted hazard is graphed in deviations (by period) from the true hazard.