Chapter 1

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1.4. a. $U_1 = 6^{0.5}4^{0.5} = 4.90$

$U_2 = 14^{0.5}16^{0.5} = 14.97$

$$MRS_j = \frac{\partial U_j / \partial c_1^j}{\partial U_j / \partial c_2^j} = \frac{\alpha c_2^j}{(1-\alpha)c_1^j}$$

with $\alpha = 0.5$, $MRS_j = \frac{c_2^j}{c_1^j}$

$$MRS_1 = \frac{4}{6} = 0.67$$

$$MRS_2 = \frac{16}{14} = 1.14$$

$MRS_1 \neq MRS_2$, not Pareto Optimal; it is possible to reallocate the goods and make one agent (at least) better off without hurting the other.

b. PS = { $c_1^j = c_2^j$, $j = 1,2 : c_1^i + c_2^i = 20, i = 1,2$ }, the Pareto set is a straight line (diagonal from lower-left to upper-right corner).

c. The problem of the agents is

$\text{Max} U^j$ s.t. $p_1 e_1^j + e_2^j = p_1 c_1^j + c_2^j$.

The Lagrangian and the FOC's are given by

$$L^j = \left(c_1^j\right)^{1/2}\left(c_2^j\right)^{1/2} + y\left(p_1 e_1^j + e_2^j - p_1 c_1^j - c_2^j\right)$$

$$\frac{\partial L^j}{\partial c_1^j} = \frac{1}{2}\left(\frac{c_2^j}{c_1^j}\right)^{1/2} - y p_1 = 0$$

$$\frac{\partial L^j}{\partial c_2^j} = \frac{1}{2}\left(\frac{c_1^j}{c_2^j}\right)^{1/2} - y = 0$$

$$\frac{\partial L^j}{\partial y} = p_1 e_1^j + e_2^j - p_1 c_1^j - c_2^j = 0$$

Rearranging the FOC's leads to $p_1 = \frac{c_2^j}{c_1^j}$. Now we insert this ratio into the budget constraints of agent 1 $p_1 6 + 4 - 2p_1 c_1^1 = 0$ and after rearranging we get $c_1^1 = 3 + \frac{2}{p_1}$. This expression can be interpreted as a demand function. The remaining demand functions can be obtained using the same steps.

$$c_2^1 = 3p_1 + 2$$

$$c_1^2 = 7 + \frac{8}{p_1}$$

$$c_2^2 = 7p_1 + 8$$

To determine market equilibrium, we use the market clearing condition $c_1^1 + c_1^2 = 20, c_2^1 + c_2^2 = 20$.

Finally we find $p_1 = 1$ and $c_1^1 = c_2^1 = 5, c_1^2 = c_2^2 = 15$.

The after-trade MRS and utility levels are:

$U_1 = 5^{0.5} 5^{0.5} = 5$

$U_2 = 15^{0.5} 15^{0.5} = 15$

$$MRS_1 = \frac{5}{5} = 1$$

$$MRS_2 = \frac{15}{15} = 1$$

Both agents have increased their utility level and their after-trade MRS is equalized.

d. $U_j(c_1^j, c_2^j) = \ln\left(\left(c_1^j\right)^{\alpha} \cdot \left(c_2^j\right)^{1-\alpha}\right) = \alpha \ln c_1^j + (1-\alpha) \ln c_2^j$,

$$MRS_j = \frac{\partial U_j / \partial c_1^j}{\partial U_j / \partial c_2^j} = \frac{\alpha c_2^j}{(1-\alpha)c_1^j}$$

Same condition as that obtained in a). This is not a surprise since the new utility function is a monotone transformation (logarithm) of the utility function used originally.

$U_1 = \ln\left(6^{0.5}4^{0.5}\right) = 1.59$

$U_2 = \ln\left(14^{0.5}16^{0.5}\right) = 2.71$

MRS's are identical to those obtained in a), but utility levels are not. The agents will make the same maximizing choice with both utility functions, and the utility level has no real meaning, beyond the statement that for a given individual a higher utility level is better.

e. Since the maximizing conditions are the same as those obtained in a)-c) and the budget constraints are not altered, we know that the equilibrium allocations will be the same too (so is the price ratio). The after-trade MRS and utility levels are:

$U_1 = \ln\left(5^{0.5}5^{0.5}\right) = 1.61$

$U_2 = \ln\left(15^{0.5}15^{0.5}\right) = 2.71$

$$MRS_1 = \frac{5}{5} = 1$$

$$MRS_2 = \frac{15}{15} = 1$$

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Chapter 3

3.1. Mathematical interpretation:
We can use Jensen's inequality, which states that if f(.) is concave, then
$$E(f(X)) \leq f(E(X))$$
Indeed, we have that
$$E(f(X)) = f(E(X)) \Leftrightarrow f'' = 0$$
As a result, when f(.) is not linear, the ranking of lotteries with the expected utility criterion might be altered.

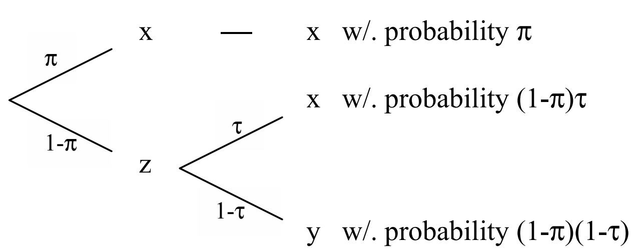
Economic interpretation:
Under uncertainty, the important quantities are risk aversion coefficients, which depend on the first and second order derivatives. If we apply a non-linear transformation, these quantities are altered. Indeed, $R_A(f(U(.))) = R_A(U(.)) \Leftrightarrow$ f is linear.

a. L = (B, M, 0.50) = 0.50×U(B) + 0.50×U(M) = 55 > U(P) = 50. Lottery L is preferred to the "sure lottery" P.

b. f(U(X)) = a+b×U(X)
L_f = (B, M, 0.50)$_f$ = 0.50×(a+bU(B)) + 0.50×(a+bU(M)) = a + b55 > f(U(P)) = a+bU(P) = a + b50. Again, L is preferred to P under transformation f.

g(U(X)) = lnU(X)
L_g = (B, M, 0.50)$_g$ = 0.50×lnU(100) +0.50×lnU(10) = 3.46 < g(U(P)) = lnU(50) = 3.91. P is preferred to L under transformation g.

3.2. Lotteries:
We show that (x,z,π) = (x,y,π + (1-π)τ) if z = (x,y, τ).



The total probabilities of the possible states are
$$\pi(x) = \pi + (1-\pi)\tau$$
$$\pi(y) = (1-\pi)(1-\tau)$$
Of course, $\pi(x) + \pi(y) = \pi + (1-\pi)\tau + (1-\pi)(1-\tau) = 1$. Hence we obtain lottery (x,y,π + (1-π)τ).

Could the two lotteries (x,z,π) and $(x,y,\pi + (1-\pi)\tau)$ with $z = (x,y, \tau)$ be viewed as non-equivalent ? Yes, in a non-expected utility world where there is a preferences for gambling. Yes, also, in a world where non-rational agents might be confused by the different contexts in which they are requested to make choices. While the situation represented by the two lotteries is too simple to make this plausible here, the behavioral finance literature building on the work of Kahneman and Tversky (see references in the text) point out that in more realistic experimental situations similar 'confusions' are frequent.

3.3 U is concave. By definition, for a concave function f(.)

$$f(\lambda a + (1-\lambda)b) \geq \lambda f(a) + (1-\lambda)f(b), \lambda \in [0,1]$$

Use the definition with $f = U$, $a = c_1$, $b = c_2$, $\lambda = 1/2$

$$U\left(\frac{1}{2}c_1 + \frac{1}{2}c_2\right) \geq \frac{1}{2}U(c_1) + \frac{1}{2}U(c_2)$$

$$U(\bar{c}) \geq \frac{1}{2}U(c_1) + \frac{1}{2}U(c_2)$$

$$2U(\bar{c}) \geq U(c_1) + U(c_2)$$

$$V(\bar{c}, \bar{c}) \geq V(c_1 + c_2)$$