Online Appendix to

Reassessing The Dependence Between Economic Growth and Financial Conditions Since 1973[*]

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[*]The views expressed in this appendix are those of the authors. No responsibility for them should be attributed to the Bank of Canada.

A.1 Data and Summary Statistics

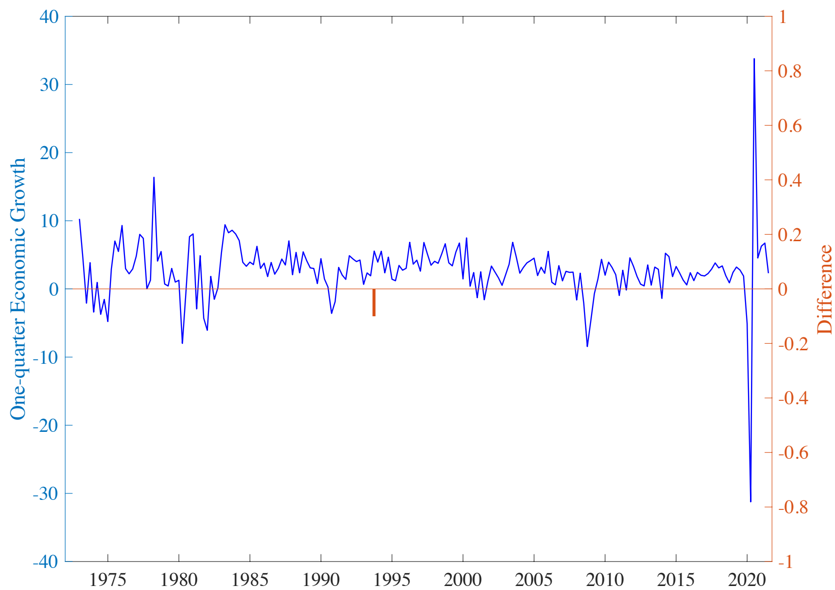
The data for our extended sample, 1973:1 to 2021:3, were downloaded from FRED on January 14th, 2022. Following ABG, we construct a quarterly series for the National Financial Conditions Index (series ID: NFCI) as the average of weekly observations. We construct quarterly measures of economic growth using series GDPC1.

ABG (2019) use FRED series A191RL1Q225SBEA to measure economic growth. This series is accurate to one decimal place. For ranked data, working with higher precision is helpful. We calculate one-quarter economic growth as:

$$\Delta y_t = 100 \times \left[\left(\frac{GDPC1_t}{GDPC1_{t-1}} \right)^4 - 1 \right]$$

Figure A1 plots our economic growth series from 1973:1 to 2021:3. It also plots the difference between our series rounded to one decimal place and FRED series A191RL1Q225SBEA. There is only one observation with a non-zero difference.

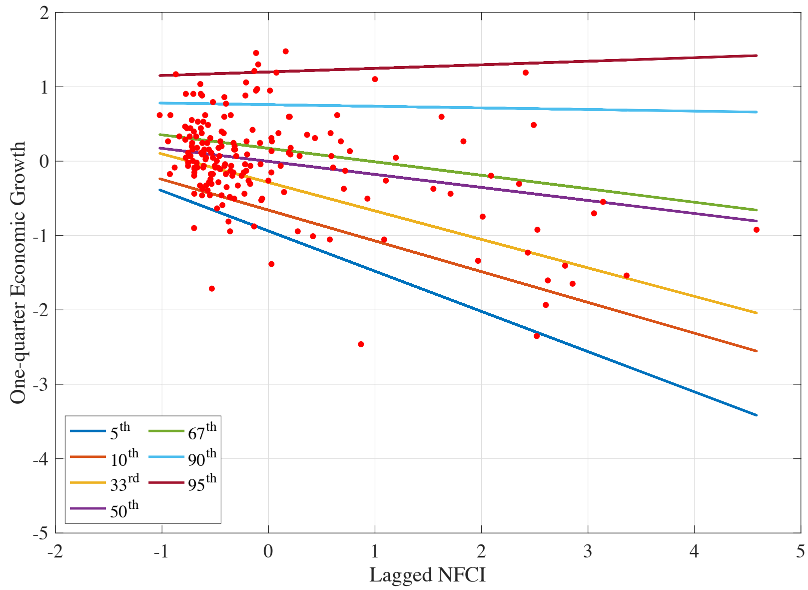
Figure A1: Economic Growth Measures, Extended Sample



Note: The blue line represents one-quarter economic growth calculated using the FRED GDPC1. The red bars represent the differences (rounded to one decimal place) between this measure and FRED A191RL1Q225SBEA.

Figure A2 displays conventional QR lines for our extended sample using ABG's measure of economic growth. These are similar to those displayed in Figure 3 of the paper, which are based on our preferred economic growth measure.

Figure A2: Conventional QR Lines, Extended Sample
Alternative Series for One-quarter Economic Growth



Note: QR lines overlaid with a scatterplot of the (standardised) data (red dots). QR estimates obtained using FRED series A191RL1Q225SBEA to represent one-quarter economic growth.

The first four rows of Table A1 display the first four sample moments for economic growth (based on FRED series GDPC1 as described above) and the NFCI. The first two columns use ABG's sample and the last two columns use our extended sample.

Table A1: Summary Statistics for Economic Growth and NFCI

	1973:1 - 2015:4		1973:1 - 2021:3	
	Growth	NFCI	Growth	NFCI
Mean	2.770	0.039	2.730	−0.026
Standard Deviation	3.265	1.031	4.567	0.985
Skewness	−0.184	1.843	−0.569	2.035
Kurtosis	5.425	5.962	28.232	6.823
Shapiro-Wilk	0.945	0.762	0.710	0.731
p-value	<0.001	<0.001	<0.001	<0.001

Although the sample means for economic growth vary little with the end date, the other sample moments display considerable variation. For example, the standard deviation of economic growth is around 40% higher for the extended sample, approximately 4.6, than for ABG's sample, where it is around 3.3. Furthermore, there is greater left skew for the extended sample, approximately -0.6, than for ABG's sample, roughly -0.2. And, the kurtosis is much stronger for the extended sample, approximately 28.3, compared with roughly 5.4 for ABG's sample. Excess kurtosis is usually defined as a statistic greater than that of a univariate Gaussian distribution, namely 3. The greater kurtosis reflects the incidence of outliers in the extended sample.

In contrast, turning to financial conditions, there is relatively little variation with the sample end date. For both ABG's sample and the extended sample, there is positive skewness and excess kurtosis, where the skew is notably higher in absolute size than for economic growth and has the opposite sign. As a rough guide, the Shapiro-Wilk tests displayed in the last two rows indicate rejection of the null hypothesis of Gaussianity, for both variables, regardless of end date. The p-values in all cases are less than 0.001.

Our quantile regression analysis is performed using standardized data to facilitate comparison with the copula quantile regression analysis.

A.2 Inference, Robustness and Monte Carlo Simulations

A.2.1 Heteroskedasticity Tests

Table A2 displays results using the Machado and Santos Silva (MSS, 2000) test for heteroskedasticity, with the null hypothesis of homoskedastic errors against the alternative that the error variance depends on lagged NFCI. The first two columns refer to ABG's sample, reporting the test statistic and p-value, respectively. The last two columns refer to the extended sample.

Table A2: Machado and Santos Silva Heteroskedasticity Test

	1973:2 - 2015:4		1973:2 - 2021:3	
τ	Statistic	p-value	Statistic	p-value
0.05	12.50	<0.001	0.31	0.579
0.10	9.07	0.003	0.77	0.379
0.33	18.25	<0.001	4.56	0.033
0.50	14.70	<0.001	5.67	0.017
0.67	10.67	0.001	4.17	0.041
0.90	3.85	0.050	1.70	0.193
0.95	1.89	0.169	1.81	0.178

Using ABG's sample, the null hypothesis is rejected at the 5% level for $\tau = 0.33$, 0.50 and 0.67. Using our extended sample, the null is again rejected at the 5% level for these three values of τ. Overall, the evidence against the null is weaker in the tails for the extended sample, which includes the pandemic observations.

Table A3 displays results for the same tests as Table A2, but based on pseudo data. The results are similar, with heteroskedasticity apparent for $\tau = 0.33$, 0.50 and 0.67, regardless of the sample end date.

Table A3: Machado and Santos Silva Heteroskedasticity Test, Pseudo Data

	1973:2 - 2015:4		1973:2 - 2021:3	
τ	Statistic	p-value	Statistic	p-value
0.05	3.48	0.062	0.93	0.335
0.10	3.39	0.065	1.62	0.203
0.33	9.71	0.002	7.99	0.005
0.50	7.84	0.005	8.69	0.003
0.67	10.42	0.001	11.37	0.001
0.90	10.09	0.002	8.73	0.003
0.95	6.15	0.013	6.08	0.014

A.2.2 Confidence Intervals for QR Coefficients

To construct confidence intervals for our QR coefficients, we use a moving block bootstrap. Hahn (1995) shows that the percentile bootstrap gives correct asymptotic coverage probabilities. Tarr (2012) studies the small sample properties and notes the robustness to heteroskedasticy, skewed covariates and heavy tailed error distribution. We draw bootstrap samples from the data in $[y, x]$ pairs where $y = \Delta y_{t+1}$ and $x = [\Delta y_t,\ NFCI_t]$.

Given that we are dealing with dependent data, we use a block bootstrap rather than resampling independent observations. See MacKinnon (2006) for a discussion of the choice of block length. With a block length of b and a sample size of T we have $T - b + 1$ overlapping blocks of $[y, x]$ pairs. We choose b to be approximately $T^{1/3}$. ABG's sample contains 171 observations and our extended sample contains 194 observations, so we set a block length of $b = 6$. We then draw 10,000 bootstrap samples consisting of T/b blocks drawn with replacement. With T/b not delivering an integer, the last block in each of our bootstrap samples contains less than six $[y, x]$ pairs. For each of the 10,000 bootstrap samples, we fit the conventional and copula QR specifications described in the paper. First, for each bootstrap sample, we estimate conventional QR. We then take ranks and use the inverse Gaussian Cumulative Distribution Function (CDF) to generate the standard Normal pseudo data and estimate the copula QR. The percentiles reported in Tables A4 and A5 are from the distributions based on 10,000 bootstrap samples.

Table A4 reports the distribution of the fitted conventional QR coefficients from the block bootstrap, for ABG's sample in the top panel, and for the extended sample in the lower panel. With ABG's sample, we find that confidence intervals widen as we move as we move away from the median—indicating lower precision in the tails. For example, for $\tau = 0.50$, the spread between the 5th and 95th percentile of the bootstrapped distribution is 0.34. For $\tau = 0.05$ and 0.95, the equivalent spreads are 0.64 and 0.57, respectively.

Table A4: Conventional QR Coefficients for Lagged NFCI
Percentiles of Bootstrapped Distribution

Quantile	5th Percentile	33rd Percentile	Sample Estimate	67th Percentile	95th Percentile
(a) ABG's sample					
$\tau = 0.05$	-0.984	-0.740	-0.762	-0.555	-0.349
$\tau = 0.10$	-0.897	-0.652	-0.600	-0.538	-0.388
$\tau = 0.33$	-0.667	-0.539	-0.455	-0.388	-0.221
$\tau = 0.50$	-0.488	-0.345	-0.270	-0.247	-0.145
$\tau = 0.67$	-0.381	-0.289	-0.251	-0.211	0.036
$\tau = 0.90$	-0.297	-0.042	0.048	0.050	0.144
$\tau = 0.95$	-0.256	-0.047	-0.015	0.035	0.317
(b) Extended sample					
$\tau = 0.05$	-0.918	-0.574	-0.540	-0.364	-0.134
$\tau = 0.10$	-0.787	-0.532	-0.415	-0.368	-0.217
$\tau = 0.33$	-0.619	-0.394	-0.384	-0.267	-0.120
$\tau = 0.50$	-0.393	-0.238	-0.184	-0.163	-0.095
$\tau = 0.67$	-0.306	-0.209	-0.183	-0.147	-0.017
$\tau = 0.90$	-0.289	-0.058	-0.027	0.054	0.176
$\tau = 0.95$	-0.413	-0.037	0.045	0.077	0.359

In the upper panel, the sample estimate of the coefficient on lagged NFCI is decreasing (in absolute value) in τ—reflecting the relative steepness of the QR lines for low τ displayed in Figure 2 of the paper.

Turning to the lower panel of Table A4, both the widening of confidence intervals as we move away from the central mass and the greater (in absolute value) coefficients for lower τ are also features for the extended sample. Finally, Table A4 shows that there is attenuation in the sample estimates of the QR coefficients as we move to the extended sample, for $\tau = 0.33, 0.50$ and 0.67, as displayed in Figure 2 of the paper.

Table A5 displays the distribution of the fitted Copula QR coefficients from the block bootstrap, for ABG's sample in the upper panel, and for the extended sample in the lower panel. In this case, for the (standard normal) pseudo data. As in Table A4, the fitted coefficient on lagged NFCI is greater (in absolute value) for lower τ, regardless of sample end date. However, there is relatively little attenuation in the coefficients as we move from ABG's original sample to our extended sample.

Table A5: Copula QR Coefficients for Lagged NFCI
Percentiles of Bootstrapped Distribution

Quantile	5th Percentile	33rd Percentile	Sample Estimate	67th Percentile	95th Percentile
(a) ABG's sample					
$\tau = 0.05$	-0.572	-0.462	-0.422	-0.398	-0.290
$\tau = 0.10$	-0.553	-0.470	-0.465	-0.410	-0.300
$\tau = 0.33$	-0.536	-0.401	-0.328	-0.314	-0.196
$\tau = 0.50$	-0.492	-0.342	-0.293	-0.245	-0.137
$\tau = 0.67$	-0.411	-0.290	-0.252	-0.179	0.056
$\tau = 0.90$	-0.205	-0.070	-0.063	0.014	0.146
$\tau = 0.95$	-0.133	0.011	-0.015	0.084	0.222
(b) Extended sample					
$\tau = 0.05$	-0.515	-0.417	-0.420	-0.330	-0.122
$\tau = 0.10$	-0.518	-0.429	-0.400	-0.357	-0.190
$\tau = 0.33$	-0.518	-0.389	-0.334	-0.301	-0.172
$\tau = 0.50$	-0.481	-0.320	-0.254	-0.220	-0.108
$\tau = 0.67$	-0.369	-0.248	-0.205	-0.152	0.055
$\tau = 0.90$	-0.167	-0.042	0.052	0.057	0.180
$\tau = 0.95$	-0.096	0.026	0.056	0.137	0.278

A.2.3 Robustness with a Covid-19 Pandemic Dummy Variable

Tables A6 and A7 display bootstrap distributions for the coefficients for lagged NFCI from the conventional and copula QRs with a dummy variable added for the Covid-19 pandemic. The upper panels in these two tables are equivalent to the lower panels of Tables A4 and A5, except that the explanatory variable vector is now $x = [\Delta y_t,\ NFCI_t,\ C19_{t+1}]$. That is, the vector includes a constant, lagged economic growth, lagged NFCI and a dummy variable for the pandemic. This dummy variable, $C19_{t+1}$, takes on the value zero for all observations up to and including 2019:4, and the value one for the observations from 2020:1 to 2021:3, inclusive.

Tables A6 and A7 are based on a modified version of our moving block bootstrap procedure. Recall that our extended sample contains 194 observations, the last seven of which cover the pandemic. We now draw 10,000 bootstrap samples consisting of $187/b$ blocks drawn with replacement from the first 187 observations of our extended sample, that is observations up to and including 2019:4. As before we set $b = 6$. Then, for each of these 10,000 bootstrap samples, we add the seven observations from 2020:1 to 2021:3 and fit the QR specifications described in the paper. This modified procedure ensures that each of our bootstrapped samples contains the seven observations for which the pandemic dummy variable is equal to one.

Table A6: Conventional QR, Extended Sample
Percentiles of Bootstrapped Distribution

Quantile	5th Percentile	33rd Percentile	Sample Estimate	67th Percentile	95th Percentile
(a) NFCI					
$\tau = 0.05$	-0.684	-0.510	-0.519	-0.388	-0.246
$\tau = 0.10$	-0.603	-0.444	-0.405	-0.370	-0.254
$\tau = 0.33$	-0.457	-0.361	-0.308	-0.254	-0.140
$\tau = 0.50$	-0.331	-0.226	-0.183	-0.171	-0.103
$\tau = 0.67$	-0.283	-0.211	-0.183	-0.156	-0.052
$\tau = 0.90$	-0.290	-0.047	0.043	0.037	0.148
$\tau = 0.95$	-0.241	-0.029	0.016	0.050	0.210
(b) Covid-19 Dummy					
$\tau = 0.05$	-7.039	-6.369	-6.083	-5.937	-5.346
$\tau = 0.10$	-7.191	-6.754	-6.664	-6.417	-5.854
$\tau = 0.33$	-1.454	-0.931	-1.035	-0.260	-0.061
$\tau = 0.50$	-0.443	-0.330	-0.290	-0.240	0.431
$\tau = 0.67$	0.358	0.453	0.477	0.518	0.623
$\tau = 0.90$	4.831	6.795	8.263	8.079	9.129
$\tau = 0.95$	4.625	6.287	7.432	7.675	8.866

Comparing panel (a) in Table A6 with panel (b) in Table A4, we see that the addition of the dummy variable has little impact on the fitted QR coefficients for the NFCI. We continue to see attenuation relative to ABG's sample.

As expected, the fitted coefficients for the dummy variable are large (in absolute size) and negative in the lower tail, for $\tau = 0.05$ and 0.10. And, large and positive in the upper tail, $\tau = 0.90$ and 0.95. In both tails, the confidence intervals exclude zero.

Table A7 reports Copula QR coefficients, including the dummy variable. Comparing panel (a) of this table with panel (b) in Table A5, we again see that the addition of the dummy variable has little impact on the fitted QR coefficients on the NFCI. Recall that the copula QR coefficients displayed little attenuation when considering the extended sample in the absence of the dummy. Table A7 shows that with consideration of the dummy variable, this feature is still apparent. For example, looking at the upper panel of Table A7, for $\tau = 0.33$, 0.50 and 0.67, with the dummy variable the sample estimates are -0.325, -0.254 and -0.203, respectively. Whereas, looking at the lower panel of Table A5, for the same value of τ, without the dummy, the estimates are -0.334, -0.254 and -0.205 respectively. As for Table A6, the confidence intervals for the dummy exclude zero in both tails.

Table A7: Copula QR Coefficients, Extended Sample
Percentiles of Bootstrapped Distribution

Quantile	5th Percentile	33rd Percentile	Sample Estimate	67th Percentile	95th Percentile
(a) NFCI					
$\tau = 0.05$	-0.516	-0.416	-0.386	-0.360	-0.265
$\tau = 0.10$	-0.509	-0.430	-0.404	-0.370	-0.259
$\tau = 0.33$	-0.506	-0.384	-0.325	-0.302	-0.175
$\tau = 0.50$	-0.477	-0.322	-0.254	-0.218	-0.111
$\tau = 0.67$	-0.367	-0.245	-0.203	-0.154	0.055
$\tau = 0.90$	-0.172	-0.047	-0.039	0.051	0.170
$\tau = 0.95$	-0.111	0.013	0.010	0.099	0.219
(b) Covid-19 Dummy					
$\tau = 0.05$	-1.254	-1.074	-1.006	-0.931	-0.676
$\tau = 0.10$	-1.449	-1.276	-1.184	-1.120	-0.880
$\tau = 0.33$	-0.782	-0.566	-0.518	-0.408	-0.189
$\tau = 0.50$	-0.641	-0.282	-0.170	-0.240	0.845
$\tau = 0.67$	0.090	0.382	0.481	0.527	0.732
$\tau = 0.90$	1.668	2.051	2.274	2.251	2.572
$\tau = 0.95$	1.020	1.680	2.006	2.036	2.429

A.2.4 Monte Carlo Simulations

In this simulation exercise, we begin by assuming a linear relationship between output growth and lagged NFCI and lagged output growth, matching the features of U.S. data considered by ABG. Then, we experiment with introducing outliers matched to the post-2015 data. We begin by briefly summarising the baseline experiment and then discuss our variation involving outliers.

A.2.4.1 Baseline Experiment

The dependence is:

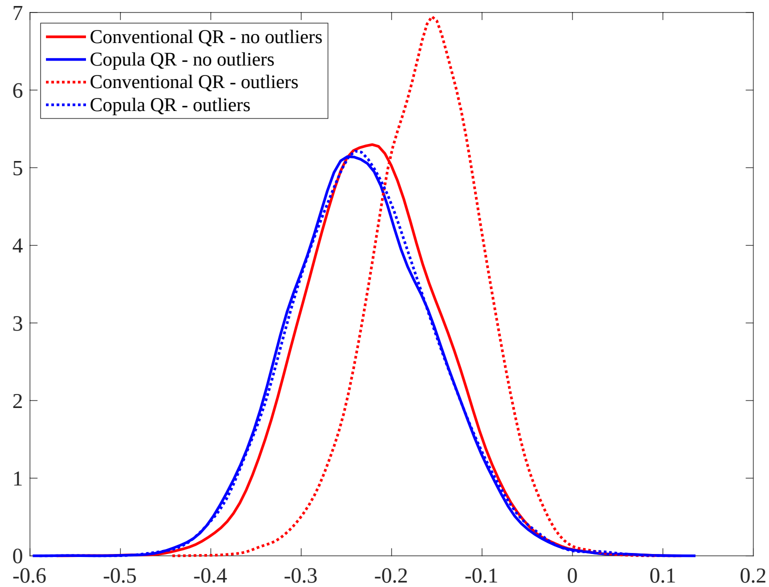
$$Y_{t+1} = a_X X_t + a_Y Y_t + \epsilon_{t+1}$$

where Y_{t+1} is the target variable in time t, $t = 1, \ldots, 200$. We assume X is iid standard normal, and a_X and a_Y denote the parameters for the regressors. We set the parameters such that $a_X = -0.25$ and $a_Y = 0.1$. Based on the discussion of heteroskedasticity by ABG, and the MSS test for heteroskedasticity on the sample data, we assume that ϵ_{t+1} is mean zero, with standard deviation $0.8795 + 0.1715X_t$.

To simulate the asymmetric distribution of output growth, we use MATLAB's pearsrnd, with parameters (0, 1, -0.75, 5), denoting mean, standard deviation, skewness and kurtosis, respectively. We then fit conventional QR and copula QR specifications to each of the 10,000 simulated samples of length 200.

Figure A3 plots the (kernel-smoothed) density for the estimated coefficients for $\tau = 0.5$, for each fitted specification. These are the depicted by the solid red line for conventional QR and the solid blue line for Copula QR. Both densities are centred close to the true value for the parameter, -0.25, with right skew. The means are -0.22 for conventional QR and -0.23 for copula QR.

Figure A3: Simulated Density of Fitted Coefficients for $\tau = 0.5$



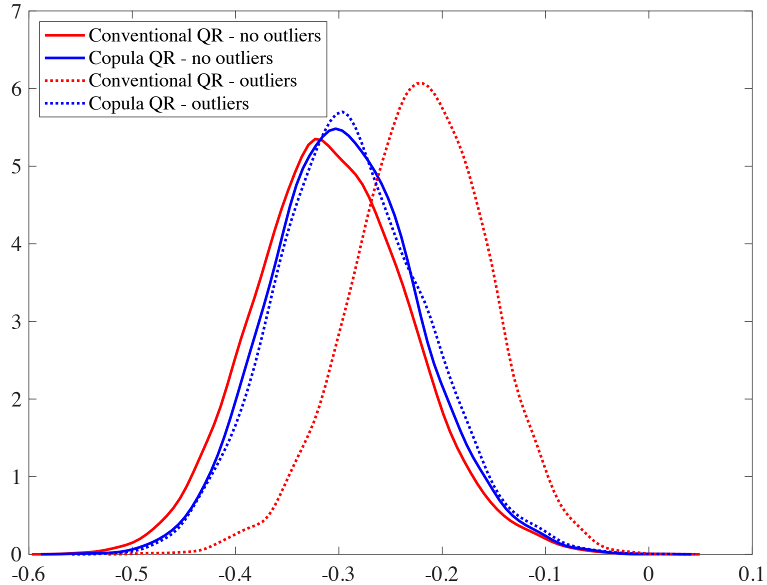
A.2.4.2 Experiment with Post-2015 Outliers

In this experiment, we match the dependence structure and the functional form of the heteroskedasticity to that in the baseline experiment. The first 85% of observations are (marginally) distributed as in the baseline experiment (using pearsrnd); whereas, the last 15% of observations, 30 observations, are (marginally) distributed to match the output growth data from 2015:4 to 2021:3, using the (inverse) ECDF. We again fit conventional QR and copula QR specifications for 10,000 simulations.

To Figure A3, we now add a plot of the (kernel-smoothed) density for the coefficients for $\tau = 0.5$, for each specification. These are the dotted red and blue lines for the conventional and copula QR, respectively. In this case, the copula QR density is centred close to the true value for the parameter, -0.23. In contrast, the conventional QR density is centred on approximately -0.16—indicating considerable attenuation (around 30%) from the true value.

We also examine the (lower tail) coefficients for both specifications for $\tau = 0.33$, which display similar attenuation characteristics. In Figure A4, we plot the corresponding densities for the $\tau = 0.33$ case, revealing considerable attenuation with conventional QR when the data include the outliers. The probability that the fitted coefficient using conventional QR exceeds the true value for $\tau = 0.5$, -0.25, is around 67%. With copula QR, the corresponding probability is around 29%.

Figure A4: Simulated Density of Fitted Coefficients for $\tau = 0.33$



Repeating the analysis with sample sizes 150, 300, 400 and 500, the simulations reveal similar attenuation of the conventional QR coefficients with outliers calibrated to the post-2015 data. In each case, copula QR comfortably outperforms conventional QR, with the distributions of the fitted parameters for $\tau = 0.5$ located close to the true value. These results are available on request.

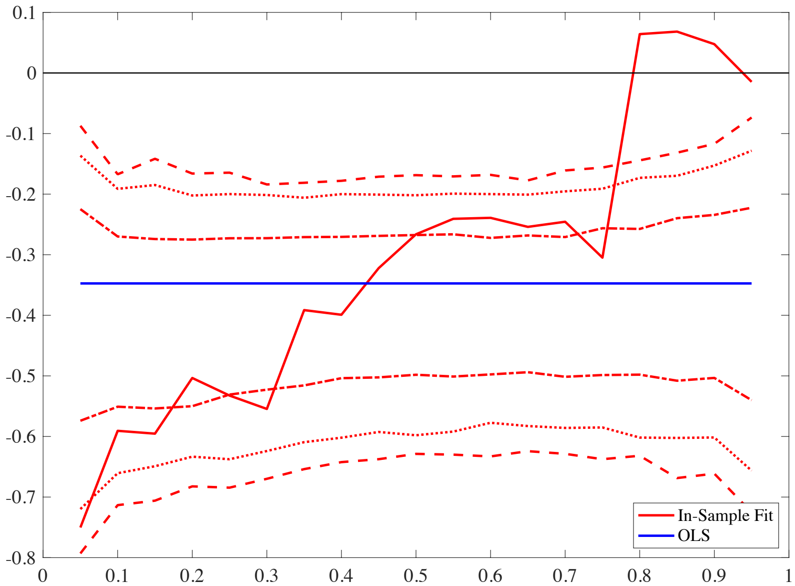
A.3 Additional Figures

A.3.1 Conventional QR and ABG's Sample

Following ABG's analysis, Figure A5 displays the fitted QR coefficients for NFCI for values of $\tau = 0.05, 0.10, \ldots 0.95$ using their sample, together with (in blue) the OLS estimate of that coefficient from a regression of output growth on lagged output growth and lagged NFCI. Following ABG, this figure displays 95%, 90% and 68% confidence intervals from a VAR(4) in output growth and the NFCI. As such, they do not reflect the precision of the fitted QR coefficients. Instead they provide a guide to whether the fitted QR coefficients resemble those from a VAR.

In Figure A5, there is variation in the QR coefficients for NFCI across τ, with the most notable inconsistencies with the VAR(4) apparent in the tails.

Figure A5: Conventional QR Coefficients for NFCI, ABG's Sample



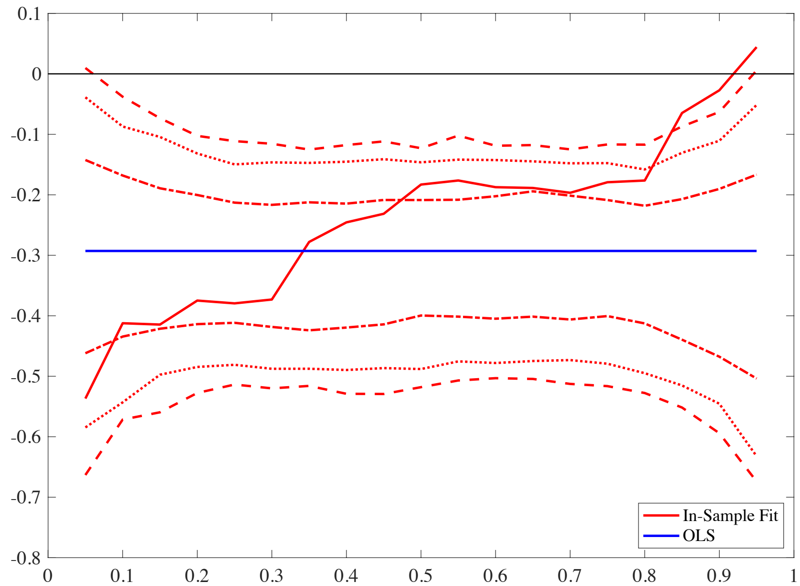
Note: QR coefficients and 95%, 90% and 68% confidence bands based on the null hypothesis that the data generating process is a linear VAR(4).

A.3.2 Conventional QR and Extended Sample

Figures A6 to A12 display additional results for the replication using our extended sample and conventional QR methods. That is, we replicate analysis from ABG's original study with the sample extended to 2021:3. We begin by using our extended sample to replicate Figure A5 in order to facilitate a comparison of the QR coefficients with variation in the sample end date. We then replicate other key figures from ABG's study with our extended sample.

Figure A6 displays QR coefficients for NFCI for $\tau = 0.05, 0.10, \ldots 0.95$ for our extended sample, and corresponds to Figure A5, which uses ABG's sample. As we report in the paper, we find attenuation in the coefficients as we move from ABG's sample to our extended sample. There is also some attenuation in the OLS coefficient, which is about -0.35 and -0.30, for ABG's sample and the extended sample, respectively. Based on the extended sample, there is still variation in the QR coefficients for NFCI across τ, with the most notable inconsistencies with the VAR(4) apparent in the tails, but the difference in Figure A6 is less marked than it is in Figure A5, which used ABG's sample.

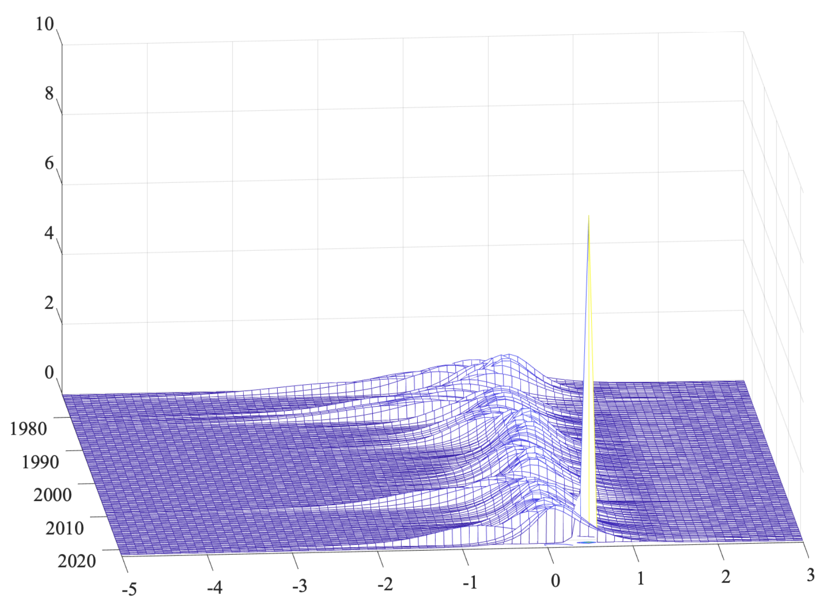
Figure A6: Conventional QR Coefficients for NFCI, Extended Sample



Note: QR coefficients and 95%, 90% and 68% confidence bands based on the null hypothesis that the data generating process is a linear VAR(4).

Figure A7 plots the distribution of one-quarter ahead economic growth, conditioning on current economic growth and current financial conditions. It corresponds to ABG's Figure 1, albeit at the one-quarter horizon rather than the one-year horizon and using the extended sample. As noted by ABG, based on their sample, there is considerable variation in the lower tail over time, and for most of the sample, relative stability in the upper tail. However, the extended sample includes the pandemic observations and this generates a nearly degenerate conditional distribution for 2020:4.

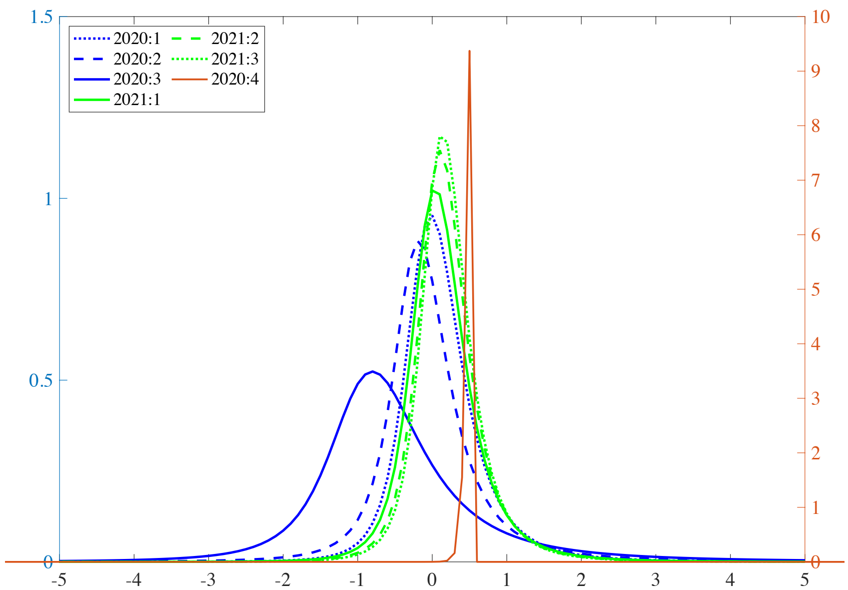
Figure A7: Conditional Distributions for Economic Growth, Extended Sample



Note: One-quarter ahead predictive distribution for economic growth, based on conventional QR with current real GDP growth and NFCI as conditioning variables.

To shed further light on this, Figure A8 plots the conditional distribution for the last seven observations of our extended sample which are associated with the pandemic. The distribution for 2020:4 is plotted using the vertical axis on the right, while the distributions for the other six quarters use the vertical axis to the left. The conditional distribution for 2020:4 has almost zero mass on economic growth more than a percentage point from its mean and displays no upside risk. Recall that here we are following ABG and using a skewed $t-$distribution to transform the empirical quantile distribution into a conditional distribution for output growth. The conditioning information includes lagged economic growth; the economic growth realisation for 2020:3 was in the region of 30%.

Figure A8: Conditional Distributions for Economic Growth During Pandemic



Notes: (1) One-quarter ahead predictive distribution of economic growth, based on QR with current real GDP growth and NFCI as conditioning variables. (2) The distribution for 2020:4 is plotted using the vertical axis on the right, while the distributions for the other six quarters use the vertical axis to the left.

Figure A9 displays downside and upside entropy based on our extended sample, and so represents an update to Panel A of ABG's Figure 9. Figure A10 displays 5% expected shortfall and longrise based on our extended sample, and so is equivalent to Panel C of their Figure 9. In each case these results are consistent with ABG's findings from their sample. That is, we see departures from Gaussianity and greater volatility for downside entropy and expected shortfall than for upside entropy and expected longrise respectively.

Figure A9: One-quarter Ahead Entropy, Extended Sample

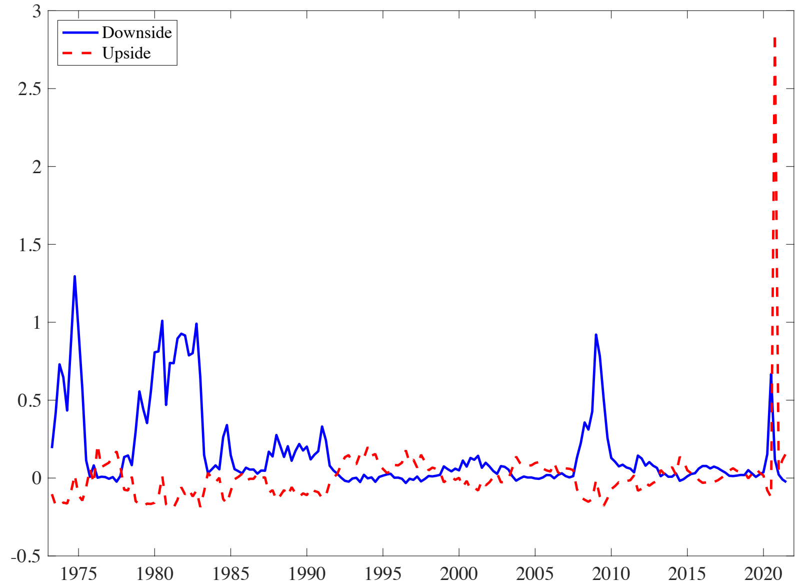
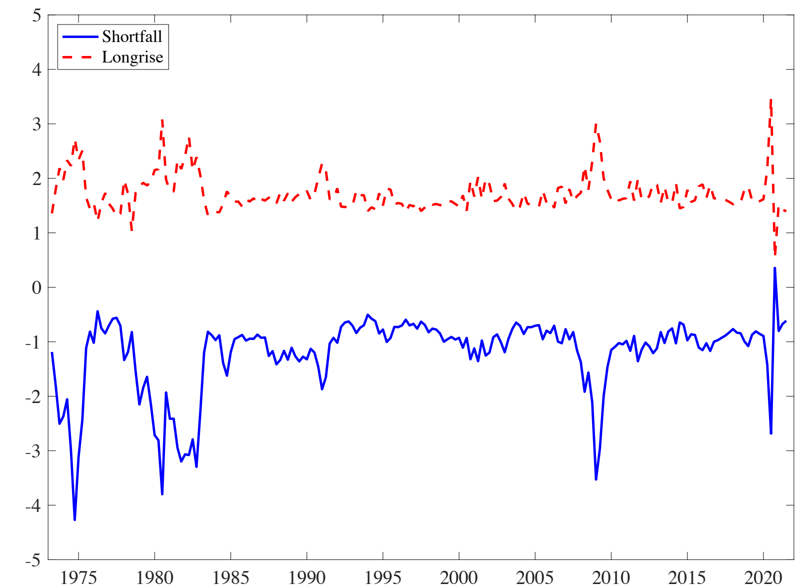


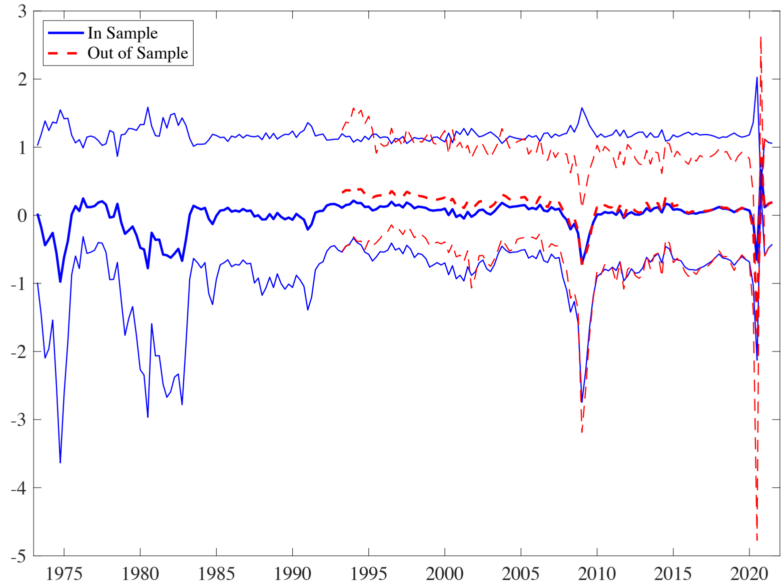
Figure A10: One-quarter Ahead 5% Expected Shortfall and Longrise, Extended Sample



We conclude this subsection with a replication of some of ABG's out-of-sample evidence using our extended sample. Figure A11 displays the 5, 50, and 95 percentiles of the in-sample and out of sample predictive densities for one-quarter ahead GDP growth, representing an update to Panel A of ABG's Figure 10. Finally, Figure A12 displays downside entropy, based on the in-sample and out-of-sample predictive densities for GDP growth at the one-quarter horizon, and so extends Panel C of ABG's Figure 10.

Figure A11 shows that, as in ABG's study, the in-sample and out of sample percentiles of the predictive densities continue to track each other closely. The only exceptions are those during the financial crisis of 2007-09 and the first quarters of the pandemic.

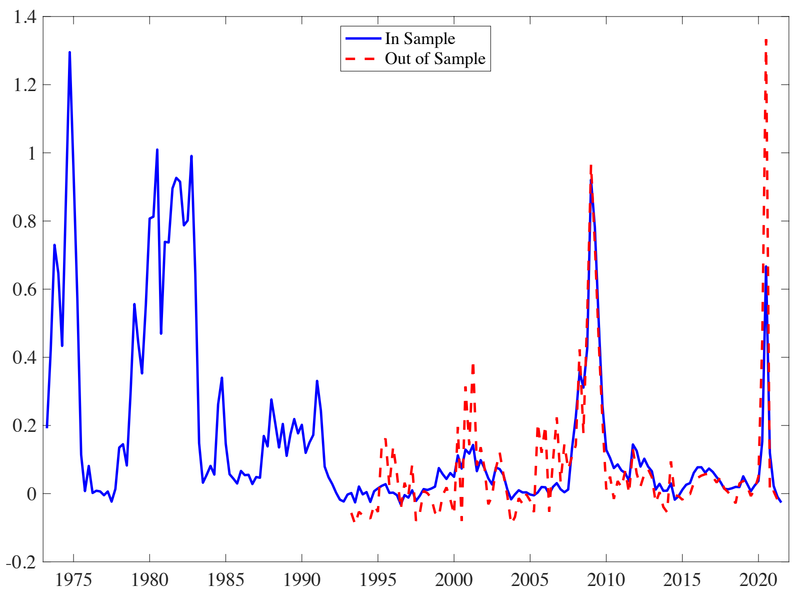
Figure A11: One-quarter Ahead Predictive Densities for Economic Growth, Extended Sample



Note: This figure reports 5, 50, and 95 percentiles of the in-sample and out of sample one-quarter ahead predictive densities for economic growth.

Figure A12 shows that in-sample and out-of-sample downside entropy are similar in the extended sample, as is the case in ABG's original sample.

Figure A12: One-quarter Ahead Downside Entropy, Extended Sample

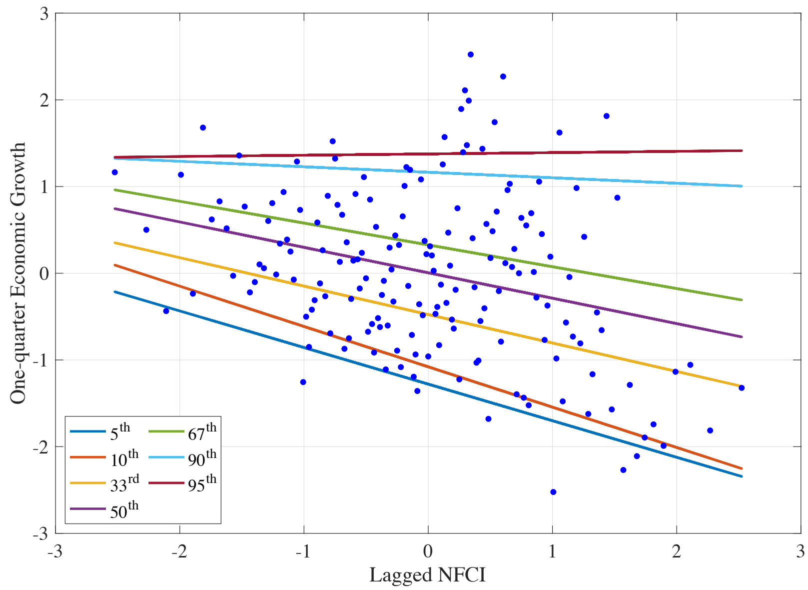


A.3.3 Copula QR, ABG's Sample and Extended Sample

Figures A13 to A17 display additional results from our broader replication using copula methods, both on ABG's original sample and our extended sample.

Figure A13 displays copula QR lines for ABG's sample ending in 2015:4. These lines are very similar to those presented in the paper for the extended sample. This reflects the minor attenuation in the coefficients for the NFCI when using pseudo data as we move to the extended sample.

Figure A13: Copula QR Lines, ABG's Sample

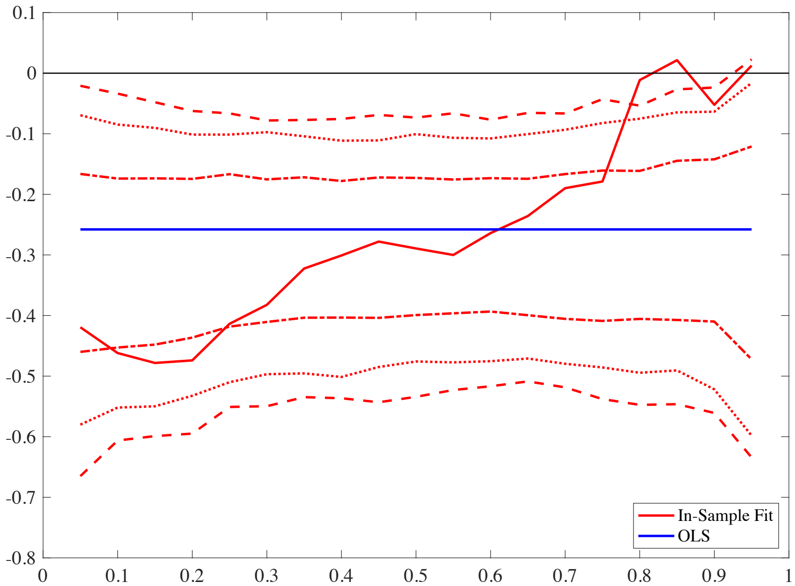


Note: QR lines overlaid with a scatterplot of the pseudo data (blue dots).

Figures A14 and A15 display copula QR coefficients for NFCI for $\tau = 0.05, 0.10, \ldots 0.95$ for ABG's sample and our extended sample. They also report (in blue) the corresponding OLS estimate from a regression of output growth on lagged output growth and lagged NFCI using pseudo data. Figure A14 uses ABG's sample and Figure A15 uses our extended sample. Both figures also report 95%, 90% and 68% bootstrapped confidence bands, based on a pseudo-data VAR(4) in output growth and the NFCI.

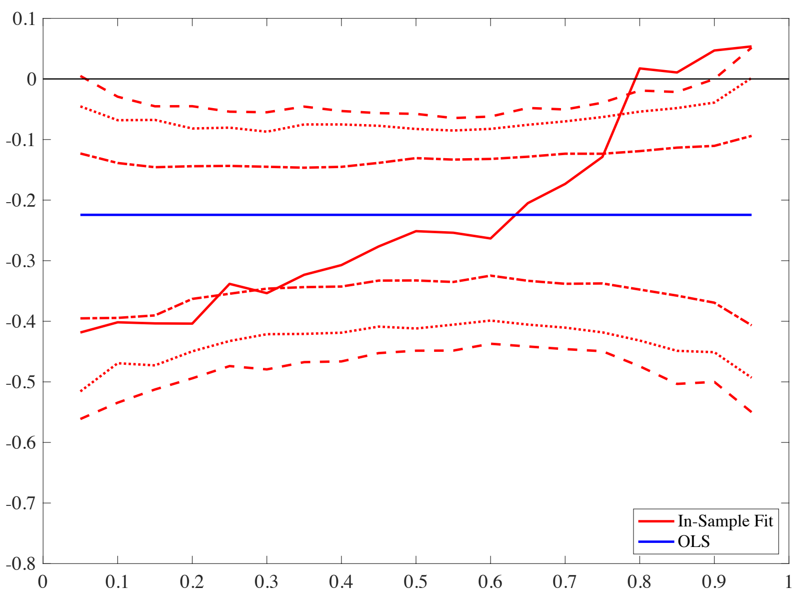
Comparing Figures A14 and A15, there is little variation stemming from the sample end date with copula QR. As with Figure A5, there is variation in the coefficients for NFCI across τ, with the most notable inconsistencies with the VAR(4) apparent in the tails for both ABG's sample and our extended sample.

Figure A14: Copula QR Coefficients for NFCI, ABG's Sample



Note: QR coefficients and 95%, 90% and 68% confidence bands based on the null hypothesis that the data generating process is a linear VAR(4).

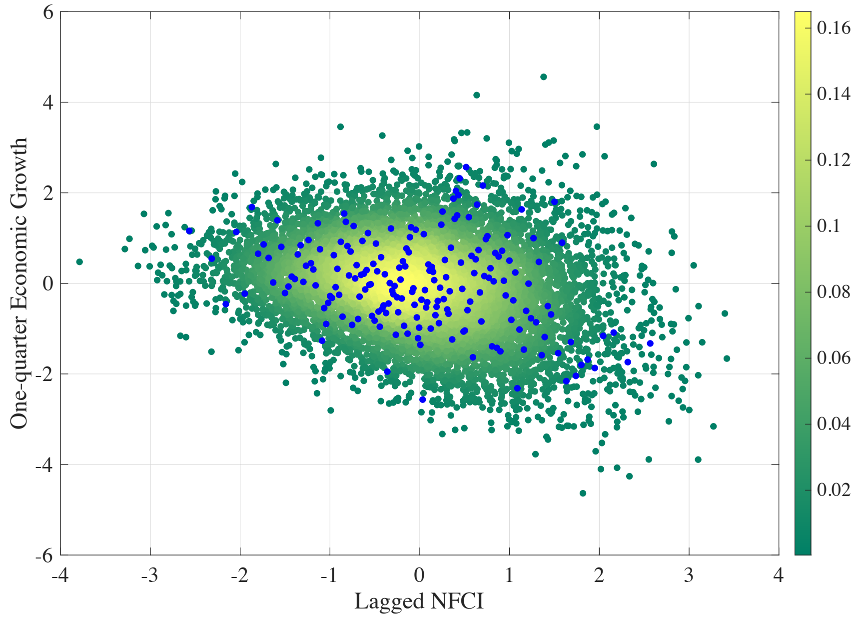
Figure A15: Copula QR Coefficients for NFCI, Extended Sample



Note: QR coefficients and 95%, 90% and 68% confidence bands based on the null hypothesis that the data generating process is a linear VAR(4).

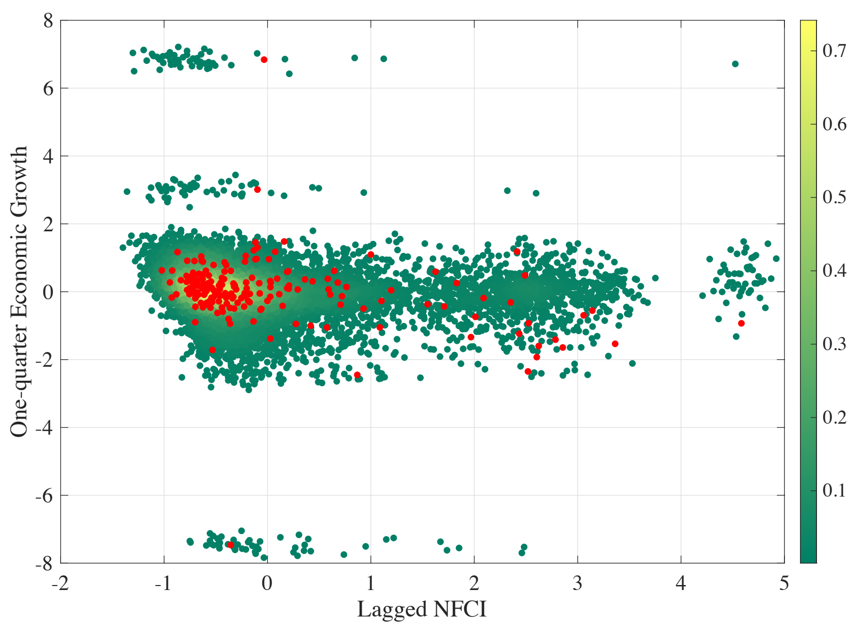
To visualise the implications of the copula model, Figure A16 plots a 10,000 swarm of simulated pseudo data from the copula density, overlaid with the (extended sample) pseudo data (in blue). The simulated data are coloured using a kernel density estimator, Nils (2020). Figure A17 plots the swarm of simulated data in the observed data space, obtained using the inverse ECDFs for each margin. The swarm plot in Figure A17 is overlaid with the (extended sample, standardised) data (in red).

Figure A16: Simulated Swarm for the Pseudo Margins of the Copula Density, Extended Sample



Note: Swarm from the copula density, coloured using a kernel density estimator. The swarm is overlaid with a scatterplot of the pseudo data (in blue).

Figure A17: Simulated Swarm for the Observed Margins of the Copula Density, Extended Sample



Note: Swarm from the copula density, coloured using a kernel density estimator. The swarm is overlaid with a scatterplot of the (standardised) observed data (in red).

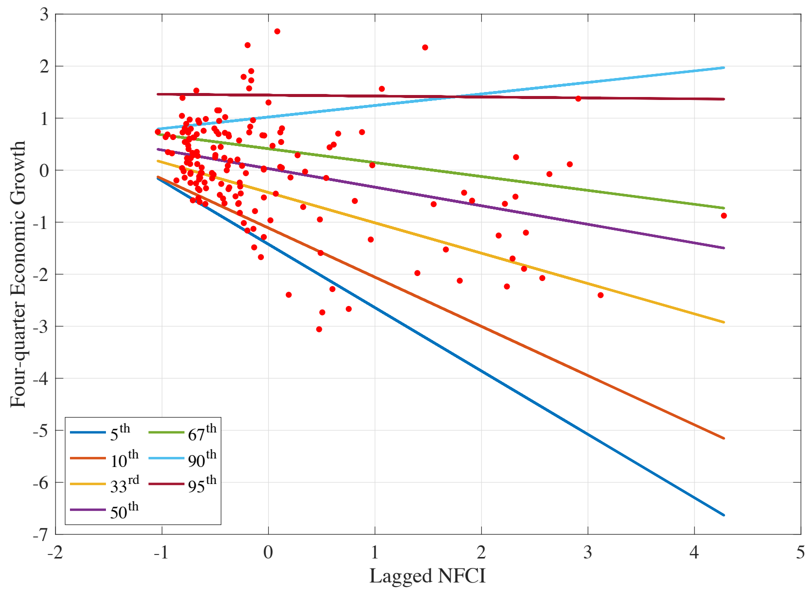
A.3.4 Conventional QR and Copula QR with Four-quarter Economic Growth

ABG also estimate their specification using economic growth over a four quarter horizon. In Figures A18 to A22, we display results using four-quarter economic growth which we calculate using FRED series GDPC1 as:

$$\Delta_4 y_t = 100 \times \left[\left(\frac{GDPC1_t}{GDPC1_{t-4})} \right) - 1 \right].$$

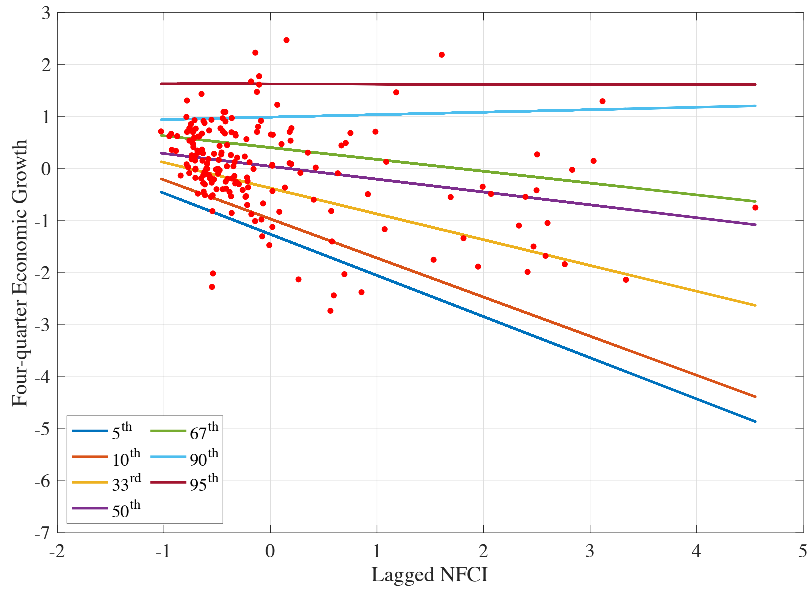
Figure A18 displays conventional QR lines for ABG's sample and Figure A19 does the same for our extended sample. Again these plots reflect the attenuation in the NFCI coefficients. For example, at $\tau = 0.50$, the fitted coefficients are -0.36 and -0.25 for ABG's sample and the extended sample, respectively.

Figure A18: Conventional QR Lines, ABG Sample
Four-quarter Economic Growth



Note: QR lines overlaid with a scatterplot of the (standardised) data (red dots).

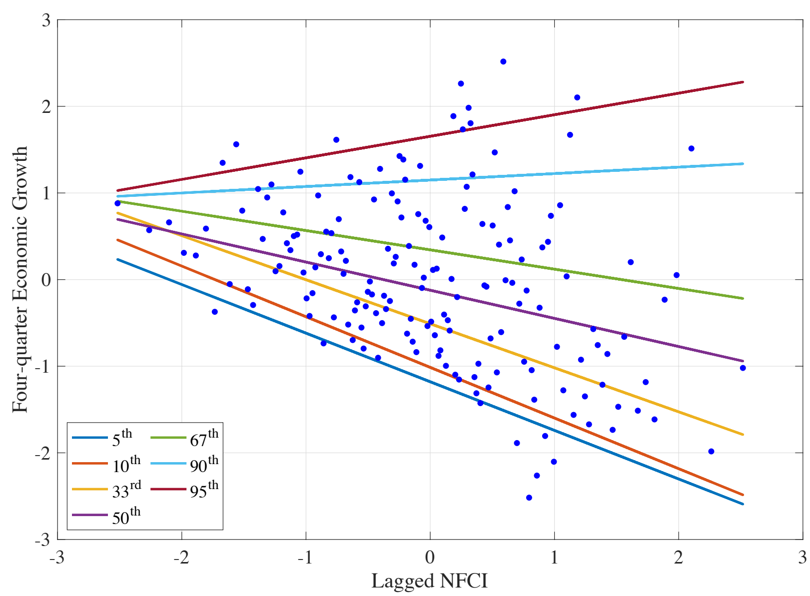
Figure A19: Conventional QR Lines, Extended Sample
Four-quarter Economic Growth



Note: QR lines overlaid with a scatterplot of the (standardised) data (red dots).

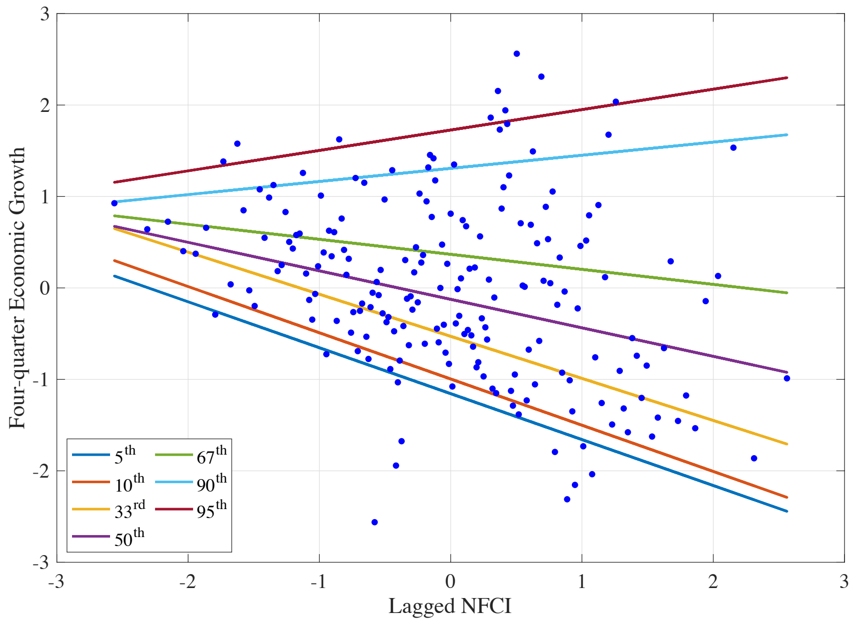
Using the four-quarter measure of economic growth, Figures A20 and A21 display copula QR lines for ABG's sample and the extended sample, respectively. There is very little difference between the two figures. For example, we see almost no attenuation at $\tau = 0.50$ with the estimated coefficient on the NFCI falling from -0.33 to -0.31 as we move from ABG's sample to our extended sample.

Figure A20: Copula QR Lines, ABG's Sample
Four-quarter Economic Growth



Note: QR lines overlaid with a scatterplot of the pseudo data (blue dots).

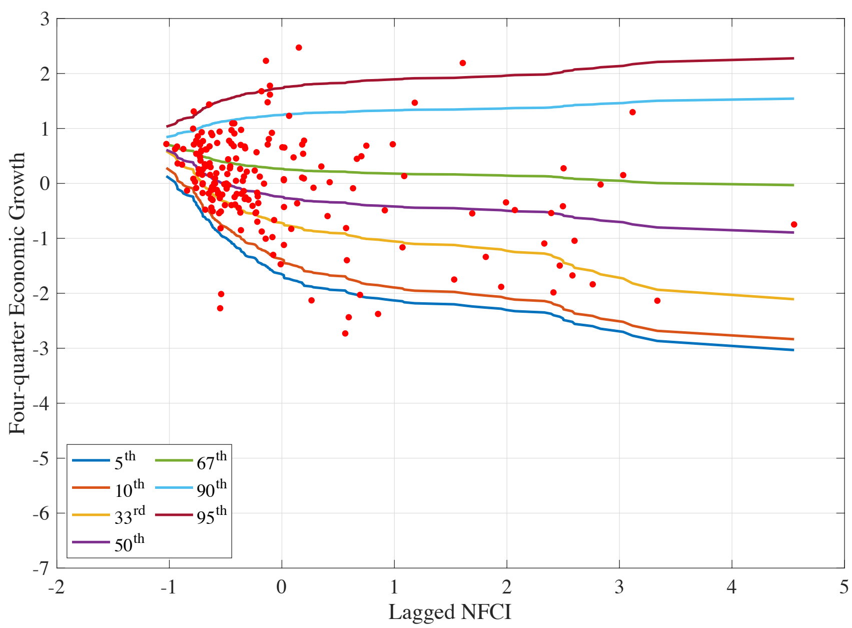
Figure A21: Copula QR Lines, Extended Sample
Four-quarter Economic Growth



Note: QR lines overlaid with a scatterplot of the pseudo data (blue dots).

Finally, using the ECDFs for both margins, Figure A22 reports the non-linear copula QR lines for four-quarter growth based on our extended sample. The slopes in the copula QR lines imply modest non-linearity.

Figure A22: Non-linear Copula QR Lines, Extended Sample
Four-quarter Economic Growth



Note: QR lines overlaid with a scatterplot of the (standardised) data (red dots).

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