Subjective Evaluations with Performance Feedback

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Abstract

Firms use subjective performance evaluations to provide employees with both incentives and feedback. This article shows that if an objective measure of performance, however imperfect, is available, subjective evaluations with incentive effects can be sustained even without repeated interaction. Although full efficiency cannot be achieved in general, it is achievable if the firm can commit to a forced distribution of evaluations and employs a continuum of workers. When the number of workers is small, a forced distribution is useful only if the objective measure is poor. The model also shows that a leniency bias in evaluations can improve incentives.

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1. Introduction

Most workers are regularly evaluated by their superiors. Such an evaluation typically includes the superior's subjective judgement about the worker's performance — for example, Gibbs et al (2009) document the use of subjective performance evaluations in the compensation packages of auto dealership managers, Levin (2003) cites survey evidence of subjective performance pay in law firms, and Eccles and Crane (1988) describe how the compensation of investment bankers depends on such subjective measures as the quality of their deals and customer satisfaction. Even the pay of the CEOs often depends on subjective assessments by the firms' boards of directors (Bushman et al, 1996; Hayes and Schaefer, 1997).

Performance evaluations usually serve multiple goals, but two of the most important are provision of incentives and performance feedback. For example, Cleveland et al (1989) report that 69% of their survey respondents considered salary administration and 53% considered performance feedback to be among the three main purposes of performance appraisals.[1] The incentive role of subjective evaluations has been studied extensively in the economics literature (e.g. MacLeod and Malcomson, 1989; Baker et al, 1994; Levin, 2003), but their feedback role, although recognized (Milgrom and Roberts, 1992; Prendergast, 2002), has been largely missing from formal models.[2]

This article integrates the feedback and the incentive roles of evaluations in a principal-agent model in which an owner/manager hires a worker for two periods. In each period, the worker performs two tasks that jointly determine his output according to a production function in which his ability and effort are complements. Worker ability is initially unknown, but the manager gets to privately observe it during the first period of production. The incentive contract then depends on an objective measure of performance and on a subjective evaluation which consists of

[1] Feedback is used to help workers identify their strengths and weaknesses, to inform them on whether their performance has been satisfactory, and to allow them to fine-tune or target their future efforts.

[2] One exception is Suvorov and van de Ven (2009), discussed in greater detail later.

the manager's message about the worker's ability. Due to multitasking distortions the objective measure is imperfect and therefore fails to achieve the first best outcome, despite both the manager and the worker being risk neutral. Subjective evaluations are thus valuable for two reasons: they provide first period incentives by complementing and correcting the distorted objective measure and they provide feedback to the worker about his productive abilities, which helps him to better allocate his second period effort. The article studies how these two roles of evaluations make the subjective scheme operational and how the interaction between subjective and objective measures of performance shapes the optimal incentive contract.

In this setting, subjective evaluations are useful for incentive provision because they are based on an undistorted measure; in fact, the principal would ideally like all of the agent's first period incentives to derive from subjective pay. I show that this is in general not possible, as the principal's freedom to design the contract is constrained by the need to ensure that the evaluations are truthful. Nevertheless, at least some incentives derived from subjective pay are always feasible, as long as the objective measure is not completely worthless. This is because under complementarity the worker's effort provision increases in his belief about his ability, so that the employer has an incentive to give the worker a good evaluation, in order to boost his effort. A properly designed reward scheme then balances the supervisor's desire to inflate the worker's self-assessment against her temptation to save on labor costs by under-reporting.

An optimal contract in this environment arises from a two-period mechanism design problem in which the principal faces her own, rather than the agent's, truthtelling constraint. Consequently, the contract is not shaped by the standard trade-off between rent extraction and allocation efficiency. Rather, the trade-off is between the efficiency of the second period objective contract and the efficiency of the first period incentives from subjective pay. In particular, for the evaluations to provide any incentives, the second period objective contract must necessarily be distorted away from what would be optimal in the absence of subjective pay. This trade-off limits the usefulness of subjective pay, preventing the optimal contract from achieving full efficiency in the first period.

The situation is different when firms can pre-commit to a specific distribution of evaluations. Such "forced distributions" are common in real world firms (one well known example is GE's "vitality curve"), but their purpose is not well understood. I show that, similar to the benefit of tournaments pointed out by Malcomson (1984), the advantage of a forced distribution is that it relaxes the principal's truthtelling constraint by making the size of the wage bill independent of individual evaluations. This eliminates the above tradeoff and allows the principal to achieve full efficiency in the first period by completely replacing the objective measure with subjective pay.

The truthtelling benefit of forced distributions does not come for free, however. A forced distribution limits the amount of information that the evaluations convey about the workers' productive capacities, which impedes the workers' ability to properly tailor their second period efforts. Crucially, this constraint gets more stifling the smaller is the number of workers. Thus, in this model the number of workers is of central importance in the choice between subjective evaluations with and without a forced distribution: When the workforce is large, a forced distribution can closely approximate the true distribution of the workers' productivities and is therefore highly informative. In this case, the optimal contract includes a forced distribution because the efficiency gain from improved first period incentives outweighs the loss from the misallocation of second period effort.

In contrast, when the number of workers is small, the choice between the two subjective schemes depends on the quality of the *objective* measure. If the objective measure is good, the main benefit of subjective evaluations is to inform the workers about their productive abilities, which favors subjective evaluations without a forced distribution. If the objective measure is poor, then it provides very inefficient incentives even if the workers are fully informed about their abilities. In this case, the main goal is to strengthen the workers' incentives, which is best achieved via evaluations with a forced distribution.

A comparison of the analyses with and without forced distributions yields insights into which firms will find forced distributions useful and how the presence of a forced distribution affects the properties of the optimal subjective scheme. In particular, the article shows that:

- forced distributions of subjective evaluations should be observed mainly in firms whose objective measures are relatively poor; in contrast, firms with access to relatively good objective measures should favor subjective evaluations without a forced distribution;
- consistent with Murphy's (1993) account of the subjective bonus scheme in the pharmaceutical company Merck, forced distributions of subjective evaluations should be more frequently observed in large organizations or teams than in small ones;
- the optimal subjective scheme that does not involve a forced distribution is convex, whereas an optimal scheme with a forced distribution does not require convexity and can be linear;
- a subjective scheme without a forced distribution should be steeper when the objective measure is of an intermediate quality than when it is either very good or very poor; the slope of the optimal subjective scheme with a forced distribution is independent of the quality of the objective measure.

An extension of the model in which in the absence of a forced distribution the supervisor is prone to inflating the evaluations — which in the literature is referred to as "leniency bias" (e.g., Prendergast, 1999) — yields an additional potentially testable prediction:

- contrary to what one might expect based on the standard theory, but consistent with the evidence in Bol (2011), a leniency bias can improve incentives and make the principal better off.

Related literature. Previous research on performance evaluations has focused primarily on how repeated interaction allows firms to overcome the reneging problem wherein supervisors are tempted to underreport workers' performance in order to save on labor costs (e.g., Bull, 1987; Baker et al, 1994; Levin, 2003). Within this literature, the present article is most closely related to Baker et al (1994), who were the first to model an interaction between objective and subjective measures.

MacLeod (2003) has generalized the logic of repeated game models by demonstrating that subjective schemes can be feasible even without infinite interaction if workers can punish a deviation from the implicit contract by imposing on the employer some type of socially wasteful cost, say, through quitting or sabotage at the firm. This model was further developed by Fuchs (2007), who extended it to a more dynamic environment, and by Rajan and Reichelstein (2009), who introduced in it objective measures of performance.

The present article complements the MacLeod/Fuchs/Rajan/Reichelstein theory by examining an alternative mechanism for sustaining subjective evaluations that does not require ex post destruction of surplus. Further, it points to the availability of an objective measure as a potentially important factor in determining the feasibility of subjective pay and to the number of workers as a determinant of whether the subjective pay scheme will include a forced distribution. Also, where Fuchs (2007) concludes that in his setting it is optimal for incentive purposes not to give the agent interim feedback about his performance, the current article provides a framework in which interim feedback is vital. This accords well with the evidence that companies cite feedback to workers as one of the main reasons for using subjective evaluations.

The article by Rajan and Reichelstein (2009) includes an analysis of a setting with two agents. The optimal scheme in this version of their model does not require surplus destruction and resembles a forced distribution of evaluations studied in the second part of this article. However, unlike in the present model, a forced distribution is necessary for Rajan and Reichelstein's scheme to work (in the absence of surplus destruction) and hence the question whether or not the firm will find a forced distribution useful cannot be addressed within their framework.

The mechanism that in the present model allows truthtelling to be sustainable is related to that studied in Suvorov and van de Ven (2009), who show that if the agent has intrinsic motivation to provide effort, the feedback role of performance assessments mitigates the reneging problem and makes subjective incentive schemes feasible even without infinite interaction. The present article does not presuppose that the agent is intrinsically motivated and instead studies the role of objective measures in sustaining subjective evaluations.

The effects of a principal's feedback on an agent's effort have also recently been studied in the context of multi-stage tournaments (Aoyagi, 2007; Goltsman and Mukherjee, 2011; Ederer, 2010). In these tournament models, feedback does not always affect effort in a desirable way and the main question is whether the agents provide more effort with or without information revelation.[3] In the present article, feedback is always useful. More to the point, this tournament literature is not about subjective evaluations, as it assumes that the feedback is contractible, which eliminates the problem of inducing the principal to reveal her information truthfully.

Noncontractible feedback is studied in Crutzen et al. (2012), who in a two-agent model study the manager's incentives to differentiate the workers by ability. In a somewhat different setting, Manso (2011) shows that feedback from supervisors is important in motivating employees to experiment with new technologies.

The model of this article is also formally related to Hermalin (1998), Benabou and Tirole (2003), and Fuchs (2013). The first two articles share with the present model the feature that a principal uses her private information about the production process to influence agents' incentives. In a setting without incentives, Fuchs (2013) shows how bonuses can help a principal to communicate

[3] An early article that addresses this question in a single-agent setting is Lizzeri et al (2002).

to the agent whether the agent is a good match for the job. More generally, the article is related to the vast literature on contracting with informed principals, such as Spier (1992), or, more recently, Halac (2012). This literature, however, focuses mostly on signaling issues, which do not arise in the present model because the principal gets informed only after the contract is signed.

The plan for the rest of the article. Section 2 describes the model and Section 3 establishes two benchmarks. Section 4 contains an analysis of the case without a forced distribution of evaluations. It provides conditions under which subjective pay is feasible, a result regarding the efficiency of subjective pay schemes, and a characterization of the optimal contract. Section 5 discusses the role of commitment and considers several extensions, including the effects of a leniency bias. Section 6 allows for evaluations with a forced distribution and compares the benefits and disadvantages of the two types of subjective schemes. Section 7 concludes.

2. The Model

Production technology. A principal (she) supervises an agent/worker (he) over two periods, $t = 1, 2$. The worker's output in period t is $y_t \in \{0, 1\}$. The probability of high output $y_t = 1$ is given by $q_t = a\mathbf{e}_t \cdot \mathbf{f}$, where $a \in \mathbb{R}_+$ is the worker's innate time-invariant ability, $\mathbf{e}_t = (e_{1t}, e_{2t}) \in \mathbb{R}_+^2$ is his 2-dimensional vector of efforts provided in period t, and $\mathbf{f} = (f_1, f_2) \in \mathbb{R}_+^2$ is the vector of marginal contributions of the worker's efforts to firm value. A key feature of this specification is that ability and effort are complements in the production function.[4]

The worker's ability is initially unknown. Both the worker and the principal only know that the ability is drawn from an interval $[0, 1]$ according to a distribution function $H(a)$ with density

[4]More generally, what is needed is for effort, total surplus, and both the objective and subjective performance measures (specified below) to be all increasing in ability.

$h(a)$, which is strictly positive and twice differentiable at each a.[5,6]

Performance measures. Neither the worker's expected contribution to firm value, q_t, nor its realization, y_t, are contractible. Instead, the worker's incentives come from two alternative sources:

Objective measures. First, there are contractible but imperfect measures of the worker's performance, $z_t \in \{0,1\}$, $t = 1,2$. The measures are imperfect due to multitasking problems similar to those studied in Feltham and Xie (1994), Datar et al (2001), and Baker (2002). Specifically, the objective measure distorts an agent's allocation of effort across tasks because it aggregates his individual efforts in a manner that differs from his contribution to the firm value.[7]

Formally, the probability that $z_t = 1$ is $p_t = a\mathbf{e}_t{\cdot}\mathbf{g}$, where $\mathbf{g} = (g_1, g) \in \mathbb{R}^2_+$ captures the marginal impact of the worker's efforts on z_1 and z_2. The measures z_t are imperfect in the sense that $\mathbf{g} \neq \alpha\mathbf{f}$ for any constant α. This makes it impossible for a contract based solely on z_t to induce the efforts that maximize the firm's value. The degree of distortion of the objective measure will be captured by the angle between $\mathbf{g}$ and $\mathbf{f}$ denoted by θ and defined by $\cos\theta = \frac{\mathbf{f}\cdot\mathbf{g}}{\|\mathbf{f}\|\|\mathbf{g}\|}$, where $\|\mathbf{f}\|$ and $\|\mathbf{g}\|$ are the lengths of the vectors $\mathbf{f}$ and $\mathbf{g}$ respectively, that is, $\|\mathbf{f}\| = \sqrt{f_1^2 + f_2^2}$ and $\|\mathbf{g}\| = \sqrt{g_1^2 + g_2^2}$. To ensure that p_t and q_t can be interpreted as probabilities, assume $\max\{\|\mathbf{f}\|, \|\mathbf{g}\|\} \leq 1$.

Without loss of generality, the focus will be on performance measures such that $\cos\theta \geq 0$. An undistorted measure has $\cos\theta = 1$ and the smaller is $\cos\theta$, the more distorted is the measure.

Subjective measures. At the end of period t, the principal receives a signal about the worker's expected contribution to the firm's period-t value. Although the main qualitative results would likely go through even if the signal were noisy, for simplicity I will assume that the signal is perfect,

[5] It would be straightforward to adapt the model so that a represents human capital that the worker develops during the first period.

[6] For some of the results, especially propositions 1 and 2, a simpler model with two ability levels would be enough. Other results, however, for example Proposition 3, as well as some of the results in the section on forced distributions of subjective evaluations, require multiple ability levels.

[7] The idea that subjective evaluations are useful because imperfect objective measures of performance can lead to dysfunctional behavior has been recognized by a number of writers (e.g., Baker et al., 1994; Prendergast, 1999) and is supported by the empirical findings in Gibbs et al (2009).

i.e., the principal privately observes $q_t = a\mathbf{e}_t \cdot \mathbf{f}$. This specification captures the idea, long present in the economics literature, that by the nature of her job a supervisor has superior information about the worker's contribution to firm value: "The employer, by virtue of monitoring many inputs, acquires special superior information about their productive talents" (Alchian and Demsetz, 1972). Alternatively, one could think of a as the quality of the principal's project and q_t as her private signal about this quality.

To ease the exposition, it will be assumed that z_1, z_2, y_1, and y_2 only become observable at the end of period 2, so that the worker cannot use z_1 and y_1 to update his belief about his ability. It should be stressed though that, at the cost of complicating the analysis, one could let the worker observe z_1 or y_1 or receive some other imperfect signal about his ability. Similar to MacLeod (2003), the optimal contract would then in general include messages from both the worker and the principal. However, the mechanism behind the main results would work the same way as in the current formulation, as long as the worker's signal is not a sufficient statistic for the principal's signal with respect to a, that is, as long as the principal's signal adds new information to what the worker already knows.

Subjective evaluations and contracting. After privately observing the worker's first period performance, the supervisor provides him with a subjective evaluation, which consists of a message $m \in [0, 1]$ about the worker's ability a.[8] This message is contractible, so that the worker's wage, w, can be written as $w = w(z_1, z_2, m)$. It will be convenient to write the general contract in terms of a base salary $s(m)$ and bonuses $b_1(m)$, $b_2(m)$, and $b_3(m)$, all of which can depend on m. The worker receives $b_1(m)$ if $z_1 = 1$, $b_2(m)$ if $z_2 = 1$, and $b_3(m)$ if $z_1 = z_2 = 1$, whereas the salary $s(m)$ is independent of z_1 and z_2. This formulation, where bonuses depend on both subjective and objective measures, is broadly consistent with bonus plans observed in the real world: Murphy and

[8] Alternatively, the message could be about the agent's first period contribution q_1.

Oyer (2003), for example, document that almost all of the 280 firms in their sample have bonus plans that depend upon achievement of a predetermined performance standard, and close to two-thirds of them adjust the bonuses based on subjective assessments of individual performance. Similarly, Gibbs et al (2009), who study incentive systems in auto dealerships, conclude that "incentive plans are a system of interrelated instruments, explicit and implicit, that are designed to work together."[9]

In the first part of the article, m will be the only contractible part of the subjective evaluation scheme. Section 6 will consider subjective evaluations with a forced distribution, where not only m but also the resulting distribution of m is contractible. The possibility that m is not contractible will be discussed in Section 5.

Preferences. Both parties are risk neutral and do not discount future income. The principal's goal is to maximize the firm's expected profit. The worker's per period reservation utility from not working is normalized to zero and his lifetime utility from being employed by the firm is $w - \Psi(\mathbf{e}_1) - \Psi(\mathbf{e}_2)$, where $\Psi(\mathbf{e}_t) = \sum_{k=1}^{2} \psi(e_{kt}) = (e_{1t}^2 + e_{2t}^2)/2$ is his disutility from effort in period $t = 1, 2$. The worker's participation constraint only needs to be satisfied at the beginning of the relationship, when the contract is signed.

Timing. At the beginning of the first period, the principal and the worker sign a contract that specifies the wage function $w(z_1, z_2, m)$. Subsequently, the worker chooses his first period effort levels, $\mathbf{e}_1$. At the end of the first period, the principal observes the worker's input q_1 and provides a performance evaluation m. At the beginning of the second period, the worker updates his belief about his own ability and exerts second period efforts $\mathbf{e}_2$. At the end of the second period, z_1 and z_2 are observed and the worker is paid $w(z_1, z_2, m)$.

[9] See also Woods (2012), who reports that in the large internal audit organization that he studied supervisors had the discretion to subjectively adjust the performance system's objective measures if they thought the measures misrepresented the employee's true performance.

3. Two benchmarks

To understand the nature of the optimization problem faced by the principal, it is helpful to start with a brief analysis of two benchmark cases.

Symmetric information.

In the first benchmark, the agent receives the same information about his performance as the principal. In this case subjective evaluations do not play any meaningful role and the agent's incentives depend solely on the objective measures z_1 and z_2. The principal's problem is then to choose a message independent contract (s, b_1, b_2, b_3) so as to maximize the expected total surplus[10]

$$E_a[q_1 - \Psi(\mathbf{e}_1) + q_2 - \Psi(\mathbf{e}_2)],$$

subject to the agent's incentive compatibility constraints for the two periods

$$\begin{aligned} \mathbf{e}_1 &= \arg\max_{\mathbf{e}_1'} E_a\left[s + b_1 p_1' + \left(b_2 + b_3 p_1'\right) p_2 - \Psi(\mathbf{e}_2) - \Psi(\mathbf{e}_1')\right]; \\ \mathbf{e}_2 &= \arg\max_{\mathbf{e}_2'} \left(b_2 + b_3 p_1\right) p_2' - \Psi(\mathbf{e}_2'), \end{aligned}$$

where $p_1' = a\mathbf{e}_1' \cdot \mathbf{g}$ and $p_2' = a\mathbf{e}_2' \cdot \mathbf{g}$.

Because conditional on the worker's ability a the realizations of z_1 and z_2 are independent of each other, one can without loss of generality set $b_3 = 0$ and treat the incentive problems in the two periods as two separate problems. Replacing the two constraints with their respective first order conditions, the principal's first period problem is then to choose b_1 so as to maximize $E_a[a\mathbf{e}_1 \cdot \mathbf{f} - \Psi(\mathbf{e}_1)]$ subject to $E(a) b_1 g_k = \psi'(e_{k1})$, and her second period problem is to maximize $a\mathbf{e}_2 \cdot \mathbf{f} - \Psi(\mathbf{e}_2)$ subject to $a b_2 g_k = \psi'(e_{k2})$, $k = 1, 2$. The only difference between these two problems

[10] As usual, the problem reduces to surplus maximization after the agent's individual rationality constraint is substituted into the principal's expected profit function.

is that in period 2 a is publicly known, whereas in period 1 only the distribution of a is known. In either case, though, the optimal bonus is the same:

$$b_1 = b_2 = b^{SB} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|} \cos\theta,$$

where the superscript SB indicates that the solution represents a second-best contract.

As discussed in Baker (2002), $\cos\theta$ in b^{SB} captures the degree of congruence between the performance measure and the firm value (the closer is $\cos\theta$ to 1, the better is the objective measure), whereas the term $\frac{\|\mathbf{f}\|}{\|\mathbf{g}\|}$ reflects scaling (i.e., it accounts for the fact that $\mathbf{f}$ and $\mathbf{g}$ can have different lengths). The benchmark bonus b^{SB} will prove useful later, in characterizing the optimal contract. For future reference, note that b^{SB} does not depend on a.

To summarize, without an informational asymmetry the problem collapses into a standard problem familiar from the literature on multitasking.

Perfect objective measures.

In the second benchmark of interest, instead of assuming that the agent observes the principal's information, assume that the measure z_t is perfectly aligned with y_t, so that $\cos\theta = 1$. In this case the optimal contract does entail subjective evaluation m, but s, b_1, b_2, and b_3 are again set to be independent of m. Under such a contract, the principal is willing to reveal her private information truthfully. The evaluations then do not have any incentive effect, but in this ideal case incentives from subjective pay are not needed, because a perfect objective measure can ensure the first-best outcome. In particular, the agent's incentive problems in the two periods can again be viewed as independent of each other and solved separately. Analogous to the solution obtained in the previous benchmark, the optimal contract is obtained as $b_1 = b_2 = b^{FB} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|}$. Note that when the vectors $\mathbf{f}$ and $\mathbf{g}$ are of the same length, this solution reduces to the standard first-best contract for risk-neutral agents, $b^{FB} = 1$.

In what follows, the objective measure is imperfect and cannot provide the first-best incentives. Additional incentives from subjective evaluations, in which the agent's pay depends on $\mathbf{e}_1$ through m, are therefore valuable. But such a subjective scheme cannot be arbitrary — it has to be incentive compatible for the principal. As we will see, this will be made possible by the fact that the evaluations also affect the agent's *second* period effort, $\mathbf{e}_2$.

Thus, when the objective measure is distorted, the agent's incentive problems in the two periods are no longer independent of each other. Rather, they are connected through the principal's truthtelling constraint and have to be solved simultaneously. I will now turn to the analysis of this problem.

4. Feasibility of subjective evaluations with incentive effects

It is clear that truthful evaluations always affect the agent's second period effort (as long as it is positive), by affecting his belief about his ability. But is it possible for subjective evaluations to also have first-period incentive effects? The analysis will start by addressing this question.

The worker's problem

Working backwards from the second period, let $x(m)$ be the worker's posterior belief about his expected ability based on his subjective evaluation m. The worker's second period problem is then to choose $\mathbf{e}_2$ so as to maximize $\beta(m)x(m)\mathbf{e}_2{\cdot}\mathbf{g} - \Psi(\mathbf{e}_2)$, where $\beta(m) \equiv b_2(m) + b_3(m)p_1$. One can think of $\beta(m)$ as a "composite bonus," but it is important to bear in mind that it depends on p_1 and hence on $\mathbf{e}_1$, even though for the sake of streamlining the exposition this dependence on $\mathbf{e}_1$ will be suppressed in the notation. The worker's second period efforts $e_{k2}(\beta, x)$ are then determined by the first order conditions

$$\beta(m)x(m)g_k = \psi'(e_{k2}) = e_{k2}, \;\; k = 1, 2.$$

The principal does not observe the worker's first period efforts, but she makes a conjecture about them, $\tilde{\mathbf{e}}_1$. She then uses her observation of $q_1 = a\mathbf{e}_1{\cdot}\mathbf{f}$ to infer the worker's ability as $a = \frac{q_1}{\tilde{\mathbf{e}}_1\cdot\mathbf{f}}$. Focusing on truth-telling and fully separating contracts, the principal's equilibrium message will be $m = \frac{q_1}{\tilde{\mathbf{e}}_1\cdot\mathbf{f}}$, which will allow the worker to infer his ability via $x(m) = m\frac{\tilde{\mathbf{e}}_1\cdot\mathbf{f}}{\mathbf{e}_1\cdot\mathbf{f}}$.[11,12] In equilibrium, the principal's conjecture will be correct, $\tilde{\mathbf{e}}_1 = \mathbf{e}_1$. The first set of incentive compatibility constraints for the principal's optimization problem is thus obtained as

$$\beta(m)mg_k = e_{k2}, \;\; k = 1, 2. \tag{1}$$

The thing to notice here is that, holding $\beta(m)$ fixed, each component of the worker's vector of efforts increases in his belief $x(m)$ and hence in the principal's evaluation m.

In period 1, the worker chooses his efforts so as to maximize his expected lifetime utility

$$E_a[s + b_1(m)a\mathbf{e}_1{\cdot}\mathbf{g} + \beta(m)a\mathbf{e}_2{\cdot}\mathbf{g} - \Psi(\mathbf{e}_2)] - \Psi(\mathbf{e}_1),$$

taking into account the effect of $\mathbf{e}_1$ on the principal's report m. In particular, for any first period effort vector $\hat{\mathbf{e}}_1$, the worker expects the evaluation $m(\hat{\mathbf{e}}_1) = \frac{a\hat{\mathbf{e}}_1\cdot\mathbf{f}}{\tilde{\mathbf{e}}_1\cdot\mathbf{f}}$. This yields the worker's first-period incentive compatibility constraint

$$\mathbf{e}_1 \in \arg\max_{\hat{\mathbf{e}}_1} E_a\left[s(\frac{a\hat{\mathbf{e}}_1{\cdot}\mathbf{f}}{\tilde{\mathbf{e}}_1{\cdot}\mathbf{f}}) + b_1(\frac{a\hat{\mathbf{e}}_1{\cdot}\mathbf{f}}{\tilde{\mathbf{e}}_1{\cdot}\mathbf{f}})a\hat{\mathbf{e}}_1{\cdot}\mathbf{g} + \beta(\frac{a\hat{\mathbf{e}}_1{\cdot}\mathbf{f}}{\tilde{\mathbf{e}}_1{\cdot}\mathbf{f}})a\mathbf{e}_2{\cdot}\mathbf{g} - \Psi(\mathbf{e}_2))\right] - \Psi(\hat{\mathbf{e}}_1) \tag{2}$$

The principal's problem

The principal's message m maximizes her expected second period profit subject to the worker's

[11] The possibility of pooling will be discussed in Section 5.

[12] Although for an arbitrary m the belief $x(m)$ depends on $\mathbf{e}_1$, I do not indicate this dependence in notation, as under truthful reporting $x(m)$ is independent of $\mathbf{e}_1$. That is, the agent cannot fool himself by providing more (or less) effort. Similarly, the effect of $\mathbf{e}_1$ on $\mathbf{e}_2$ via m is also not indicated.

incentive compatibility constraint (1). Combined with the requirement of truthtelling, this yields the following incentive compatibility constraint for the principal:

$$a \in \arg\max_{m} a\mathbf{e}_2(m)\cdot\mathbf{f} - \beta(m)a\mathbf{e}_2(m)\cdot\mathbf{g} - s(m) - b_1(m)a\mathbf{e}_1\cdot\mathbf{g}. \tag{3}$$

The principal's general problem is then to maximize the total surplus from the employment relationship according to the following program:

$$\text{(P):} \quad \max_{s(.),b_1(.),b_2(.),b_3(.),\mathbf{e}_1,\mathbf{e}_2} E_a[a\mathbf{e}_1\cdot\mathbf{f} - \Psi(\mathbf{e}_1) + a\mathbf{e}_2(m)\cdot\mathbf{f} - \Psi(\mathbf{e}_2(m))] \quad \text{subject to (1) - (3).}$$

I will say that subjective evaluations provide incentives when the marginal effect of the agent's first-period effort on his expected pay is higher (at least for some effort levels strictly) when his pay depends on m than when it does not. Recalling that θ denotes the angle between $\mathbf{f}$ and $\mathbf{g}$, the first result provides conditions under which subjective evaluations with incentive effects are feasible.[13]

Proposition 1. *If* $\cos\theta = 0$*, no subjective evaluation scheme with incentive effects is feasible. If* $\cos\theta > 0$*, then a subjective scheme that is both truthful and provides incentives for first period effort is feasible.*

In most of the existing literature, subjective evaluations with incentive effects are feasible only if the principal and the agent interact repeatedly (e.g., Baker et al, 1994; Levin, 2003) or if agents can take ex post inefficient actions that destroy surplus (MacLeod, 2003). Proposition 1 shows that neither repeated interaction nor surplus destruction are needed for subjective evaluations to have incentive effects.

The logic behind the proposition is related to the idea of countervailing incentives in Lewis

[13] All proofs are in Appendix A.

and Sappington (1989). Specifically, because the subjective measure of the worker's performance depends on both his actions and his underlying type (ability), the principal faces two opposing temptations. On the one hand, she wants to give the worker a bad evaluation in order to save on the wage bill. This is the standard consideration, extensively studied in the previous literature. On the other hand, because the worker's second period effort increases in m, the principal is tempted to boost the worker's self-assessment through a good evaluation. A truthtelling wage scheme balances these two temptations in such a way that they offset each other.

Critically, the second effect is only present if the worker's output in period 2 depends on m. For a subjective scheme to work, it is therefore important that the principal has access to some objective measure, however imperfect. This distinguishes the present theory from the theories in which subjective evaluations are supported by repeated interaction or surplus destruction. In these alternative theories, objective measures are not needed; in fact, they can render subjective evaluations infeasible, as emphasized by Baker et al (1994). In the present framework, the worker provides no valuable effort in period 2 if the objective measure is useless ($\cos\theta = 0$) or if no objective measure is available, so there is no point trying to influence his belief. In such a situation, evaluations can be truthful only if they do not affect the worker's pay, which means they cannot have any incentive effect. This is why the result in Proposition 1 depends on $\cos\theta$.

As I have already mentioned, the larger is $\cos\theta$, the less distorted is the objective measure. At one extreme, $\cos\theta = 1$ and the measure z_t is perfectly aligned with y_t. As discussed in the analysis of this benchmark case, the first-best outcome is feasible in this ideal situation and is achieved by a contract in which the agent's pay does not dependent on the evaluation he receives. At the other extreme, $\cos\theta = 0$ and the performance measure elicits no valuable effort. Consequently, (3) can hold only if the worker's wage is independent of m, which leads to part (i) in the proposition.

Recall that the mechanism that makes subjective evaluations feasible in this model does not require that the manager is better informed than the worker. But it does require that the manager has at least some private information about the worker's performance. The empirical importance of this is that we should expect subjective evaluations to be used in situations where principals have private information about the worker's performance. This is in line with the evidence in the management literature, such as Cleveland et al (1989) mentioned in the Introduction, that documents that many firms consider performance feedback to be among the main purposes of performance appraisals. Moreover, this literature maintains that feedback conveys information to the worker (e.g., Alvero et al, 2001, and the references therein), which implies that in these firms managers have information that workers do not possess.

Limits on efficiency

Constraint (1) makes it clear that full efficiency cannot be achieved in the second period. This is because $\mathbf{e}_2$ is induced only through the objective measure z_2, which provides distorted incentives: actual efforts are proportional to $\mathbf{g}$, whereas efficient efforts are proportional to $\mathbf{f}$. The best the principal can do in period 2 is to set β equal to the second best bonus $b^{SB} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|}\cos\theta$.

What about the first period efforts? The benefit of subjective evaluations is that in period 1 incentives from the distorted measure z_1 are at least partly replaced by incentives from the undistorted measure m. Does this imply that the optimal contract will elicit the first best vector of efforts $\mathbf{e}_1^{FB}$?[14] As the next result shows, the answer is No.

Proposition 2. *The optimal contract elicits vectors of efforts* $\mathbf{e}_1^*$ *and* $\mathbf{e}_2^*$ *such that* $\mathbf{e}_1^* \neq \mathbf{e}_1^{FB}$ *and* $\mathbf{e}_2^* \neq \mathbf{e}_2^{FB}$.

Even though a subjective scheme that elicits the efficient efforts in period 1 might be feasible, Proposition 2 says that the principal will not find such a scheme optimal. This result may seem

[14] Because a is not known when $\mathbf{e}_1$ is chosen, vector $\mathbf{e}_1^{FB}$ is defined by $\psi'(e_{k1}^{FB}) = E(a)f_k$. At the end of period 1, the evaluations reveal a to the worker, so $\mathbf{e}_2^{FB}$ is defined by $\psi'(e_{k2}^{FB}(a)) = af_k$.

surprising because at the time of contracting there is no informational asymmetry and the principal can hold the worker down to his reservation utility. But the reasoning is simple: Because $\mathbf{e}_1^{FB}$ does not depend on $\mathbf{g}$, any scheme that elicits $\mathbf{e}_1^{FB}$ requires that the contract is independent of z_1. The principal is thus left with two measures, m and z_2, which do not give her enough degrees of freedom to solve the three agency problems she faces: the worker's two moral hazard problems and her own truth-telling problem. Put differently, the principal has two functions she can control, $s(m)$ and $\beta(m)$, tied down by two constraints, (3) and $\mathbf{e}_1 = \mathbf{e}_1^{FB}$. The functions that satisfy these two constraints do not in general optimize the worker's second period incentives. Consequently, starting from $\mathbf{e}_1 = \mathbf{e}_1^{FB}$, a small change in the contract that moves $\beta(m)$ towards b^{SB} increases the principal's overall payoff, as it generates a first order improvement in period 2 incentives, but only a second order loss due to the period 1 deviation from $\mathbf{e}_1^{FB}$.

Proposition 2 will prove useful in Section 6, where the current setting is compared with a setting in which the principal commits to a specific distribution of subjective evaluations.

Optimal contract

In general, the worker's first period efforts depend on b_1 and b_3 directly and on all of s, b_1, b_2, and b_3 indirectly, through the effects of $\mathbf{e}_1$ on the evaluation m. Furthermore, $\mathbf{e}_2$ can depend on $\mathbf{e}_1$ through β. A wage scheme that allows for all of s, b_1, b_2, and b_3 to depend on m therefore has complicated effects on both $\mathbf{e}_1$ and $\mathbf{e}_2$. Fortunately, Lemma 1 below simplifies the problem significantly. It shows that if the optimal contract is piecewise differentiable (assumed throughout the rest of this section[15]) the only parts of the contract that need to be allowed to depend on m

[15] Piecewise differentiability is a common assumption in optimal control problems. Although in many mechanism design problems the optimal contract can be shown to be monotonic, which ensures that it is differentiable almost everywhere, in the current setting monotonicity of the optimal contract cannot be ascertained ex ante.

are $s(m)$ and $b_2(m)$. Specifically, consider the following augmented version of problem (P):

$$\text{(P'):} \qquad \max_{b_1, b_2(m), \mathbf{e}_1} E_a \left[a\mathbf{e}_1 \cdot \mathbf{f} - \Psi(\mathbf{e}_1) + \|\mathbf{g}\|^2 a^2 b_2(a) \left[b^{SB} - \frac{b_2(a)}{2} \right] \right]$$

$$\text{subject to} \qquad e_{1k} = E_a \left[ab_1 g_k + \frac{\|\mathbf{g}\|^2 f_k}{\mathbf{e}_1 \cdot \mathbf{f}} a^2 \left[b^{SB} - b_2(a) \right] \left[b_2(a) + ab_2'(a) \right] \right]. \tag{4}$$

Problem (P') was obtained from problem (P) by taking advantage of the quadratic cost function and by (i) setting $b_3(m) = 0$ and $b_1(m) = b_1$, where b_1 is a constant, (ii) substituting (1) into the objective function and into the remaining constraints, and (iii) substituting $s'(m)$ from the first order condition for (3) into the first order condition for (2) (see the proof of Lemma 1 for details). Steps (ii) and (iii) are self-explanatory. The logic behind setting $b_3(m) = 0$ is that the agent's incentive problems in the two periods are tied only through the principal's truthtelling constraint (3) — otherwise, they are independent of each other, as discussed in the analysis of the benchmarks. This independence means that conditioning the agent's bonus on the joint realizations of the objective measures across the two periods is of no help in improving his incentives. Finally, b_1 can be set constant without loss of generality because $s(m)$ is enough to capture any incentive effects m can have on the agent's first period efforts.

Lemma 1. *Suppose b_1^*, $b_2^*(.)$, $\mathbf{e}_1^*$, and $\mathbf{e}_2^*$ solve the amended problem (P'). Suppose also $ab_2^*(a) \left[b^{SB} - b_2^*(a) \right]$ is non-decreasing in a. Then b_1^*, $b_2^*(.)$, $\mathbf{e}_1^*$, and $\mathbf{e}_2^*$ solve the original problem (P).*

To guarantee that the condition in the lemma is satisfied, so that the solution to the simplified problem (P') also solves (P), some restrictions on the density function $h(a)$ are needed. Roughly speaking, the prior beliefs about the worker's ability should be sufficiently diffused, so as to not place a disproportional weight on any particular ability level.[16] The often used uniform distribution

[16] More formally, let $\varepsilon(a) \equiv \frac{ah'(a)}{h(a)}$ be the elasticity of $h(.)$ at a and let $M \equiv \max_{a \in [0, \bar{a}]} |\varepsilon'(a)|$. It can be shown that $ab_2^*(a) \left[b^{SB} - b_2^*(a) \right]$ is non-decreasing in a if M is sufficiently small.

satisfies this requirement, so for simplicity the discussion in this section will proceed under the assumption that $H(.)$ is uniform.[17]

A formal analysis of problem (P') is contained in Appendix B. It characterizes the optimal bonuses b_1^* and $b_2^*(m)$ and shows that b_2^* is independent of m when H is uniform (except at $m = 1$). Thus, the optimal contract is effectively separated into a subjective part, consisting of $s(m)$, and an objective part, consisting of the fixed bonuses b_1 and b_2. This makes it possible to characterize the optimal subjective scheme $s(.)$:

With b_1 and b_2 constant, the principal's truthtelling problem (3) reduces to

$$a \in \arg\max_m a\mathbf{e}_2{\cdot}\mathbf{f} - b_2 a\mathbf{e}_2{\cdot}\mathbf{g} - s(m) \text{ subject to (1),}$$

which yields the first order condition $s'(m) = ab_2\left(\mathbf{g}\cdot\mathbf{f} - b_2\left\|\mathbf{g}\right\|^2\right)$. The proof of Proposition 3 verifies that this condition describes the principal's optimum. Imposing truthtelling then yields a differential equation that implicitly defines the optimal subjective scheme $s^*(m)$:

$$s^{*\prime}(m) = mb_2\left(\mathbf{g}\cdot\mathbf{f} - b_2\left\|\mathbf{g}\right\|^2\right).$$

Together with the analysis of problem (P') in Appendix B, this leads to the following result.

Proposition 3. *When $H(.)$ is uniform, it is optimal to set*

$$b_1^* > 0,$$

$$b_2^* = \kappa b^{SB} \text{ a.e., } \textit{and} \tag{5}$$

$$s^*(m) = \frac{m^2}{2}\left\|\mathbf{g}\right\|^2 b_2^*\left(b^{SB} - b_2^*\right) + D, \tag{6}$$

[17] An earlier version of this article derived the optimal contract under a more general distribution and showed that it has similar properties. The main difference is that the optimal bonus b_2 is in general a function of m, which, as will be seen in Proposition 3, is not true under uniform distribution.

where κ and D are constants. Moreover, $\kappa \in (\frac{1}{2}, 1)$.

Proposition 3 provides several insights into the economics of the model. First, (6) reveals that to be truthful, the wage scheme $s(m)$ must be not only increasing but also strictly convex in the subjective evaluation m. This skewness of the subjective pay reflects the production complementarities between ability and effort. Intuitively, the more productive is the worker, the bigger is the principal's potential gain from misleading him about his ability through a good evaluation. To balance this temptation, the "price" for increasing the evaluation must be higher for higher ability workers, which leads to convexity of the pay scheme.

Second, (5) tells us that in the presence of subjective evaluations the principal optimally weakens the agent's formal second period incentives: $b_2^* < b^{SB}$ almost everywhere. This is not because subjective pay has second period incentive effects that substitute for formal contracts. Rather, the purpose of weakening the second period formal contract is to ensure that the evaluations induce *first period* effort. To see this, notice that an increase in the agent's belief about his ability has the same effect on his second period incentives as an increase in b_2, and recall that the second period surplus is maximized when $b_2 = b^{SB}$. Hence, if b_2 were equal to b^{SB}, there would be no further benefit to strengthening the second period incentives and the principal would have no desire to induce a higher $\mathbf{e}_2$ through a good evaluation. The only way to ensure truthful evaluation would therefore be to make $s(m)$ independent of m, in which case the evaluations would provide no incentives. Thus, to ensure that the evaluations have incentive effects, the principal scales back the second period formal contract. This makes it desirable for her to boost $\mathbf{e}_2$ by providing good evaluations, which in turn requires $s(m)$ to depend on m because otherwise the principal's truthtelling constraint would not hold. An $s(m)$ that depends on m then provides first period incentives. However, because incentives in period 2 come solely from b_2, it is not optimal to scale b_2 all the way back to zero. Consequently, $b_2^* > 0$.

Third, the optimal contract retains some formal incentives also in the first period, $b_1^* > 0$, even though subjective evaluations, being undistorted, provide more efficient incentives. The logic is similar to that behind Proposition 2: The first period incentive effects of subjective evaluations come at the cost of further muting second period effort. To limit this cost, first period incentives are supplemented by incentives from the objective measure.

Fourth, Proposition 3 shows that the first period incentives from subjective pay are actually non-monotonic in the quality of the objective measure, as captured by $\cos\theta$. When $\cos\theta = 0$, we have $b_2^* = 0$, and (6) reduces to $s^*(m) = D$. This confirms more directly the result of Proposition 1, which says that subjective evaluations can have incentive effects only if the objective measure is not useless. In this case, telling the worker that he is a high type entails no benefit to the principal, which means that evaluations can be truthful only if they do not affect the worker's pay. Such a scheme, however, has no incentive value. A similar conclusion obtains when $\cos\theta = 1$. Then $b_2^* = b^{SB} = b^{FB}$, so that (6) again yields $s^*(m) = D$, that is, a constant wage with no subjective incentives. These observations yield a potentially testable prediction that the optimal subjective bonus scheme is steeper when the objective measure is of an intermediate quality than when it is either very good or very poor.

Finally, note that $e_{k2}^*(m) = mb_2^*(m)g_k$ increases in m. An implication is that good evaluations are followed by good performance and bad evaluations are followed by poor performance. This prediction is supported by several studies in the management and social psychology literatures, which find that positive feedback accompanied by extrinsic rewards improves subsequent motivation (e.g., Rosenfield et al, 1980; Eisenberger et al, 1999; Fang and Gerhart, 2012).

5. Discussion of the base model and extensions

Non-contingent objective bonus

Under the uniform distribution, the optimal contract was shown to be strictly separated into a part that rewards objective performance (b_1 and b_2) and a part that rewards subjective performance ($s(m)$). This is not true for other distributions, but if one imposed such a separation between objective and subjective bonuses as a constraint (say, because such contracts are simpler and easier to implement), then for *any* distribution function the optimal contract would look like the one described in Proposition 3. The only difference would be that the scalar κ would generically have a different value. This result is summarized as follows.

Proposition 4. *For any distribution function $H(.)$, if b_1 and b_2 are restricted to be independent of m, the optimal contract sets $b_1^* > 0$, $b_2^* = \kappa b^{SB}$, and $s^*(m) = \frac{m^2}{2} \|\mathbf{g}\|^2 b_2^* \left(b^{SB} - b_2^*\right) + D$, where D is a constant and $\kappa \in (\frac{1}{2}, 1)$.*

Learning

In the equilibrium of the present model, the worker learns his ability by the end of the first period. Such perfect learning simplifies things, but the framework could be extended to allow for gradual learning. In principle, the worker can learn from two sources: (a) from the principal's feedback and (b) from observing his own output.

Starting with (a), feedback from the principal currently leads to perfect learning at the end of the first period for two reasons. The first is that the model only has two periods. In a setting with more periods, the principal might actually find it optimal to insert some noise into her communication with the agent, in order to slow down his learning, as perfect learning makes subjective evaluations infeasible in all subsequent periods.

The second reason the current model exhibits perfect learning is that the principal's information about the worker's ability is perfect. In a more realistic setting, the principal's signals would be noisy, so that every period she would learn something new about the worker. In such an alternative setting, the principal's evaluation could be truthful in every period without leading to perfect learning and without destroying the viability of subjective evaluations in later periods.

As for (b), this channel is shut down in the current model by assumption, but one could allow the worker to observe at the end of each period a signal about his performance. It seems safe to conjecture that as long as such signals are imperfect, subjective evaluations would remain feasible. Perfect learning would destroy subjective evaluations in all subsequent periods for a similar reason that it destroys incentives in Holmström's (1999) career concerns model, but as demonstrated by Holmström, this could be remedied by assuming that the worker's ability evolves over time.

Leniency bias

Some observers suggest that an important problem with subjective evaluations is that supervisors are prone to a leniency bias, i.e., a preference for giving good evaluations (see, e.g., Prendergast, 1999). Such a bias can be readily incorporated into the present model: Suppose the principal derives utility $v(m)$ from giving an evaluation m, where the preference for good evaluations is captured by $v'(m) > 0$ for all m. The only part in the setup of the optimization problem (P) that needs to be adjusted to account for this is the truthtelling constraint (3), which is now written as

$$a \in \arg\max_{m} v(m) + a\mathbf{e}_2{\cdot}\mathbf{f} - \beta(m)a\mathbf{e}_2{\cdot}\mathbf{g} - s(m) - b_1(m)a\mathbf{e}_1{\cdot}\mathbf{g}. \tag{7}$$

Following the steps in the proof of Lemma 1 and making use of the result that the optimal bonuses b_1 and b_2 are constant, it can be verified that the new term $v(m)$ above translates into an additional term $\frac{f_k}{\mathbf{e}_1{\cdot}\mathbf{f}}E_a\left(av'(a)\right)$ in constraint (4) to problem (P'), which now becomes

$$e_{1k} = E(a)b_1g_k + \frac{f_k}{\mathbf{e}_1{\cdot}\mathbf{f}}\left\|\mathbf{g}\right\|^2 E(a^2)b_2\left(b^{SB} - b_2\right) + \frac{f_k}{\mathbf{e}_1{\cdot}\mathbf{f}}E_a\left(av'(a)\right) \tag{8}$$

Thus, for a given b_1 and b_2, the direct effect of the bias is to strengthen the first-period incentives from subjective pay. This is intuitive: Even without a bias, the principal likes to give good evaluations because they induce the agent to work harder; the salary function $s(.)$ is therefore designed to be increasing, so as to make good evaluations costly. A leniency bias merely magnifies this effect and makes the salary function $s(.)$ steeper.

Of course, the principal might adjust b_1 and b_2 so as to mute this direct effect on $s(.)$. Tracing the precise impact of a leniency bias on the optimal contract $(b_1^*, b_2^*, s^*(m))$ is beyond the scope of this article, but three interesting observations follow rather easily:

First, unless $E_a\left(av'(a)\right)$ is too large, a leniency bias makes the principal better off. This is because a steeper function $s(m)$ provides stronger incentives. To see this more formally, start with b_1^* and b_2^* that are optimal in the absence of a bias, set $b_1 = b_1^*$, and increase b_2 to a level b_2^{**} such that $\|\mathbf{g}\|^2 E(a^2) b_2^{**}\left(b^{SB} - b_2^{**}\right) + E_a\left(av'(a)\right) = \|\mathbf{g}\|^2 E(a^2) b_2^*\left(b^{SB} - b_2^*\right)$, which is possible if $E_a\left(av'(a)\right)$ is not too large. Such a contract leaves the first period efforts unchanged, as is apparent from (8), but increases efficiency in the second period because b_2 is closer to b^{SB}. The expected total surplus therefore increases and the principal is better off.[18] This conclusion fits well with the empirical findings of Bol (2011), who examines the incentive plan of a financial service provider and documents that leniency bias positively affects employees' incentives. Bol's view is that this finding contradicts the standard agency theory, but the current model shows that this need not be the case.

Second, the right amount of leniency bias can in fact ensure full efficiency in period 1 and the second best outcome in period 2. To see this, suppose that $E_a\left(av'(a)\right) = E(a)\mathbf{e}_1^{FB} \cdot \mathbf{f}$. Then setting $b_1 = 0$ and $b_2 = b^{SB}$ reduces (8) to $e_{1k} = E(a) f_k$, which is the condition that gives the first best efforts in period 1. The second best outcome in period 2 then follows from $b_2 = b^{SB}$.

Finally, a leniency bias makes subjective evaluations with incentive effects feasible even in

[18] With a leniency bias such that $E_a\left(av'(a)\right) > E(a)\mathbf{e}_1^{FB} \cdot \mathbf{f}$, truthtelling would necessitate a salary function $s(m)$ so steep that the first period efforts would exceed their first-best levels. This could make the principal worse off than in the absence of a leniency bias.

the absence of an objective measure. This can be readily seen from the principal's truthtelling constraint (7): Without an objective measure, we have $b_1 = b_2 = e_{2k} = 0$, $k = 1, 2$, so that (7) becomes $a \in \arg\max_m v(m) - s(m)$. Truthtelling is therefore ensured by setting $s(m) = v(m)$ and, given that $s'(m) = v'(m) > 0$, such a scheme provides first period incentives.

Optimality of separation

The analysis of Section 4 was conducted under the assumption that the contract entails no pooling, i.e., the worker always learns his ability precisely. It can be readily seen that from the point of view of second period efficiency, pooling is never desirable, as it prevents the worker from tailoring his effort to his ability. It is conceivable, though, that pooling improves the first period incentive effects of the subjective scheme. If this were the case, the overall desirability of full separation would depend on the tradeoff between the incentive gain from pooling and the second period loss due to inefficient allocation of effort. A complete characterization of the optimal pooling contract and its comparison with a fully separating contract is not undertaken here. However, relatively simple logic shows that if the objective measure is sufficiently good, then even if an optimal contract were to require some pooling, the measure of types that are pooled would approach zero.

Proposition 5. *For any $\rho > 0$, there exists a $\bar{c} < 1$ such that for $\cos\theta \geq \bar{c}$, the measure of agents whose evaluations are pooled under the optimal contract is less than ρ.*

The intuition is that when $\cos\theta$ is close to one, the separating contract provides incentives that are already quite efficient, so that even if there were additional efficiency gains from pooling, they would have to be small. By the same token, the effort level in the second period is large and therefore the cost of misallocating effort through pooling is also large. Taken together, these two arguments imply that extensive pooling cannot be optimal when the objective measure is sufficiently good.

No commitment

The assumption that the firm can make the worker's pay contractually contingent on the evaluations is not unrealistic. Subjective evaluation schemes are often well defined in advance and adherence to such schemes might be verifiable. But even if commitment of this sort were not possible, the subjective evaluation scheme characterized above could still work. Without commitment, the setting is formally a signaling game and as such can have multiple equilibria. For the purposes of this analysis, the most interesting among them is a separating Perfect Bayesian Equilibrium (PBE), in which the principal reveals her information truthfully.

To see that such a separating PBE exists, suppose that at the end of period one, the principal can pay the worker a wage s (to which she cannot commit) in addition to giving him an evaluation m. Assume also that $s \geq 0$, i.e., at this stage the agent cannot be forced to transfer money to the principal.[19] Then even without committing to it ex ante, the principal may have an incentive to convey her information by paying more to higher ability workers, as long as the workers interpret this signal correctly. In particular, suppose that upon being paid a wage s, the agent's belief x is given by $x = m$ if $s = s^{*-1}(m)$ for some $m \in [0, 1]$ and by $x = 0$ otherwise, where $s^*(m)$ is as in (6). Then Proposition 3 tells us that if faced with a worker of ability a, the principal prefers the evaluation $m = a$ and wage $s^*(a)$ to any other evaluation $m = a' \in [0, 1]$ and wage $s^*(a')$. The only deviation one therefore needs to worry about is where the principal decides to pay a wage that is not in the range of $s^*(.)$. This, however, can be prevented by setting $D = s^*(0) = 0$ — any deviation from $s^*(a)$ then necessarily involves paying the agent more than $s^*(1)$.[20] Given the specified beliefs, this is dominated by paying $s^*(1)$. Thus, the above beliefs, together with the wage scheme (6) and $D = 0$, support a separating PBE of this signalling game.

[19] In the previous section, it was assumed that all of the wages are paid at the end of the second period, but it would be without loss of generality to allow the part of the wage that only depends on m to be paid at the end of the first period.

[20] Setting $D = 0$ is always feasible: Although in the previous analysis D was lumped together with the agent's base salary and hence determined by his participation constraint, conceptually, these two wage components can be separated. When $D = 0$, the participation constraint determines the agent's base salary.

6. Forced distributions

Some companies, for example GE, Intel, Ford, Goodyear, EDS, and others (Lawler, 2003), adopt forced distributions of subjective evaluations (FDSE), where they commit to a pre-specified distribution of evaluations. This section provides justification for such practice. It also shows that whether an FDSE improves upon the subjective scheme studied in Section 4 depends critically on the quality of the objective measures z_1 and z_2 and on the number of employees. Accordingly, I proceed by exploring optimal FDSE contracts in two alternative settings: In the first one, the firm employs a continuum of workers (Proposition 6); in the second, there is a finite number of workers (Proposition 7). The overall optimal contract is then obtained by comparing the optimal contracts with and without a forced distribution, which is the focus of propositions 8 and 9.

Continuum of workers

Under a forced distribution, the firm's total wage bill associated with the evaluations is always constant, whether the evaluations are truthful or not. Misreporting therefore affects the firm's profit only through the effects it has on the workers' actions.[21]

Truthtelling. The main benefit of an FDSE is that it allows the principal to eliminate the truthtelling constraint (3). To see this, suppose the firm employs a measure one of agents whose abilities are drawn independently from $[0, 1]$ according to the cumulative distribution $H(a)$ with density $h(a)$. Suppose also that the firm pre-commits to an FDSE under which a fraction $h(m)$ of the workers get evaluation m. Then the principal has no incentive to misreport because misreporting does not affect the wage bill, but hurts her second period expected profit by preventing the workers from tailoring their efforts to their abilities.

To make this argument formally, suppose the principal's evaluation strategy upon inferring that

[21] In this respect, an FDSE game is similar to a cheap talk game, and, as is common in cheap talk games, has multiple equilibria, including a babbling equilibrium in which evaluations are completely uninformative.

a worker has ability a is to report $m \in [0,1]$ according to the probability density function $\sigma(m|a)$. An agent with evaluation m then forms a posterior belief $h(a|m)$ about the distribution of his true ability according to

$$h(a|m) = \frac{\sigma(m|a)h(a)}{\int_0^1 \sigma(m|\tau)h(\tau)d\tau} = \frac{\sigma(m|a)h(a)}{h(m)}, \tag{9}$$

where the second equality exploited that the evaluations must adhere to the forced distribution. The worker's expected ability conditional on evaluation m, $x(m)$, is then

$$x(m) = \int_0^1 ah(a|m)da, \tag{10}$$

and his optimal second period efforts $\mathbf{e}_2(m)$ are given by the first order conditions $e_{2k}(m) = \beta(m)x(m)g_k$, $k = 1, 2$.

Ignoring $s(m)$, the firm's second period expected profit from a worker of ability a is therefore

$$\begin{aligned} E_\sigma \pi_2(a) &= \int_0^1 \left[a\mathbf{e}_2{\cdot}\mathbf{f} - \beta(m)a\mathbf{e}_2{\cdot}\mathbf{g}\right]\sigma(m|a)dm \\ &= \int_0^1 a\left\|\mathbf{g}\right\|^2 x(m)\beta(m)\left[b^{SB} - \beta(m)\right]\sigma(m|a)dm, \end{aligned}$$

so that its total expected second period profit is

$$E\pi_2 \equiv E_a E_\sigma \pi_2(a) = \int_0^1 \int_0^1 a\left\|\mathbf{g}\right\|^2 x(m)\beta(m)\left[b^{SB} - \beta(m)\right]\sigma(m|a)h(a)dmda. \tag{11}$$

As will become apparent shortly, $\beta(m)$ can be set constant here w.l.o.g. Thus, let $\beta(m) = b_2$, where $b_2 \leq b^{SB}$ is a constant. Using (9) and (10), $E\pi_2$ can then be written as

$$\begin{aligned} E\pi_2 &= \left\|\mathbf{g}\right\|^2 b_2\left(b^{SB} - b_2\right)\int_0^1 \left[\int_0^1 ah(a|m)da\right]x(m)h(m)dm \\ &= \left\|\mathbf{g}\right\|^2 b_2\left(b^{SB} - b_2\right)E_m\left[x(m)\right]^2. \end{aligned}$$

Now, any improvement in the informativeness of the principal's reporting strategy σ, in particular a switch to truthful reporting, induces a mean-preserving spread of the agents' posteriors x (Marschak and Miyasawa, 1968). This increases the principal's expected second period profit, because $E\pi_2$ is an expectation of a convex function of the posteriors. The principal's profit is therefore maximized under truthful evaluations. That an equilibrium with truthful evaluations indeed exists is shown in the proof to Proposition 6 below.

First period incentives. Observe that the principal's incentives to provide truthful evaluations under the FDSE depend neither on $b_1(m)$ nor on the exact shape of $s(m)$; the only constraint on the contract is that $b_2 \leq b^{SB}$. The principal can therefore choose $s(.)$, b_1, and $b_2 \leq b^{SB}$ so as to optimize the workers' incentives. In the second period, the best she can do is to set $b_2 = b^{SB}$. I will now show that in the first period, it is possible to achieve the efficient vector of efforts $\mathbf{e}_1^{FB}$.

Because $\mathbf{e}_1^{FB}$ does not depend on $\mathbf{g}$, set $b_1(m) = 0$ for all m. Given that b_2 is constant, $\mathbf{e}_1$ then depends solely on $s(m)$. Now, recall that when evaluations are truthful, a worker who provides effort $\hat{\mathbf{e}}_1$ expects his evaluation to be $m = \frac{a\hat{\mathbf{e}}_1 \cdot \mathbf{f}}{\tilde{\mathbf{e}}_1 \cdot \mathbf{f}}$, where $\tilde{\mathbf{e}}_1$ is the principal's conjecture about $\mathbf{e}_1$. The first best outcome in period 1 is therefore obtained by setting $s(m) = m\mathbf{e}_1^{FB} \cdot \mathbf{f}$. The worker's first period expected utility is thus $E_a\left[a\hat{\mathbf{e}}_1 \cdot \mathbf{f}\frac{\mathbf{e}_1^{FB} \cdot \mathbf{f}}{\tilde{\mathbf{e}}_1 \cdot \mathbf{f}} - \Psi(\mathbf{e}_1)\right]$, so that his optimal efforts are given by the first order condition

$$E(a)f_k\frac{\mathbf{e}_1^{FB} \cdot \mathbf{f}}{\tilde{\mathbf{e}}_1 \cdot \mathbf{f}} = \psi'(e_{1k}) = e_{1k} \ , \quad k = 1, 2.$$

The equilibrium requirement $\tilde{\mathbf{e}}_1 = \mathbf{e}_1$ then yields $\mathbf{e}_1 = \mathbf{e}_1^{FB}$ as an equilibrium outcome.

Proposition 6. *Suppose the firm employs a continuum of workers. There exists a forced distribution of subjective evaluations (FDSE) that induces a Perfect Bayesian Equilibrium in which the evaluations truthfully reveal each worker's ability. An optimal contract sets* $b_1 = b_3 = 0$*,*

$b_2 = b^{SB}$, and $s(m) = m\mathbf{e}_1^{FB} \cdot \mathbf{f}$, and achieves the first best outcome in $t = 1$ and the second best outcome in $t = 2$. This contract strictly dominates the optimal contract without FDSE.

Proposition 6 shows that when the firm employs a large number of workers, a forced distribution improves efficiency by ensuring that the subjective pay scheme is incentive compatible. The subjective scheme can therefore be used solely to shape the workers' first period incentives. This has two effects on the optimal contract. First, it allows the firm to provide fully efficient incentives in period 1 by completely removing from the contract the distortive objective measure z_1 and replacing it with undistorted incentives from subjective evaluations. Second, it eliminates the dependence of $s(.)$ on $\beta(.)$, thus freeing $\beta(.)$ to be used solely for the purpose of second period incentives, which improves efficiency in period 2.

Finite number of workers

Now suppose the firm employs $n \geq 2$ workers, where $n < \infty$. In this case, it is not possible for a forced distribution to replicate the true distribution of abilities $H(.)$. Nevertheless, an FDSE again relaxes the principal's truthtelling constraint and, as will be shown shortly, allows for the first best to be achieved in period 1. Hence, $\beta(m)$ will again be optimally set equal to b^{SB}. Also as before, $\mathbf{e}_1 = \mathbf{e}_1^{FB}$ requires $b_1(m) = 0$.

In this case, an FDSE scheme entails (i) n possible evaluations, $m_1 \leq m_2 \leq ... \leq m_n$, (ii) the corresponding salaries $s_j \equiv s(m_j)$, $j = 1, 2, ..., n$, and (iii) a commitment by the firm to assign each evaluation to exactly one worker.[22] Clearly, for n finite, this scheme cannot fully reveal the workers' true abilities. However, arguments similar to those behind Proposition 6 imply that the principal will assess the workers truthfully in the sense that she will assign evaluation m_n to the highest ability worker, m_{n-1} to the second highest, and so on.[23]

[22] This allows for the possibility that multiple workers get the same evaluation. For example, if $m_1 = m_2 = ... = m_n$, then all workers effectively receive the same evaluation. In this particular case, evaluations do not convey any information.

[23] Two or more workers having the same ability is a zero probability event and will be ignored.

To see that one can find a salary scheme $\{s_j\}_{j=1}^n$ that elicits $\mathbf{e}_1^{FB}$, suppose the FDSE entails only two possible salaries, s^H and $s^L < s^H$, and define $\triangle s \equiv s^H - s^L$. Let salary s^L be attached to the first r lowest evaluations $m_1, m_2, ..., m_r$, and salary s^H to the evaluations $m_{r+1}, ..., m_n$. Consider a worker i of ability a and denote by $H_{(r)}(a)$ the c.d.f. of the event that at least r workers other than i have abilities less than a and let $h_{(r)}(a)$ be the corresponding density function.[24] Then if worker i provides effort $\mathbf{e}_1^i$ and anticipates that all the other workers provide the efforts $\mathbf{e}_1^{FB}$, he expects the high salary s^H with probability

$$\Pr\{m_i \geq m_r\} = \begin{cases} \int_0^{1\frac{\mathbf{e}_1^i \cdot \mathbf{f}}{\mathbf{e}_1^{FB} \cdot \mathbf{f}}} \left[1 - H\left(\frac{a\mathbf{e}_1^{FB} \cdot \mathbf{f}}{\mathbf{e}_1^i \cdot \mathbf{f}}\right)\right] h_{(r)} da & \text{for } \mathbf{e}_1^i \cdot \mathbf{f} \leq \mathbf{e}_1^{FB} \cdot \mathbf{f} \\ \int_0^1 \left[1 - H\left(\frac{a\mathbf{e}_1^{FB} \cdot \mathbf{f}}{\mathbf{e}_1^i \cdot \mathbf{f}}\right)\right] h_{(r)} da & \text{for } \mathbf{e}_1^i \cdot \mathbf{f} \geq \mathbf{e}_1^{FB} \cdot \mathbf{f}. \end{cases}$$

Worker i's problem is to maximize $s^L + \triangle s \Pr\{m_i \geq m_r\} - \Psi(\mathbf{e}_1^i)$, which yields

$$e_{1k}^i = \psi'(e_{1k}^i) = \begin{cases} \triangle s \int_0^{1\frac{\mathbf{e}_1^i \cdot \mathbf{f}}{\mathbf{e}_1^{FB} \cdot \mathbf{f}}} a \frac{f_k \mathbf{e}_1^{FB} \cdot \mathbf{f}}{(\mathbf{e}_1^i \cdot \mathbf{f})^2} h\left(\frac{a\mathbf{e}_1^{FB} \cdot \mathbf{f}}{\mathbf{e}_1^i \cdot \mathbf{f}}\right) h(a) da & \text{for } \mathbf{e}_1^i \cdot \mathbf{f} \leq \mathbf{e}_1^{FB} \cdot \mathbf{f} \\ \triangle s \int_0^1 a \frac{f_k \mathbf{e}_1^{FB} \cdot \mathbf{f}}{(\mathbf{e}_1^i \cdot \mathbf{f})^2} h\left(\frac{a\mathbf{e}_1^{FB} \cdot \mathbf{f}}{\mathbf{e}_1^i \cdot \mathbf{f}}\right) h(a) da & \text{for } \mathbf{e}_1^i \cdot \mathbf{f} \geq \mathbf{e}_1^{FB} \cdot \mathbf{f}. \end{cases}$$

Setting $\mathbf{e}_1^i = \mathbf{e}_1^{FB}$, we see that $e_{1k}^i = e_{1k}^{FB}$ if $\triangle s \frac{f_k}{\mathbf{e}_1^{FB} \cdot \mathbf{f}} \int_0^1 a h(a) h_{(r)}(a) da = E(a) f_k$, which is achieved by choosing $\triangle s = \frac{E(a)\mathbf{e}_1^{FB} \cdot \mathbf{f}}{\int_0^1 a h(a) h_{(r)}(a) da}$.

Proposition 7 below summarizes the analysis of the case with a finite number of workers.

Proposition 7. *Suppose the firm employs $n \geq 2$ workers. There exists an FDSE scheme that induces a Perfect Bayesian Equilibrium in which the evaluations are truthful in the sense that the highest ability worker receives the highest evaluation, the second highest ability worker receives the second highest evaluation, and so on. An optimal contract sets $b_1 = b_3 = 0$, $b_2 = b^{SB}$, and entails two salary levels, s^L and $s^H = s^L + \frac{E(a)\mathbf{e}_1^{FB} \cdot \mathbf{f}}{\int_0^1 a h(a) h_{(r)}(a) da}$. This contract*

[24] In other words, $H_{(r)}(a)$ is the cdf of the rth order statistic: $H_{(r)}(a) = \sum_{i=r}^{n-1} \binom{n-1}{i} H^i(a)[1 - H(a)]^{n-1-i}$.

achieves full efficiency in $t = 1$ and provides the second-best level of incentives in $t = 2$.

The above analysis suggests that the main difference between the cases with n workers and a continuum of workers (and, similarly, between the cases with n workers and $n' > n$ workers) is in how precise is the information the workers get from the evaluations. When there is a continuum of workers, each worker learns his exact ability. When the number of workers is finite, the workers' information remains coarse in the second period, because they only learn their rank out of n workers. This coarseness of beliefs is a source of a second period inefficiency, as it prevents the workers from fully tailoring their efforts to their abilities. However, the information content of the evaluations increases with the number of workers, because it is more informative to know how one ranks among $n+1$ workers than to know how one ranks among n workers. In particular, as $n \to \infty$, each worker's estimate of his ability converges to his true ability a. These observations, combined with the last claim in Proposition 6, lead to the next result.

Proposition 8. *The (per worker) efficiency of FDSE increases with the number of workers. Furthermore, for any given $H(.)$ and $\cos\theta$, there exists an $n^* \geq 2$ such that for all $n \geq n^*$, FDSE dominates subjective evaluations without a forced distribution.*

Thus, according to this proposition, subjective schemes with forced distributions are particularly attractive to large firms.

The role of the contractible measures

As shown above, FDSE is always optimal if the firm employs sufficiently many workers, but this leaves open the question whether subjective evaluations without a forced distribution can be optimal when the number of workers is small. Proposition 8 implies that to answer this question, it is enough to consider $n = 2$. Thus, the remainder of this section will concentrate on a setting with two workers.

As already pointed out, the advantage of FDSE is that it allows the firm to achieve full efficiency in $t = 1$ and to set $\beta(m) = b^{SB}$ in $t = 2$. The downside is that the agents' information about their abilities remains coarse in the second period, which distorts almost every agent's effort choice from the level appropriate for his ability. This trade-off leads to Proposition 9 below.

Proposition 9. *When $n = 2$, FDSE is optimal if $\cos\theta \leq c^*$, whereas subjective evaluations without a forced distribution are optimal if $\cos\theta \geq c^{**}$, where $0 < c^* \leq c^{**} < 1$.*

The proposition says that the relative benefits of FDSE depend on the quality of the objective measure z. This result is intuitive. When the objective measure is poor ($\cos\theta \leq c^*$), a contract based solely on this measure provides poor incentives. The additional incentives from subjective evaluations are therefore highly valuable. This favors FDSE, as FDSE induces fully efficient effort in the first period and hence improves efficiency substantially. Moreover, because the second period incentives are severely distorted (as $\cos\theta$ is small), leaving the workers with less precise information about their abilities does not much hurt efficiency in this period. This limits the efficiency loss from adopting FDSE.

In contrast, when the objective measure is good ($\cos\theta \geq c^{**}$), the contract provides relatively efficient incentives in both periods even without subjective pay. The main benefit of subjective evaluations is then in informing the workers about their abilities, which is better achieved through evaluations without a forced distribution. Thus, when the contractible measure is good, subjective evaluations without a forced distribution are optimal.

Empirical implications

As pointed out by Indjejikian and Nanda (2002), firms rarely publicly disclose detailed information about their bonus plans. The amount of empirical evidence on the structure and properties of subjective incentive schemes is therefore quite limited. Nevertheless, the model yields several clear-cut empirical predictions that could be useful in guiding future empirical studies on this topic.

One was already mentioned when discussing the effects of a leniency bias: (i) A leniency bias on the part of the supervisor can improve incentives and efficiency, which is consistent with evidence in Bol (2011). In addition, a comparison of the analyses with and without a forced distribution yields the following empirically relevant implications:

(ii) As noticed in the discussion of Proposition 3, the subjective part of a contract that does not involve a forced distribution should be convex, in order to ensure the principal's truthtelling. In contrast, when the contract involves an FDSE, then a linear subjective scheme can be optimal, as shown in Proposition 6. Under FDSE there may exist multiple optimal schemes, including non-linear ones, but the general point that in the absence of a forced distribution the truthtelling constraint adds convexity to the subjective scheme appears to be a robust and testable prediction.

(iii) The discussion of Proposition 3 also points out that in the absence of an FDSE the subjective bonus scheme should be steeper when the objective measure is of an intermediate quality than when it is either very good or very poor. An FDSE, on the other hand, entails no such relationship between the slope of the subjective scheme and the quality of the objective measure, as can be seen from Proposition 6.

(iv) Large organizations or teams should more frequently implement forced distributions of subjective evaluations than small ones. This prediction follows from Proposition 8 and is consistent with Murphy's (1993) account of the subjective bonus scheme in the pharmaceutical company Merck, according to which a forced distribution was only applicable to groups with more than 100 employees. The prediction is also indirectly supported by the evidence that larger organizations are more likely to adopt formal appraisal systems (Murphy and Cleveland, 1995).

(v) Finally, Proposition 9 implies that firms with poor objective measures should favor forced distributions of subjective evaluations, whereas firms with relatively good objective measures should not use forced distributions.

7. Conclusion

Firms that use subjective performance evaluations typically use them with multiple goals in mind. Economists have traditionally focused on the incentive effects of subjective evaluations, mostly overlooking their other functions. This article brings to forefront the feedback role of evaluations, which appears to be of equal, if not greater, importance to real world firms as their incentive role. In the model, the feedback and the incentive roles of subjective evaluations are complementary in the optimal contract: when both are present, subjective evaluations are feasible where they could not be sustained otherwise.

The feedback from the evaluations improves efficiency by informing workers about their abilities, which allows them to better choose their optimal actions. Because higher ability workers optimally provide more effort, the principal has a motivation to give good evaluations, which makes truthful evaluations possible. The article shows that truthful subjective evaluations are always feasible if there exists some, albeit imperfect, verifiable measure of performance. However, the need to ensure that the evaluations are truthful means that the optimal contract never fully replaces the imperfect objective measure with subjective pay. Instead, subjective and objective pay are intertwined in the optimal contract, and the contract's exact shape depends upon the quality of the objective measure. In particular, the strength of the incentives from subjective pay is limited by the quality of the objective measure — when the objective measure is poor, subjective evaluations can only have weak incentive effects.

The article also explains the benefits and the costs of a forced distribution of evaluations, that is, of the ability to commit to a specific distribution to which the evaluations must adhere. It shows that a forced distribution of subjective evaluations is better than a subjective scheme without a forced distribution when the number of employees is sufficiently large or when the objective performance measure is poor.

Although it expands the view of subjective evaluations beyond that in traditional economic models, the model of this article is far from capturing the variety of purposes for which subjective appraisals are used in practice. Building a more comprehensive economic model of performance evaluations that would incorporate additional reasons real world firms find performance evaluations useful (such as improved job matching or ensuring the employees' ongoing development) could be a fruitful topic for future research.

A. Appendix A: Proofs

Proof of Proposition 1. The principal's period 2 expected revenue from a worker of ability a is $ETR_2 = a\mathbf{e}_2{\cdot}\mathbf{f}$. Using $e_{k2} = \beta(m)mg_k$ from (1) yields $ETR_2 = am\beta(m)\mathbf{g}\cdot\mathbf{f} = am\beta(m)\left\|\mathbf{g}\right\|\left\|\mathbf{f}\right\|\cos\theta$. If $\cos\theta = 0$, then $ETR_2 = 0$, so that truthtelling requires

$$[b_2(q_1) + b_3(q_1)a\mathbf{e}_1{\cdot}\mathbf{g}]\, a\mathbf{e}_2{\cdot}\mathbf{g} + s(q_1) + b_1(q_1)p_1 \leq \left[b_2(q_1') + b_3(q_1')a\mathbf{e}_1{\cdot}\mathbf{g}\right] a\mathbf{e}_2'{\cdot}\mathbf{g} + s(q_1') + b_1(q_1')p_1 \quad \text{(A1)}$$

for all a and a', where $q_1 = a\mathbf{e}_1{\cdot}\mathbf{f}$ and $q_1' = a'\mathbf{e}_1{\cdot}\mathbf{f}$.

Now, for subjective pay to provide incentives, the effect of the agent's effort on his expected pay must be higher (at least for some effort levels) when his pay depends on m than when it does not. Formally, consider two first period effort vectors $\mathbf{e}_1$ and $\mathbf{e}_1'' \leq \mathbf{e}_1$, i.e. $e_{1k}'' \leq e_{1k}$ for $k = 1, 2$, with at least one strict inequality. Let $w(m) = (s(m), b_1(m), b_2(m), b_3(m))$ be a contract that depends on subjective evaluations and let $w'' = (\bar{s}'', \bar{b}_1'', \bar{b}_2'', \bar{b}_3'')$ be a contract where $\bar{s}'', \bar{b}_1'', \bar{b}_2''$, and $\bar{b}_3''$ are all constant, with $q_1'' = a\mathbf{e}_1''{\cdot}\mathbf{f}$, $\bar{s}'' = s(q_1'')$, $\bar{b}_1'' = b_1(q_1'')$, $\bar{b}_2'' = b_2(q_1'')$, and $\bar{b}_3'' = b_3(q_1'')$. The evaluations can have a positive incentive effect only if there exists some $\mathbf{e}_1'' \leq \mathbf{e}_1$ such that if the worker's effort increases from $\mathbf{e}_1''$ to $\mathbf{e}_1$, his expected pay increases more under contract $w(m)$ than under w'' :

$$\begin{aligned} & E_a\left[\left[b_2(q_1)+b_3(q_1)a\mathbf{e}_1\cdot\mathbf{g}\right]a\mathbf{e}_2\cdot\mathbf{g}+s(q_1)+b_1(q_1)p_1\right]-E_a\left[\left[b_2(q_1'')+b_3(q_1'')a\mathbf{e}_1''\cdot\mathbf{g}\right]a\mathbf{e}_2''\cdot\mathbf{g}+s(q_1'')+b_1(q_1'')p_1''\right] \\ > \; & E_a\left[\left[\bar{b}_2+\bar{b}_3a\mathbf{e}_1\cdot\mathbf{g}\right]a\mathbf{e}_2\cdot\mathbf{g}+\bar{s}+\bar{b}_1p_1\right]-E_a\left[\left[\bar{b}_2+\bar{b}_3a\mathbf{e}_1''\cdot\mathbf{g}\right]a\mathbf{e}_2\cdot\mathbf{g}+\bar{s}+\bar{b}_1p_1''\right], \end{aligned}$$

where $p_1''=a\mathbf{e}_1''\cdot\mathbf{g}$ and $e_{k2}''=\left[b_2(q_1'')+b_3(q_1'')a\mathbf{e}_1''\cdot\mathbf{g}\right]ag_k$. Rearranging, this condition yields

$$E_a\left[\left[b_2(q_1)+b_3(q_1)a\mathbf{e}_1\cdot\mathbf{g}\right]a\mathbf{e}_2\cdot\mathbf{g}+s(q_1)+b_1(q_1)p_1\right]>E_a\left[\left[b_2(q_1'')+b_3(q_1'')a\mathbf{e}_1\cdot\mathbf{g}\right]a\mathbf{e}_2''\cdot\mathbf{g}+s(q_1'')+b_1(q_1'')p_1\right],$$

which contradicts (A1). Hence, when $\cos\theta=0$, subjective pay cannot induce effort.

The second claim will be proven by constructing a contract with subjective evaluations that are truthful and improve incentives whenever $\cos\theta>0$. In particular, let $b_1\geq 0$ and $b_2\in(0,b^{SB})$ be independent of m, let $b_3=0$ (so that $\beta=b_2$), and let $s(m)=\frac{1}{2}m^2\left\|\mathbf{g}\right\|^2 b_2\left(b^{SB}-b_2\right)+D$, where D is a constant. Now, plug the expression for $s(m)$ into the principal's second period profit $\pi_2=a\mathbf{e}_2\cdot\mathbf{f}-\beta a\mathbf{e}_2\cdot\mathbf{g}-s(m)$ and use $e_{k2}=\beta m g_k$ to get

$$\pi_2=am\left\|\mathbf{g}\right\|^2 b_2\left(b^{SB}-b_2\right)-\frac{1}{2}m^2\left\|\mathbf{g}\right\|^2 b_2\left(b^{SB}-b_2\right)-D.$$

Maximization with respect to m yields $a\left\|\mathbf{g}\right\|^2 b_2\left(b^{SB}-b_2\right)=m\left\|\mathbf{g}\right\|^2 b_2\left(b^{SB}-b_2\right)$, which demonstrates that this contract induces truthful evaluations.[25] To see that it improves incentives, observe that for any first period effort vector $\hat{\mathbf{e}}_1$, the worker expects evaluation $m(\hat{\mathbf{e}}_1)=\frac{a\hat{\mathbf{e}}_1\cdot\mathbf{f}}{\mathbf{e}_1\cdot\mathbf{f}}$. Hence, $E_a\frac{\partial s(m)}{\partial e_{1k}}=E_a[a\frac{f_k}{\mathbf{e}_1\cdot\mathbf{f}}s'(m)]=b_2\left(b^{SB}-b_2\right)\frac{f_k}{\mathbf{e}_1\cdot\mathbf{f}}\left\|\mathbf{g}\right\|^2 E_a[am(\hat{\mathbf{e}}_1)]>0$. Q.E.D.

Proof of Proposition 2. From (1), the second period efforts are $e_{2k}=a\beta(m)g_k$, whereas the efficient efforts are $e_{2k}^{FB}=af_k$. Thus, $\mathbf{e}_2^*=\mathbf{e}_2^{FB}$ is possible only if $\beta(m)g_1=f_1$ and $\beta(m)g_2=f_2$

[25] The second order condition is satisfied because $\frac{\partial^2\pi_2}{\partial m^2}=-\frac{\left\|\mathbf{g}\right\|^2}{(\mathbf{e}_1\cdot\mathbf{f})^2}b_2\left(\beta^{SB}-b_2\right)<0$.

for all m. This is precluded by the assumption that $\mathbf{f}$ and $\mathbf{g}$ are linearly independent.

Next consider $\mathbf{e}_1$. Because $\cos\theta < 1$, $\mathbf{e}_1 = \mathbf{e}_1^{FB}$ requires that $b_1(m) = 0$ a.e. Suppose that a $\beta(m)$ and $s(m)$ that elicit $\mathbf{e}_1^{FB}$ exist (if not, then we are done) and denote them as $\hat{\beta}(m)$ and $\hat{s}(m)$. Assume first that $\hat{\beta}(m)$ maximizes the expected second period surplus $ETS_2 = E_a[a\mathbf{e}_2^*\cdot\mathbf{f} - \Psi(\mathbf{e}_2^*)]$ subject to (1), so that $\hat{\beta}(m) = b^{SB} = \frac{\mathbf{g}\cdot\mathbf{f}}{\|\mathbf{g}\|^2} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|}\cos\theta$. The truthtelling constraint (3) then becomes

$$
\begin{aligned}
a &\in \arg\max_m a\mathbf{e}_2\cdot\mathbf{f} - \hat{\beta}(m)a\mathbf{e}_2\cdot\mathbf{g} - b_1(m)a\mathbf{e}_1\cdot\mathbf{g} - \hat{s}(m) \\
&= \arg\max_m a\|\mathbf{g}\|^2 m\hat{\beta}(m)\left[b^{SB} - \hat{\beta}(m)\right] - b_1(m)a\mathbf{e}_1\cdot\mathbf{g} - \hat{s}(m) \\
&= \arg\max_m -\hat{s}(m).
\end{aligned}
$$

This can only hold if $\hat{s}(m) = \hat{s}$, where $\hat{s}$ is a constant. Hence, the whole contract is independent of m in this case, so that (2) reduces to

$$\mathbf{e}_1^* \in \arg\max_{\mathbf{e}_1} E_a[\hat{s} + b^{SB}a\mathbf{e}_2\cdot\mathbf{g} - \Psi(\mathbf{e}_2^*)] - \Psi(\mathbf{e}_1).$$

This yields $e_{11}^* = e_{12}^* = 0$, contrary to the assumption that $\mathbf{e}_1 = \mathbf{e}_1^{FB}$.

Thus, if $\mathbf{e}_1 = \mathbf{e}_1^{FB}$ then $\hat{\beta}(m) \neq b^{SB}$, which means that the first period surplus, ETS_1, is maximized with respect to β, but ETS_2 is not. The standard variational argument therefore implies that a small change in β can increase the total surplus, because its positive effect on ETS_2 is of first order magnitude, whereas its negative effect on ETS_1 is of second order magnitude. Consequently, $\hat{\beta}(m)$ cannot be optimal. Thus, it must be $\mathbf{e}_1 \neq \mathbf{e}_1^{FB}$. Q.E.D.

Proof of Lemma 1. Plugging $e_{k2} = \beta(m)mg_k$ into the objective function and into (2) and (3), and using $\mathbf{g}\cdot\mathbf{f} = \|\mathbf{g}\|\,\|\mathbf{f}\|\cos\theta = \|\mathbf{g}\|^2 b^{SB}$, problem (P) can be written as

$$\max_{s(.),b_1(.),\beta(.),b_3(.),\mathbf{e}_1} E_a\left[a\mathbf{e}_1^*\cdot\mathbf{f}-\Psi(\mathbf{e}_1^*)+\beta(a)a^2\left\|\mathbf{g}\right\|^2\left[b^{SB}-\frac{\beta(a)}{2}\right]\right] \tag{A2}$$

subject to $\hat{m}=\frac{a\hat{\mathbf{e}}_1\cdot\mathbf{f}}{\hat{\mathbf{e}}_1\cdot\mathbf{f}}$, $\beta(m)=b_2(m)+b_3(m)a\mathbf{e}_1\cdot\mathbf{g}$, and

$$\mathbf{e}_1 \in \arg\max_{\hat{\mathbf{e}}_1} E_a\left[s(\hat{m})+b_1(\hat{m})a\hat{\mathbf{e}}_1\cdot\mathbf{g}+\frac{1}{2}\beta^2(\hat{m})a^2\left\|\mathbf{g}\right\|^2\right]-\Psi(\hat{\mathbf{e}}_1); \tag{A3}$$

$$a = \arg\max_m a\left\|\mathbf{g}\right\|^2 m\beta(m)\left[b^{SB}-\beta(m)\right]-b_1(m)a\mathbf{e}_1\cdot\mathbf{g}-s(m). \tag{A4}$$

Given piecewise differentiability in m, the first order condition for (A3) is[26]

$$e_{1k}^*=E_a\left[b_1(m)ag_k+\left[s'(m)+b_1'(m)a\mathbf{e}_1\cdot\mathbf{g}\right]\frac{af_k}{\mathbf{e}_1\cdot\mathbf{f}}+a^2\left\|\mathbf{g}\right\|^2\beta(m)\left[b_3(m)ag_k+\beta'(m)\frac{af_k}{\mathbf{e}_1\cdot\mathbf{f}}\right]\right] \tag{A5}$$

except for the points of non-differentiability. Similarly, the first order condition for (A4) is

$$s'(m)+b_1'(m)a\mathbf{e}_1\cdot\mathbf{g}=a\left\|\mathbf{g}\right\|^2\left[\beta(m)\left[b^{SB}-\beta(m)\right]+m\beta'(m)\left[b^{SB}-2\beta(m)\right]\right]. \tag{A6}$$

Substituting (A6) into (A5), imposing truthtelling, $m=a$, and rearranging yields

$$e_{1k}^*=E_a\left[\left[b_1(a)ag_k+\left\|\mathbf{g}\right\|^2\beta(a)b_3(a)a^3g_k+\frac{\left\|\mathbf{g}\right\|^2 f_k}{\mathbf{e}_1\cdot\mathbf{f}}a^2\left[b^{SB}-\beta(a)\right]\left[\beta(a)+a\beta'(a)\right]\right]\right]. \tag{A7}$$

The optimization problem can therefore be written as

$$\max_{s(.),b_1(.),b_2(.),b_3(.),\mathbf{e}_1} E_a\left[a\mathbf{e}_1^*\cdot\mathbf{f}-\Psi(\mathbf{e}_1^*)+\beta(m)a^2\left\|\mathbf{g}\right\|^2\left[b^{SB}-\frac{\beta(m)}{2}\right]\right] \tag{A8}$$

subject to $\beta(m)=b_2(m)+b_3(m)a\mathbf{e}_1\cdot\mathbf{g}$, $\beta'(m)=b_2'(m)+b_3'(m)a\mathbf{e}_1\cdot\mathbf{g}$, and (A7).

[26] In obtaining (A5), recall that the notation $\beta(m)$ suppresses the fact that β depends on $\mathbf{e}_1$ not only indirectly, through m, but also directly.

Note that because neither the objective function in (A8) nor any of its constraints depend on $b_1'(a)$, one can without loss of generality set $b_1(m) = b_1$, where b_1 is a constant such that $b_1 E(a) = E(b_1(a)a)$. Next observe that $b_3(m)$ enters only through the term $\|\mathbf{g}\|^2 \beta(a) b_3(a) a^3 g_k$ in (A7). It is therefore again w.l.o.g. to replace $b_3(a)$ with $\hat{b}_3 = 0$, if b_1 is replaced with $\hat{b}_1 \equiv b_1 + \frac{E_a[\|\mathbf{g}\|^2 \beta(a) b_3(a) a^3]}{E(a)}$ and $b_2(m)$ is replaced with $\hat{b}_2(m)$ such that

$$E_a \left[a \left[b^{SB} - \hat{b}_2(a) \right] \left[\hat{b}_2(a) + a\hat{b}_2'(a) \right] \right] = E_a \left[a \left[b^{SB} - \beta(a) \right] \left[\beta(a) + a\beta'(a) \right] \right].$$

This converts problem (A8) into problem (P') in the text.

Now, by construction, any solution to problem (A8) also solves (A2)-(A4) if it induces truthtelling. One thus only needs to find a salary function $s(m)$ such that the truthtelling constraint (A4) holds. Let $\Phi(m) \equiv \frac{\partial \left[m\beta(m) \left[b^{SB} - \beta(m) \right] \right]}{\partial m}$ and let $s(m)$ be given by $s'(m) = m \|\mathbf{g}\|^2 \Phi(m)$. Then for $b_1 = const$ and $b_3 = 0$, the first order condition for (A4) is

$$a \|\mathbf{g}\|^2 \Phi(m) - s'(m) = \|\mathbf{g}\|^2 \Phi(m)(a - m) = 0,$$

which yields $m = a$. Moreover, $m = a$ is the maximum if $\Phi(m) \geq 0$ for all m, because then $\|\mathbf{g}\|^2 \Phi(m)(a - m) \geq 0$ for all $m < a$ and $\|\mathbf{g}\|^2 \Phi(m)(a - m) \leq 0$ for all $m > a$. On the other hand, if $\Phi(m') < 0$ for some m', then $\frac{\partial [\Phi(m)(a-m)]}{\partial m}|_{a=m'} = -\Phi(m') > 0$, which means that the local second order condition does not hold at m'. To sum up, one can find a salary function $s(m)$ such that (A4) holds if and only if $ab_2(a) \left[b^{SB} - b_2(a) \right]$ is non-decreasing in a. Q.E.D.

Proof of Proposition 3. Appendix B shows that $b_1^* > 0$ (see the analysis of Problem (P2)), that $b_2^* = \kappa b^{SB}$, where $\kappa \equiv \frac{1-2\lambda}{1-\lambda}$ (see (B16)), and that $\lambda \in (0, \frac{1}{3})$, which implies $\kappa \in (\frac{1}{2}, 1)$ (see the last paragraph in the appendix). To show that $s^*(m) = \frac{1}{2} m^2 \|\mathbf{g}\|^2 b_2^* \left(b^{SB} - b_2^* \right) + D$ induces truthtelling, rewrite constraint (3) as in the proof of Lemma 1 to get

$$a \in \arg\max_{m} a \left\|\mathbf{g}\right\|^2 m b_2^* \left(b^{SB} - b_2^*\right) - b_1^* a \mathbf{e}_1 \cdot \mathbf{g} - s^*(m). \tag{A9}$$

Plugging $s^*(m) = \frac{1}{2}m^2 \left\|\mathbf{g}\right\|^2 b_2^* \left(b^{SB} - b_2^*\right) + D$ into (A9), the objective function becomes

$$\left\|\mathbf{g}\right\|^2 b_2^* \left(b^{SB} - b_2^*\right) \left(am - \frac{m^2}{2}\right) - D - b_1^* a \mathbf{e}_1 \cdot \mathbf{g}. \tag{A10}$$

When $a < 1$, Proposition 3 says that $0 < b_2^* < b^{SB}$, which means that (A10) is strictly maximized at $m = a$. When $a = 1$, then $b_2^* = b^{SB}$, so that $m = 1$ is weakly optimal. Q.E.D.

Proof of Proposition 4. For b_2 independent of a, problem (P1) in Appendix B becomes

$$\max_{b_2} A b_2 \left(b^{SB} - \frac{b_2}{2}\right) \qquad \text{subject to} \qquad b_2 \left(b^{SB} - b_2\right) = C/A,$$

where $A \equiv \int_0^1 a^2 h(a) da$. Now, the constraint only admits two values of b_2 and it is straightforward to verify that the larger one, $b_2^* = \frac{b^{SB} + \sqrt{(b^{SB})^2 - 4C/A}}{2}$, solves the overall problem. We can thus write $b_2^* = \kappa b^{SB}$, where $\kappa \equiv \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{4C}{A(b^{SB})^2}} \in (\frac{1}{2}, 1)$.[27] The expression for $s^*(m)$ is established the same way as in the proof of Proposition 3. Q.E.D.

Proof of Proposition 5. Suppose there is an interval $[a_1, a_2] \subseteq [0, 1]$ in which the principal sends the same message to all agents from $[a_1, a_2]$. (A proof for multiple pooling regions would follow similar steps.) The possible benefit of such a pooling contract is that it might improve the strength of the agent's incentives; the cost is that in the second period the agent cannot tailor his effort to his exact ability. Start with the second period cost. The total second period surplus from employing

[27] Obviously, the square root is a real number only if C is not too large. But C is endogenous in the overall optimization problem, so this constraint is ensured by an appropriate choice of the first period bonus b_1 (taken as given here).

the agents in the interval $[a_1, a_2]$ is given by

$$TS_2(a_1, a_2) = \int_{a_1}^{a_2} (a\mathbf{e}_2.\mathbf{f} - \Psi(\mathbf{e}_2))\, dH(a),$$

where $e^*_{k2} = \beta(m)x(m)g_k$, $k = 1, 2$. Under full separation, $x(m) = m\frac{\tilde{\mathbf{e}}_1\cdot\mathbf{f}}{\hat{\mathbf{e}}_1\cdot\mathbf{f}}$ and $\beta(m) = b^*_2(m)$ as given by 1. Under pooling, for the evaluation $\hat{m}$ that indicates that goes with $a \in [a_1, a_2]$, the worker's belief is $\hat{a} \equiv E(a|a_1\frac{\tilde{\mathbf{e}}_1\cdot\mathbf{f}}{\hat{\mathbf{e}}_1\cdot\mathbf{f}} \leq a \leq a_2\frac{\tilde{\mathbf{e}}_1\cdot\mathbf{f}}{\hat{\mathbf{e}}_1\cdot\mathbf{f}})$ and $\beta(\hat{m}) = \hat{\beta}$, where $\hat{\beta}$ is a constant. The second period cost from pooling the agents in $[a_1, a_2]$ is therefore

$$\triangle TS_2(a_1, a_2) = \int_{a_1}^{a_2} \left[a^2 b^*_2(m)\mathbf{g}.\mathbf{f} - \frac{1}{2}\left(b^*_2(m)\right)^2 a^2 \|\mathbf{g}\|^2\right] dH(a) - \int_{a_1}^{a_2} \left(a\hat{a}\hat{\beta}\mathbf{g}.\mathbf{f} - \frac{1}{2}\hat{\beta}^2\hat{a}^2 \|\mathbf{g}\|^2\right) dH(a).$$

Let ρ be the measure of pooled agents under the optimal pooling contract, i.e., $\rho \equiv \int_{a_1}^{a_2} dH(a)$. Using $\lim_{\cos\theta\to 1} b^*_2(m) = b^{FB} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|}$ and $\hat{a} = \frac{1}{\rho}\int_{a_1}^{a_2} a dH(a)$ yields

$$\begin{aligned}
\lim_{\cos\theta\to 1} \triangle TS_2(a_1, a_2) &= \frac{1}{2}\|\mathbf{f}\|^2 \int_{a_1}^{a_2} a^2 dH(a) - \left[\hat{a}\hat{\beta}\|\mathbf{g}\|\|\mathbf{f}\| \int_{a_1}^{a_2} a dH(a) - \frac{1}{2}\hat{\beta}^2\hat{a}^2\|\mathbf{g}\|^2\rho\right] \\
&= \frac{1}{2}\|\mathbf{f}\|^2 \int_{a_1}^{a_2} a^2 dH(a) - \hat{a}^2\rho\|\mathbf{g}\|^2\hat{\beta}\left(b^{FB} - \frac{\hat{\beta}}{2}\right) \\
&\geq \frac{1}{2}\rho\|\mathbf{f}\|^2\left[\frac{1}{\rho}\int_{a_1}^{a_2} a^2 dH(a) - \hat{a}^2\right] = \frac{1}{2}\rho\|\mathbf{f}\|^2 Var(a|a_1 \leq a \leq a_2),
\end{aligned}$$

where the inequality follows from $\hat{\beta}\left(b^{FB} - \frac{\hat{\beta}}{2}\right) \leq \frac{1}{2}\left(b^{FB}\right)^2$.

Now, $\lim_{\cos\theta\to 1} b^*_2 = b^{FB}$ implies that the total surplus from the separating contract converges to the first best surplus as $\cos\theta \to 1$, which means that the benefit from improving the agents' incentives through pooling converges to zero. Consequently, $\lim_{\cos\theta\to 1}\triangle TS_2(a_1, a_2)$ must also be zero. This in turn requires $\lim_{\cos\theta\to 1}\rho = 0$, which proves the claim. Q.E.D.

Proof of Proposition 6. The only two claims that do not immediately follow from the analysis in the text are that (i) there exists a PBE with truthful evaluations and (ii) the optimal contract with FDSE strictly dominates all contracts without FDSE. Claim (ii) follows from the fact, established in Proposition 2, that the optimal contract in the absence of FDSE entails $\mathbf{e}_1^* \neq \mathbf{e}_1^{FB}$ and $\mathbf{e}_2^* \neq \mathbf{e}_2^{FB}$.

To see that (i) holds, suppose the worker believes that the evaluations are truthful. Then $x(m) = m$ and, from (11), the firm's expected second period profit (again ignoring $s(m)$) is $E\pi_2 = \|\mathbf{g}\|^2 b_2 \left(b^{SB} - b_2\right) \int_0^1 amh(a)da$. Truthtelling is a PBE if $m = a$ for all a maximizes $E\pi_2$ subject to the following constraint implied by the forced distribution:

$$\int_0^1 (m^2 - a^2)h(a)da = 0. \tag{A11}$$

This is an isoperimetric optimal control problem with Hamiltonian $H = amh + \omega(m^2 - a^2)h$, where ω is the multiplier (a constant) for the state variable m. By Pontryiagin's maximum principle, the optimum is given by $H_m = ah + 2\omega mh = 0$ and by (A11). Restricting attention to $m \in [0, 1]$, this yields $\omega = -\frac{1}{2}$ and $m = a$. Q.E.D.

Proof of Proposition 9. Denote the two workers as A and B and consider first the FDSE contract. With two workers, there are two possible evaluations, m_L and $m_H > m_L$. Suppose, without loss of generality, that worker A got the evaluation m_H and worker B the evaluation m_L. The expected ability of worker A is then $x_H \equiv x(m_H) = E(a_A|a_A > a_B)$ and the expected ability of worker B is $x_L \equiv x(m_L) = E(a_B|a_A > a_B)$, with $x_H > x_L$.

According to Proposition 7, the optimal contract under FDSE achieves the first best in $t = 1$ and sets $\beta(m) = b^{SB}$ in $t = 2$. Thus, in $t = 1$, both workers provide the efficient efforts $\mathbf{e}_1^{FB} = E(a)\mathbf{f}$, whereas in $t = 2$ the worker who got the evaluation m_i, $i = L, H$, chooses his effort in task $k = 1, 2$ according to $e_{2k}^i = b^{SB} x_i g_k$, so that the vector of his efforts is $\mathbf{e}_2^i = b^{SB} x_i \mathbf{g}$. The expected total surplus under FDSE, TS^{FDSE}, is therefore given by

$$\begin{aligned} TS^{FDSE} &= 2E_a[a\mathbf{e}_1^{FB}\cdot\mathbf{f} - \Psi(\mathbf{e}_1^{FB})] + x_H\mathbf{e}_2^H\cdot\mathbf{f} + x_L\mathbf{e}_2^L\cdot\mathbf{f} - \Psi(\mathbf{e}_2^H) - \Psi(\mathbf{e}_2^L) \\ &= 2E_a\left[aE(a)\mathbf{f}\cdot\mathbf{f} - \frac{1}{2}[E(a)]^2\,\mathbf{f}\cdot\mathbf{f}\right] + b^{SB}\left(x_H^2 + x_L^2\right)\mathbf{g}\cdot\mathbf{f} - \left[b^{SB}\right]^2\frac{x_H^2 + x_L^2}{2}\mathbf{g}\cdot\mathbf{g} \\ &= [E(a)]^2\,\|\mathbf{f}\|^2 + b^{SB}\left(x_H^2 + x_L^2\right)\|\mathbf{f}\|\,\|\mathbf{g}\|\cos\theta - \left[b^{SB}\right]^2\frac{x_H^2 + x_L^2}{2}\,\|\mathbf{g}\|^2 \\ &= [E(a)]^2\,\|\mathbf{f}\|^2 + \frac{x_H^2 + x_L^2}{2}\,\|\mathbf{f}\|^2\cos^2\theta, \end{aligned}$$

where the last equality used $b^{SB} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|}\cos\theta$. Thus, we have $\lim_{\cos\theta\to 0} TS^{FDSE} = [E(a)]^2\,\|\mathbf{f}\|^2$.

Without FDSE, the evaluations reveal the workers' precise abilities, so that $e_{2k}^* = ab^{SB}g_k$, but the first period efforts are inefficient, $\mathbf{e}_1^* \neq \mathbf{e}_1^{FB}$. In this case, the expected total surplus, TS^0, is

$$\begin{aligned} TS^0 &= 2E_a[a\mathbf{e}_1^*\cdot\mathbf{f} - \Psi(\mathbf{e}_1^*) + a\mathbf{e}_2^*\cdot\mathbf{f} - \Psi(\mathbf{e}_2^*)] \\ &= 2E_a[a\mathbf{e}_1^*\cdot\mathbf{f} - \Psi(\mathbf{e}_1^*) + b^{SB}a^2\mathbf{g}\cdot\mathbf{f} - \frac{1}{2}\left[b^{SB}\right]^2 a^2\mathbf{g}\cdot\mathbf{g}] \\ &= 2E_a[a\mathbf{e}_1^*\cdot\mathbf{f} - \Psi(\mathbf{e}_1^*)] + E(a^2)\,\|\mathbf{f}\|^2\cos^2\theta, \end{aligned}$$

where the last equality again used $b^{SB} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|}\cos\theta$.

To prove that there exists a $c^* \in (0,1)$ such that for $\cos\theta \leq c^*$ a contract with FDSE is better than one without FDSE, it is enough to show that $\lim_{\cos\theta\to 0} TS^{FDSE} > \lim_{\cos\theta\to 0} TS^0$, or

$$[E(a)]^2\,\|\mathbf{f}\|^2 > \lim_{\cos\theta\to 0} 2E_a[a\mathbf{e}_1^*\cdot\mathbf{f} - \Psi(\mathbf{e}_1^*)]. \tag{A12}$$

But the expression $2E_a[a\mathbf{e}_1\cdot\mathbf{f} - \Psi(\mathbf{e}_1)]$ on the RHS of (A12) is the first period expected surplus, which by definition is maximized at $\mathbf{e}_1 = \mathbf{e}_1^{FB}$, when its value is equal to $[E(a)]^2\,\|\mathbf{f}\|^2$. Thus, a weak inequality version of (A12) clearly holds.

To see that (A12) holds strictly, refer to the analysis in Appendix B. In particular, $\lim_{\cos\theta\to 0} b^{SB} =$

0 implies that both $\delta^*(a)$, given by (B16), and $\delta^{*\prime}(a)$ converge to zero as $\cos\theta \to 0$. Consequently, the first order condition (B1) converges to $e_{1k} = E(a)b_1 g_k$, $k = 1, 2$. That is, in the limit, if the worker has any incentive to provide first period efforts, it comes solely from the objective measure. But the best such objective contract (i.e., the one that maximizes expected surplus) is the one with $b_1 = b^{SB}$, under which efforts are given by $e_{1k}^{SB} = E(a)b^{SB}g_k$. In other words, the RHS of (A12) is bounded above by $\lim_{\cos\theta\to 0} 2E_a[a\mathbf{e}_1^{SB}\cdot\mathbf{f} - \Psi(\mathbf{e}_1^{SB})]$. But given that $\lim_{\cos\theta\to 0} e_{1k}^{SB} = \lim_{\cos\theta\to 0} E(a)b^{SB}g_k = 0$ for $k = 1, 2$, it must be that $\lim_{\cos\theta\to 0} 2E_a[a\mathbf{e}_1^{SB}\cdot\mathbf{f} - \Psi(\mathbf{e}_1^{SB})] = 0$. This in turn implies that the RHS of (A12) is zero. Thus, $\lim_{\cos\theta\to 0} TS^0 = 0 < \lim_{\cos\theta\to 0} TS^{FDSE}$, which implies that there exists a $c^* > 0$ such that $TS^0 < TS^{FDSE}$ for all $\cos\theta \leq c^*$, as claimed.

To obtain the claim that there exists a $c^{**} \in [c^*, 1)$ such that for $\cos\theta \geq c^*$ any FDSE contract is dominated by a scheme without FDSE, let $\cos\theta \to 1$. Then $\lim_{\cos\theta\to 1} b^{SB} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|}$, so that

$$\lim_{\cos\theta\to 1} TS^{FDSE} = [E(a)]^2 \|\mathbf{f}\|^2 + \frac{x_H^2 + x_L^2}{2} \|\mathbf{f}\|^2 .$$

Turning to TS^0, this surplus cannot be less under the optimal contract than under the contract that (i) is independent of m (and therefore induces truthful evaluations) and (ii) sets both the first period and the second period bonuses equal to b^{SB}. But given that $\lim_{\cos\theta\to 1} b^{SB} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|} = b^{FB}$, the total surplus under this alternative contract must converge to the first best surplus $2E_a[a\mathbf{e}_1^{FB}\cdot\mathbf{f} - \Psi(\mathbf{e}_1^{FB}) + a\mathbf{e}_2^{FB}\cdot\mathbf{f} - \Psi(\mathbf{e}_2^{FB})]$. Using $e_{1k}^{FB} = E(a)f_k$ and $e_{2k}^{FB}(a) = af_k$, this implies

$$\begin{aligned}\lim_{\cos\theta\to 1} TS^0 &\geq 2E_a[a\mathbf{e}_1^{FB}\cdot\mathbf{f} - \Psi(\mathbf{e}_1^{FB}) + a\mathbf{e}_2^{FB}\cdot\mathbf{f} - \Psi(\mathbf{e}_2^{FB})] \\ &= [E(a)]^2 \|\mathbf{f}\|^2 + E(a^2) \|\mathbf{f}\|^2 .\end{aligned}$$

Now, the true distribution of a is a mean-preserving spread of the beliefs $x(m_H)$ and $x(m_L)$; it therefore must be $E(a^2) > \frac{x_H^2 + x_L^2}{2}$, which yields $\lim_{\cos\theta\to 1} TS^0 > \lim_{\cos\theta\to 1} TS^{FDSE}$. Consequently, there exists a $c^{**} < 1$ such that $TS^0 > TS^{FDSE}$ for all $\cos\theta \geq c^{**}$. Q.E.D.

B. Appendix B: Analysis of problem (P')

It will prove useful to restate the problem in terms of $\delta(a) \equiv ab_2(a)$:

$$\text{(P'):} \qquad \max_{b_1,\delta(m),\mathbf{e}_1} \int_0^1 \left[a\mathbf{e}_1 \cdot \mathbf{f} - \Psi(\mathbf{e}_1) + \|\mathbf{g}\|^2 \delta(a) \left[ab^{SB} - \frac{\delta(a)}{2}\right]\right] da$$

subject to $\delta(a) \geq 0$, $\delta(0) = 0$, and

$$e_{1k} = \int_0^1 \left[ab_1 g_k + \frac{\|\mathbf{g}\|^2 f_k}{\mathbf{e}_1 \cdot \mathbf{f}} a \left[ab^{SB} - \delta(a)\right] \delta'(a)\right] da, \quad k = 1, 2. \tag{B1}$$

Constraint (B1) can be rearranged as follows:

$$\int_0^1 a \left[ab^{SB} - \delta(a)\right] \delta'(a) da = \frac{\mathbf{e}_1 \cdot \mathbf{f}}{\|\mathbf{g}\|^2} \frac{[e_{1k} - E(a) b_1 g_k]}{f_k}, \quad k = 1, 2. \tag{B2}$$

Recalling that $b_3(m) = 0$, so that $\delta(a) = ab_2(a)$ is independent of $\mathbf{e}_1$, (B2) shows that problem (P') is separable into two self-contained problems: (P1) Optimization over $\delta(m)$, taking b_1 and $\mathbf{e}_1$ as given, and (P2) optimization over b_1 and $\mathbf{e}_1$, taking into account the effect on $\delta(m)$.

Problem (P1): Optimization with respect to $\delta(m)$.

Step 1. Problem setup. Note that in the context of Problem (P1), (B2) is just a single constraint: Because the L.H.S. of (B2) does not depend on k, the R.H.S. cannot depend on k either, i.e. it must be $\frac{\mathbf{e}_1 \cdot \mathbf{f}}{\|\mathbf{g}\|^2} \frac{[e_{1k} - E(a) b_1 g_k]}{f_k} = C$ for $k = 1, 2$, where C is a constant. The choice of C is analyzed in Problem (P2) below; here, C is treated as exogenous. Thus, (ignoring the constant $\|\mathbf{g}\|^2$) the problem of optimizing with respect to $\delta(m)$ can be stated as

$$\text{(P1):} \qquad \max_{\delta(a)} \int_0^1 \delta(a) \left[ab^{SB} - \frac{\delta(a)}{2} \right] da$$

$$\text{subject to} \qquad \int_0^1 a \left[ab^{SB} - \delta(a) \right] \delta'(a) da = C. \tag{B3}$$

It will be proven in Problem (P2) and taken here as given that $C > 0$.

Program (P1) is an isoperimetric optimal control problem, i.e., an optimal control problem with an integral constraint. To formulate it as a proper optimal control problem, define a new control variable $u(a) = \delta'(a)$ and a new state variable $y(a) = \int_0^a t \left[tb^{SB} - \delta(t) \right] u(t)dt$. This transforms (P1) to a problem with a control variable u and state variables y and δ:

$$\text{(P1'):} \qquad \max_{\delta, y, u} \int_0^1 \delta(a) \left[ab^{SB} - \frac{\delta(a)}{2} \right] da$$

subject to

$$y'(a) = a \left[ab^{SB} - \delta(a) \right] u(a); \tag{B4}$$

$$\delta'(a) = u(a); \tag{B5}$$

$$y(0) = 0; \quad y(1) = C; \tag{B6}$$

$$\delta(a) \geq 0; \quad \delta(0) = 0. \tag{B7}$$

Step 2. Necessary conditions. Let $\lambda(a)$ and $\mu(a)$ be the multiplier functions that go with y and δ respectively and $\eta(a)$ the multiplier that goes with the inequality in (B7). The generalized Hamiltonian for this problem is then

$$H(a, y, \delta, u, \lambda, \mu) = \delta \left(ab^{SB} - \frac{\delta}{2} \right) + \lambda a \left(ab^{SB} - \delta \right) u + \mu u + \eta \delta.$$

Pontryiagin's maximum principle says that any solution to (P1'), denoted by $\delta^*(a)$, $y^*(a)$, $u^*(a)$, $\lambda^*(a)$, $\mu^*(a)$, must satisfy (B4)-B(7), plus

$$\begin{aligned}
u &= \arg\max H = \arg\max \delta\left(ab^{SB} - \frac{\delta}{2}\right) + \lambda a\left(ab^{SB} - \delta\right)u + \mu u + \eta\delta && \text{(B8)}\\
\lambda' &= -H_y = 0 && \text{(B9)}\\
\mu' &= -H_\delta = -\left(ab^{SB} - \delta\right) + \lambda a u - \eta && \text{(B10)}\\
\eta\delta &= 0 && \text{(B11)}
\end{aligned}$$

and the transversality condition $\mu(1) = 0.$ (B12)

Given that H is affine in u, the above is a singular control problem with an unbounded control. This suggests that the solution entails a singular arc on some interval $I \subset [0,1]$. Along this arc, the solution must lie on the singular surface defined by $H_u = 0$, $\frac{d}{da}H_u = 0$, ..., and $\frac{d^r}{da^r}H_u = 0$, where r is the order of the singular arc, i.e., the smallest positive integer r such that $\frac{\partial}{\partial u}\left(\frac{d^r}{da^r}H_u\right) \neq 0$ (see, e.g. Chachuat, 2007). Noting that (B9) implies that λ is a constant, we have

$$H_u = \lambda a\left(ab^{SB} - \delta\right) + \mu = 0. \tag{B13}$$

Using (B5) and (B10), this yields

$$\begin{aligned}
\frac{d}{da}(H_u) &= \lambda\left(2ab^{SB} - \delta - a\delta'\right) + \mu'\\
&= \lambda\left(2ab^{SB} - \delta - au\right) - \left(ab^{SB} - \delta\right) + \lambda a u - \eta\\
&= \lambda\left(2ab^{SB} - \delta\right) - \left(ab^{SB} - \delta\right) - \eta = 0, && \text{(B14)}
\end{aligned}$$

which does not depend on u. Next,

$$\frac{d^2}{da^2}(H_u) = \lambda\left(2b^{SB} - u\right) - b^{SB} + u = 0, \tag{B15}$$

so that $\frac{\partial}{\partial u}\left(\frac{d^2}{da^2}H_u\right) = 1 - \lambda$. This is different from 0 as long as $\lambda \neq 1$, which will be confirmed later. The singular arc is thus given by (B13)-(B15).

Solving for δ from (B14) then yields $\delta^*(a) = ab^{SB}\frac{1-2\lambda}{1-\lambda} + \frac{\eta}{1-\lambda}$. Combined with constraint (B11), this means that

$$\delta^*(a) = \max\left\{0, ab^{SB}\frac{1-2\lambda}{1-\lambda}\right\}. \tag{B16}$$

The optimal λ^* is then found by plugging (B16) into (B3) and solving for λ. It will be shown later that a solution to (B3) exists iff $C \in [0, C^{\max}]$, where $C^{\max} > 0$, and that $\lambda^* \in \left(0, \frac{1}{3}\right)$.

Step 3. End points. Equation (B16) satisfies the requirement that $\delta(0) = 0$, but not the transversality condition (B12): Substituting (B12) to (B13) and using the fact that u is unbounded implies that $\delta(1) = b^{SB}$. This, however, is not compatible with (B16) and $\lambda^* \in \left(0, \frac{1}{3}\right)$. Consequently, the optimal solution has an impulse at $a = 1$: The optimal δ is given by (B16) for $a \in I = [0, 1)$ and is then transported via an impulse to $\delta(1) = b^{SB}$ at $a = 1$.[28]

Step 4. Legendre-Clebsch condition. The generalized Legendre-Clebsch condition for the above singular arc to be a maximum requires that $\frac{\partial}{\partial u}\left(\frac{d^2}{da^2}H_u\right) > 0$, or $\lambda < 1$, which holds because $\lambda^* \in \left(0, \frac{1}{3}\right)$.

Step 5. The condition imposed by Lemma 1. Lemma 1 says that for the solution to problem (P1) to be a part of the solution to the original problem (P), $ab_2(a)\left[b^{SB} - b_2(a)\right]$ must be non-decreasing in a. Using (B16) and $\delta(a) = ab_2(a)$, we have that b_2^* is constant in a almost everywhere.

Problem (P2): Optimization with respect to e_1, b_1, and C.

Step 1. Problem setup. Let $V(\delta^*, \lambda^*, C)$ be the principal's optimal value function from problem

[28] On impulses in singular optimal control problems see e.g. Bryson and Ho (1975).

(P1), i.e., $V(\delta^*, \lambda^*, C) \equiv \int_0^1 \delta^*(a) \left[ab^{SB} - \frac{\delta^*(a)}{2}\right] da$, and denote by π her total expected profit over the two periods. Problem (P2) can then be written as

$$\text{(P2):} \qquad \max_{b_1, \mathbf{e}_1, C} \pi = \max_{b_1, \mathbf{e}_1, C} E(a)\mathbf{e}_1 \cdot \mathbf{f} - \Psi(\mathbf{e}_1) + \|\mathbf{g}\|^2 V(\delta^*, \lambda^*, C)$$

$$\text{subject to} \qquad \begin{aligned} e_{1k} &= E(a) b_1 g_k + \frac{C \|\mathbf{g}\|^2}{\mathbf{e}_1 \cdot \mathbf{f}} f_k, \quad k = 1, 2; \\ C &\geq 0; \end{aligned} \tag{B17}$$

subject to $\delta(a)$ being determined by (B3) and B(6); and subject to C being feasible (i.e., such that the set of $\delta(a)$ that satisfy (B3) is non-empty). It will be shown in the last section of this appendix that the feasibility constraint for C can be expressed, for some $C^{\max} > 0$, as

$$C \leq C^{\max}. \tag{B18}$$

Step 2. FOC. The dynamic Envelope Theorem for the fixed endpoint class of optimal control problems (e.g., Theorem 9.1 in Caputo, 2005) implies $\frac{\partial V(\delta^*, \lambda^*, C)}{\partial C} = -\lambda^*$. Let μ_C be the Lagrange multiplier associated with constraint (B18). The first order conditions for (P2) are then

$$\begin{aligned} \frac{\partial \pi}{\partial C} &= \sum_{k=1}^{2} \frac{\partial e_{1k}}{\partial C} \left[E(a) f_k - e_{1k}\right] - \lambda^* \|\mathbf{g}\|^2 + \mu_C = 0, \\ \frac{\partial \pi}{\partial b_1} &= \sum_{k=1}^{2} \frac{\partial e_{1k}}{\partial b_1} \left[E(a) f_k - e_{1k}\right] = 0, \quad \text{and} \\ \left(C^{\max} - C\right) \mu_C &= 0, \quad \mu_C \geq 0, \end{aligned} \tag{B19}$$

where, from (B17), $\dfrac{\partial e_{1k}}{\partial C} = \dfrac{\|\mathbf{g}\|^2 f_k}{\mathbf{e}_1 \cdot \mathbf{f} + \|\mathbf{g}\|^2 \frac{C f_k^2}{\mathbf{e}_1 \cdot \mathbf{f}}} > 0$ and $\dfrac{\partial e_{1k}}{\partial b_1} = \dfrac{E(a) g_k \mathbf{e}_1 \cdot \mathbf{f}}{\mathbf{e}_1 \cdot \mathbf{f} + \|\mathbf{g}\|^2 \frac{C f_k^2}{\mathbf{e}_1 \cdot \mathbf{f}}} > 0.$

Step 3. $C^* > 0$. To see this, suppose $C = 0$. Then $\frac{\partial e_{1k}}{\partial C} = \frac{\|\mathbf{g}\|^2 f_k}{\mathbf{e}_1 \cdot \mathbf{f}}$, $\frac{\partial e_{1k}}{\partial b_1} = E(a) g_k$, $e_{1k} = E(a) b_1 g_k$, and (from (B3) and (B16)) $\lambda^* = 0$, so that (B19) yields $b_1 = b^{SB} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|} \cos\theta$ and $\frac{\partial \pi}{\partial C}$ becomes

$$\begin{aligned}
\frac{\partial \pi}{\partial C}|_{C=0} &= E(a) \frac{\|\mathbf{g}\|^2}{\mathbf{e}_1 \cdot \mathbf{f}} \sum_{k=1}^{2} \left[f_k^2 - b^{SB} f_k g_k \right] + \mu_C \\
&= E(a) \frac{\|\mathbf{g}\|^2}{\mathbf{e}_1 \cdot \mathbf{f}} \left[\|\mathbf{f}\|^2 - b^{SB} \|\mathbf{f}\| \|\mathbf{g}\| \cos\theta \right] + \mu_C \\
&= E(a) \frac{\|\mathbf{g}\|^2 \|\mathbf{f}\|^2}{\mathbf{e}_1 \cdot \mathbf{f}} \left(1 - \cos^2\theta \right) + \mu_C > 0.
\end{aligned}$$

Hence, it must be $C^* > 0$.

Step 4. $C^* < \frac{E(a)\mathbf{e}_1 \cdot \mathbf{f}}{\|\mathbf{g}\|^2}$. Suppose $C \geq \frac{E(a)\mathbf{e}_1 \cdot \mathbf{f}}{\|\mathbf{g}\|^2}$. Then (B17) implies $e_{1k} \geq E(a) b_1 g_k + E(a) f_k$, $k = 1, 2$. Plugging this to $\frac{\partial \pi}{\partial C}$ in (21) yields $\frac{\partial \pi}{\partial C} \leq -\sum_{k=1}^{2} \frac{\partial e_{1k}}{\partial C} E(a) b_1 g_k - \lambda^* \|\mathbf{g}\|^2 + \mu_C$. Now suppose for the moment that (P2) is not constrained by (B18). Then $\mu_C = 0$ and the above implies $\frac{\partial \pi}{\partial C} < 0$ for all $C \geq \frac{E(a)\mathbf{e}_1 \cdot \mathbf{f}}{\|\mathbf{g}\|^2}$, where the inequality follows from $\lambda^* > 0$ for $C > 0$, established in the next section. Therefore, it must be $C^* < \frac{E(a)\mathbf{e}_1 \cdot \mathbf{f}}{\|\mathbf{g}\|^2}$ if C is unconstrained and hence also if C is constrained by (B18).

Step 5. $b_1^* > 0$. Suppose $b_1 = 0$. Because $C < \frac{E(a)\mathbf{e}_1 \cdot \mathbf{f}}{\|\mathbf{g}\|^2}$, (B17) then implies $e_{1k} < E(a) f_k$, $k = 1, 2$, so that, from (B19), $\frac{\partial \pi}{\partial b_1}|_{b_1=0} > 0$. Hence, it must be $b_1^* > 0$.

Technical details regarding λ^* and constraint (B18)

Recalling that $\delta^*(a)$ is given by (B16), define

$$V(\lambda) \equiv \int_0^1 \delta^*(a) \left[a b^{SB} - \frac{\delta^*(a)}{2} \right] da, \text{ and} \tag{B20}$$

$$Z(\lambda) \equiv \int_0^1 a \left[a b^{SB} - \delta^*(a) \right] \delta^{*\prime}(a) da. \tag{B21}$$

That is, $V(\lambda)$ is the principal's optimal value function from problem (P1) as a function of λ, and

$Z(\lambda)$ is the L.H.S. of constraint (B3) evaluated at $\delta^*(a)$. Also, note that

$$\delta^*(a) \;=\; 0 \text{ if } \lambda \in [\frac{1}{2}, 1) \tag{B22a}$$

$$\delta^*(a) \;>\; 0 \text{ otherwise.} \tag{B22b}$$

Step 1. Shape of $V(\lambda)$. Let $\hat{V}(\lambda) \equiv \delta^*(a)\left[ab^{SB} - \frac{\delta^*(a)}{2}\right]$. By (B22), $\hat{V}(\lambda) = 0$ for $\lambda \in [\frac{1}{2}, 1)$, and hence also $V(\lambda) = 0$. Next, suppose $\lambda \notin [\frac{1}{2}, 1]$. Substituting (B16) into (B20) yields

$$\hat{V}(\lambda) = \frac{1}{2}\left(b^{SB}\right)^2 a^2 \left[1 - \frac{\lambda^2}{(1-\lambda)^2}\right] = \frac{1}{2}\left(b^{SB}\right)^2 a^2 \left[1 - \frac{1}{\left(\frac{1}{\lambda} - 1\right)^2}\right],$$

so that $\frac{\partial \hat{V}(\lambda,\varepsilon)}{\partial \lambda} < 0$ if $0 < \lambda < \frac{1}{2}$ and $\frac{\partial \hat{V}(\lambda,\varepsilon)}{\partial \lambda} > 0$ if $\lambda < 0$ or if $\lambda > 1$. Furthermore, we have $\hat{V}(\lambda) = 0$ iff $\lambda^2 = (1-\lambda)^2$, i.e., iff $\lambda = \frac{1}{2}$. Finally, note that $\hat{V}(\lambda)$ is continuous in λ except at $\lambda = 1$, and that $\lim_{\lambda \uparrow 1} \hat{V}(\lambda) = \lim_{\lambda \downarrow 1} \hat{V}(\lambda) = -\infty$ and $\hat{V}(0) = \frac{1}{2}\left(b^{SB}\right)^2 a^2 > 0$.[29] Hence,

$$V'(\lambda) > 0 \text{ for } \lambda < 0,\;\; V'(\lambda) < 0 \text{ for } 0 < \lambda < \frac{1}{2},\;\; V'(\lambda) = 0 \text{ for } \frac{1}{2} \leq \lambda < 1,\;\; V'(\lambda) \geq 0 \text{ for } \lambda \geq 1 \tag{B23}$$

and $V(\lambda) < 0$ for $1 < \lambda$, $V(\lambda) = 0$ for $\frac{1}{2} \leq \lambda < 1$, and $V(\lambda) > 0$ for $\lambda_0 \leq \lambda < \frac{1}{2}$, where $\lambda_0 < 0$.

Step 2. Shape of $Z(\lambda)$. Let $\hat{Z}(\lambda) \equiv \left[ab^{SB} - \delta^*(a)\right]\delta^{*\prime}(a)$. Then (B22a) implies $\hat{Z}(\lambda) = 0$ for $\lambda \in [\frac{1}{2}, 1)$, so that $Z(\lambda) = 0$ for all $\lambda \in [\frac{1}{2}, 1)$. Next, suppose $\lambda \notin [\frac{1}{2}, 1]$. Then substituting (B16) into (B21) yields

$$\hat{Z}(\lambda) = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2}. \tag{B24}$$

We thus have $Z(\lambda) > 0$ iff $\lambda \in \left(0, \frac{1}{2}\right)$.

Step 3. Constraint (B18). The above implies that for $C > 0$, (B21) can hold only if $\lambda \in \left(0, \frac{1}{2}\right)$.

[29] $\lambda \uparrow 1$ indicates convergence of λ to 1 from below; similarly, $\downarrow$ indicates convergence from above.

Furthermore, $Z(\lambda)$ is continuous on $\left[0, \frac{1}{2}\right]$, so $\max Z(\lambda)$ on $\left[0, \frac{1}{2}\right]$ exists and is positive. Denote this maximum as $C^{\max}$. Then by continuity, for every $C \in [0, C^{\max}]$ there exists a $\lambda \in \left[0, \frac{1}{2}\right]$ such that (B21) holds, whereas if there is a λ such that (B21) holds for $C > C^{\max}$, this λ cannot be a part of the solution to (P1). Consequently, the feasibility constraint for C can be expressed as $0 \leq C \leq C^{\max}$.

Step 4. Solution to (B3). Finally, (B23) says that $V'(\lambda) \leq 0$ on $\left(0, \frac{1}{2}\right)$. Hence, if multiple λ solve (B3), then λ^* is the smallest of them. But, from (B24), $Z(0) = 0$ and $Z'(\lambda) \leq 0$ for all $\lambda \in \left(\frac{1}{3}, \frac{1}{2}\right)$. The smallest λ that solves (B3) therefore cannot exceed $\frac{1}{3}$. That is, $\lambda^* \leq \frac{1}{3}$.

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