Online Appendix for "Bargaining and Reputation: an Experiment on Bargaining in the Presence of Behavioral Types"

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April 1, 2014

Abstract

This is the online appendix for *Embrey et al.* (2014). Four sections are included, which provide further details for the theory, experimental design and results sections of *Embrey et al.* (2014), as well as providing a detailed description of theoretical extensions aimed at incorporating observed deviations from the baseline theory.

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1 Theory

This section provides further details of the relevant theoretical results for a symmetric version of the stylized bargaining and reputation model of *Abreu and Gul* (2000).[1] First, the setup and the description of the equilibrium is repeated. Although included in the main paper, this ensures the section is self-contained. Subsection 1.2 details the system of equations that are used to find a numerical solution for equilibrium behavior by rational types. Subsection 1.3 provides further details of equilibrium behavior for the special case where all types are aggressive. Subsections 1.4, 1.5 and 1.6 provide further details of the key equilibrium predictions stated in the paper. Subsection 1.7 details the numerical strategy used to solve for the equilibrium strategy for a general set of behavioral types. The final subsection provides some numerical examples.

1.1 Set Up and Equilibrium Play

Two agents bargain over a pie of size one in two stages. In the first stage each player simultaneously announces a demand α^i (i.e. the faction of the pie they would like). If the two demands are compatible (i.e. $\alpha^1 + \alpha^2 \leq 1$) then the game ends immediately.[2] If the two demands are incompatible, the game proceeds to stage two, where a continuous-time concession game with an infinite horizon starts. That is, for each point in time, $t \in [0, \infty)$, both players can choose to accept (i.e. concede) or hold out. If player i concedes, she receive $1 - \alpha^j$, while if j concedes, player i receives α^i. Preferences of agents are risk neutral, with a common discount factor r. Thus, if an agreement is reached at time t in which an agent receives a share x, then their payoff is $e^{-rt}x$.

In addition, there is some probability that a player may face a behavioral type who is obstinate in their demands. Define $C := \{\alpha_1, ..., \alpha_K\}$ as the set of behavioral types, with $\alpha_i < \alpha_{i+1}$, for $i = 1, ..., K-1$, and $\alpha_K \geq \frac{1}{2}$. An α_k-type always demands α_k and only accepts an offer that gives them at least

[1]What is contained here is implicit in *Abreu and Gul* (2000). Some results for equilibrium announcements have been taken from *Abreu and Sethi* (2003), where the authors use the symmetric version of the model.

[2]If the announcements sum to strictly less than one, a sharing rule is used for the remainder. While other sharing rules can be accommodated, in the experiments we divide the remainder equally.

α_k. The probability that a player is an α_k-type is z_k, for $k = 1, ..., K$. The probability that a player is rational is denoted by $z_0 = 1 - \sum_{k=1}^{K} z_k$. This distribution over types is summarized by the vector $z = \{z_1, ..., z_K\}$. Last, a behavioral type is defined as *aggressive* if it is incompatible with all other behavioral types in C and weakly incompatible with itself.

A key property of the equilibrium is that a rational player would only choose a demand that mimics some behavioral type (i.e. $\alpha^i \in C$ for $i = 1, 2$). Consequently, players can be identified by the element of C that they announce in the first stage, $\alpha_k, \alpha_l \in C$. In a symmetric equilibrium, define μ_k to be the probability that a rational player announces demand α_k. Given this symmetric equilibrium, the probability that a player is irrational given an announcement α_k is given by

$$\overline{\pi}_k = \frac{z_k}{z_k + z_0 \mu_k} \tag{1}$$

In equilibrium, with a general set C, rational players will employ a mixed strategy over announcing different types in the first stage. If the set C contains a type, α, such that $\alpha \leq \frac{1}{2}$, there is a possibility that this type will not be replicated in equilibrium. However, if a behavioral type is replicated, then all more aggressive types are also replicated. As such, the support of the equilibrium mixing strategy, μ, will be of the form $\{\alpha_R, ..., \alpha_K\}$, where $1 \leq R < K$. Ensuring that rational players are indifferent between announcing any α_k (for $k = R, ..., K$), along with the μ being a probability measure (and therefore summing to one), yields the $(K - R + 2)$ equations needed to solve for μ and the expected payoff for rational players.[3]

Suppose a rational player announced α_k and faces an opponent who has announced α_l, where $\alpha_k + \alpha_l > 1$, causing the players to move on to the concession stage.[4] The unique equilibrium play in the incomplete information war of attrition game is given by a mixed strategy over the time of concession.

[3]This is a system of non-linear equations that only has a numerical solution, except for trivial cases. See subsection 1.7 for details of the numerical strategy used to solve this system of equations.

[4]Note that the player who announced α_l could be either an α_l-type or a rational player who has mimicked the α_l-type.

The α_k player concedes with constant hazard rate, λ_{kl}, given by[5]

$$\lambda_{kl} = \frac{r(1-\alpha_k)}{\alpha_k + \alpha_l - 1} \tag{2}$$

over the interval $[0, T_0]$, where $T_0 = \min(T_{kl}, T_{lk})$ and $T_{kl} = \frac{-\ln(\overline{\pi}_k)}{\lambda_{kl}}$ and $T_{lk} = \frac{-\ln(\overline{\pi}_l)}{\lambda_{lk}}$. Thus, equilibrium is generally characterized by inefficient delay. Concession by the α_k rational player is governed by the distribution function $\frac{\hat{F}_{kl}}{1-\overline{\pi}_k}$, where

$$\hat{F}_{kl}(t) = \begin{cases} 1 - c_{kl}e^{-\lambda_{kl}t}, & \text{for } t \in [0, T_0] \\ 1 - \overline{\pi}_k, & \text{for } t > T_0 \end{cases} \tag{3}$$

and $c_{kl} = \overline{\pi}_k e^{\lambda_{kl}T_0}$ and $(1-c_{kl})(1-c_{lk}) = 0$. Note that the distribution function is expressed in terms of $\hat{F}_{kl}$ for notational convenience: a rational player who announced α_l, when their opponent announced α_k, faces a "mixed" strategy over the time of concession given by $\hat{F}_{kl}$ (i.e. unconditional on the α_k player being rational). Figure 1 provides an illustration of this concession behavior in a subgame following announcement α_l and α_k, where $\alpha_l + \alpha_k > 1$ and $\alpha_l < \alpha_k$.

The value of T_{kl} is a measure of the α_k rational player's "strategic" weakness when facing an α_l player: if $T_{kl} > T_{lk}$, then the α_k rational player will have to concede at time $t = 0$ with strictly positive probability (mass), given by $q_{kl} := (1 - c_{kl})$. Such concession is referred to as *initial* concession. Concession resulting from the continuous part of the distribution function is referred to as *interior* concession.

1.2 Setting up the System of Equations

Consider a general set of behavioral types $C = \{\alpha_1, ..., \alpha_K\}$. Denote by the index p the smallest aggressive demand.[6] As mentioned in section 1.1 above, for such a general set C the equilibrium first stage announcement strategy by rational players will be to employ a mixed strategy over a set

[5]So long as it remains possible that their opponent is a rational-type, a rational player who announced α_k is indifferent between conceding and not conceding at a time t if $r(1-\alpha_l) = [\alpha_k - (1-\alpha_l)]\lambda_{lk}$, where λ_{lk} is the hazard rate for concession by the opponent unconditional on knowing whether the opponent is rational or not. Equation 2 ensures this indifference holds.

[6]If there are no aggressive types in the set C set $p = K$.

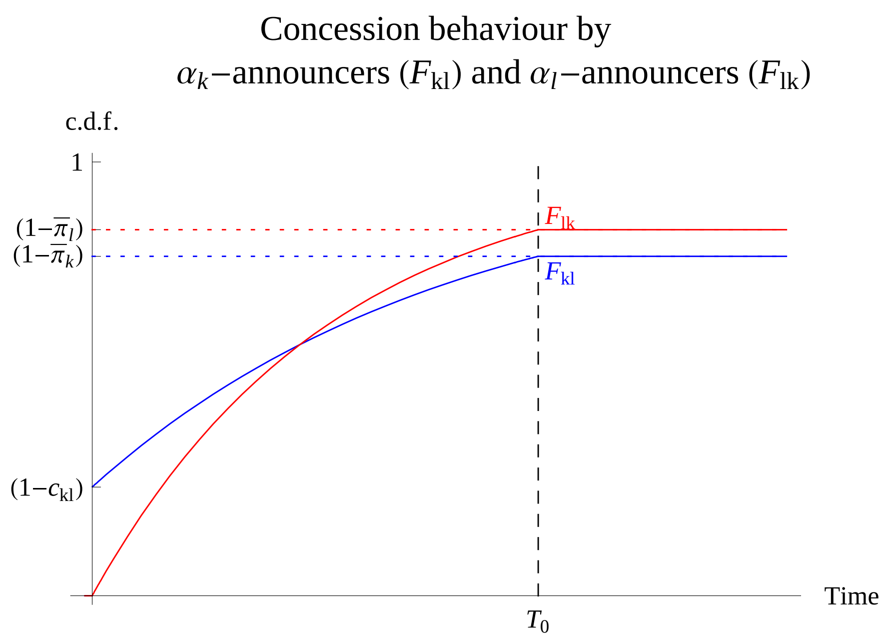


Figure 1: Example of Concession Behavior in an Asymmetric Subgame

$\{\alpha_R, ..., \alpha_p, ..., \alpha_K\}$, where $1 \leq R \leq p \leq K$. For $(\mu_R, ..., \mu_K)$ to be an equilibrium, rational players need to be indifferent between announcing demands $\alpha_R, ..., \alpha_K$ and have no incentive to announce a demand $\alpha_1, ..., \alpha_{R-1}$, given a rational opponent employs the mixed strategy $(\mu_R, ..., \mu_K)$.

The expected payoff to a rational player for making an announcement α_i when their opponent announces α_j is as follows:[7]

$$EP\left[\alpha_i|\alpha_j\right] = \begin{cases} \frac{1}{2} + \frac{\alpha_i - \alpha_j}{2} & \text{, if } \alpha_i + \alpha_j \leq 1 \\ (1 - \alpha_j) & \text{, if } \alpha_i + \alpha_j > 1, T_{ij} \geq T_{ji} \\ (1 - \alpha_j) + (1 - c_{ji})(\alpha_i + \alpha_j - 1) & \text{, if } \alpha_i + \alpha_j > 1, T_{ij} < T_{ji} \end{cases}$$

The above expression is used to calculate the expected payoff of announcing a demand α_i in stage 1, and to build a system of equations that need to be satisfied in order for the mixed strategy $(\mu_R, ..., \mu_K)$ to form an equilibrium, in which the ex-ante expected value to a rational player is v:[8]

$$\begin{bmatrix} EP\left[\alpha_R\right] - v \\ \vdots \\ EP\left[\alpha_K\right] - v \\ \left(\sum_{i=R}^{K} \mu_i\right) - 1 \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{K} (z_j + z_0\mu_j)\, EP\left[\alpha_R|\alpha_j\right] - v \\ \vdots \\ \sum_{j=1}^{K} (z_j + z_0\mu_j)\, EP\left[\alpha_K|\alpha_j\right] - v \\ \left(\sum_{i=R}^{K} \mu_i\right) - 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad (4)$$

When there are more than two aggressive types, i.e. $p < K$, the size of the system of equations can be reduced. This can be done by taking advantage of the fact that an aggressive type is never conceded to initially by another behavioral type (and being aggressive means that all other types are incompatible and consequently result in a second stage). If there are at least two such types, enforcing that neither type is ever conceded to initially guarantees that a rational player is indifferent between choosing either demand in stage 1; the expected payoff in any subgame following announcing either

[7]When $\alpha_i + \alpha_j > 1$, the subsequent subgame is a war of attrition. Rational players must be indifferent between conceding and not conceding for $t \in (0, T_0)$. If their opponent would never concede initially to them, i.e. $T_{ij} \geq T_{ji}$, then their expected payoff must be whatever they would get if they conceded instantly. If their opponent concedes initially with strictly positive probability, i.e. $T_{ij} < T_{ji}$, then their expected payoff is a convex combination of of what they would get by conceding instantly and what they would get if the other conceded instantly, with weight $(1 - c_{ji})$ on the latter payoff.

[8]Note that μ_i and μ_j enter c_{ji} in a non-linear fashion. Further note that there are $K - R + 2$ equations, including the condition that the probabilities sum to one, and $K - R + 2$ unknowns, including the unknown value v.

demand is always $1-\alpha_j$, where α_j is the announcement made by the other player. Let α_p and α_k be two aggressive types, with $k>p$. In any subgame in which an α_p-announcer and an α_k-announcer meet, the equilibrium mixing strategy over behavioral types must be such that μ_p and μ_k imply $T_{pk}=T_{kp}$:

$$\begin{aligned} -\frac{\ln \overline{\pi}_p}{\lambda_{pk}} &= -\frac{\ln \overline{\pi}_k}{\lambda_{kp}} \\ \Longrightarrow & \\ \overline{\pi}_k &= \overline{\pi}_p^{\frac{1-\alpha_k}{1-\alpha_p}} \\ \Longrightarrow & \\ \mu_k &= \left(\frac{z_k}{z_0}\right)\left[\left(\frac{z_p+z_0\mu_p}{z_p}\right)^{\frac{1-\alpha_k}{1-\alpha_p}}-1\right] \end{aligned}$$

That is, we have $\mu_k = g(\mu_p, z_p, z_k, z_0)$, for $k=p+1,...,K$, which guarantees the expected payoff to announcing α_k equals the expected payoff to announcing α_p. The system of equations can then be simplified to a system of $p-R+2$ equations in $p-R+2$ unknowns (these are $\mu_R,...,\mu_p$ and v):

$$\begin{bmatrix} \sum_{j=1}^{K}(z_j+z_0\mu_j)\,EP\left[\alpha_R|\alpha_j\right]-v \\ \vdots \\ \sum_{j=1}^{K}(z_j+z_0\mu_j)\,EP\left[\alpha_p|\alpha_j\right]-v \\ \left(\sum_{i=R}^{p}\mu_i\right)+\left(\sum_{p+1}^{K} g(\mu_p,z_p,z_k,z_0)\right)-1 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

replacing μ_k with $g(\mu_p, z_p, z_k, z_0)$, for $k=p+1,...,K$, where ever it appears in the expressions $\{EP[\alpha_i|\alpha_k]\}_{i=R,..,p}$.

1.3 Special Case: Only Aggressive Types

1.3.1 Equilibrium Play

Assume the set C is such that $\alpha_1 \geq \frac{1}{2}$. That is, all types are aggressive. In this case, all behavioral types are replicated by rational players with positive probability, $\mu_k>0$ for all k, and no player concede initially in any subgame (that is $T_{kl}=T_{lk}$, for all k,l). In this special case, the system of equations that are used to solve for μ can be further simplified to finding the root of the following equation:

$$\left(\sum_{k=1}^{K} g\left(\mu_1, z_1, z_k, z_0\right)\right) - 1 \quad = \quad 0$$

where

$$\begin{aligned}\mu_k &= g\left(\mu_1, z_1, z_k, z_0\right) \\ &= \tilde{z_k}\left[\left(1 + \frac{\mu_1}{\tilde{z_1}}\right)^{\frac{1-\alpha_k}{1-\alpha_1}} - 1\right]\end{aligned}$$

and

$$\tilde{z_k} \quad := \quad \frac{z_k}{z_0}$$

The result that $T_{kl} = T_{lk}$, for all k, l, gives the following simple observation: For any $\alpha_k, \alpha_l \in C$,

$$\overline{\pi}_k = \overline{\pi}_l^{\frac{\lambda_k}{\lambda_l}} = \overline{\pi}_l^{\frac{1-\alpha_k}{1-\alpha_l}}$$

Incorporating the definition of $\lambda_{kl} = \frac{r(1-\alpha_k)}{\alpha_k+\alpha_l-1}$ and the definition of $\overline{\pi}_k = \frac{z_k}{z_k+z_0\mu_k} = \frac{1}{1+z_0\frac{\mu_k}{z_k}}$ to the above, gives the following useful observation on equilibrium play in the first stage.

Observation 1 *In equilibrium, given announcement pair α_k and α_l, the following are equivalent*

$$\begin{aligned}\alpha_k > \alpha_l &\iff \lambda_{kl} < \lambda_{lk} \\ &\iff \overline{\pi}_k > \overline{\pi}_l \\ &\iff \frac{\mu_k}{\mu_l} < \frac{z_k}{z_l}\end{aligned}$$

That is the ratio of "rational" players mimicking the more aggressive type to those mimicking the less aggressive type is smaller than the ratio of more aggressive behavioral types in the population to less aggressive types in the population.

Since the equilibrium mixing strategy, μ, is such that there is no time zero concession with strictly positive probability mass for any subgame allows the following analysis of concession behavior (irrespective of the particulars of the numerical solution of μ). The probability that a player, which has announced α_k and faces an opponent who has announced α_l, will concede in the second stage is given by

$$\begin{aligned} & p\left(\alpha_k \text{ announcer conceding}\right) \\ = \; & \left(\tfrac{\lambda_{kl}}{\lambda_{kl}+\lambda_{lk}}\right)\left[1-e^{-(\lambda_{kl}+\lambda_{lk})T_0}\right] \end{aligned}$$

Note that this probability is unconditional on knowing that the α_k announcer is rational. This leads to the following observation:

Observation 2 *In equilibrium, for any announcements pair $\alpha_k, \alpha_l \in C$,*

$$\begin{aligned} & p\left(\alpha_l \textit{ announcer conceding}\right) > p\left(\alpha_k \textit{ announcer conceding}\right) \\ \iff & \\ & \alpha_l < \alpha_k \end{aligned}$$

Observations 1 and 2 permit an interpretation of the equilibrium behavior of rational actors: while for the most part they restrain themselves in the behavioral type that they replicate (i.e. not going for the "greedier" types so often), when they are aggressive in the initial stage, they remain aggressive (in probabilistic terms) in the concession stage.[9]

1.3.2 Comparative Static

This subsection analyzes the predicted change in μ_k as $\tilde{z}_k := \frac{z_k}{z_0}$ is changed (keeping z_l unchanged for $l \neq k$). This comparative static is the marginal equivalent of replacing a human subject (whose behavior would be drawn from $(z_0, z_1, ..., z_K)$) with a computer player that plays a fixed rule α_k.[10] The

[9] It should be noted that observation 2 also holds for a general set C (i.e. when not all types are aggressive) if the probabilities are conditional on there being no initial concession.

[10] Consider a treatment, such as is implemented in our experimental sessions, where two out of sixteen possible subjects are in fact computer players set up to behave like α_k-types. In the new treatment with computer players

$$\begin{aligned} pr\,(\text{being rational}) &= pr\,(\text{not a computer AND being rational}) \\ &= pr\,(\text{not a computer})\, pr\,(\text{rational}) \\ &= \frac{13}{15} z_0 \end{aligned}$$

implicit function theorem is applied to the system

$$G(\mu_1, \tilde{z}) := \left(\sum_{i=1}^{K} \tilde{z}_i \left[\left(1+\frac{\mu_1}{\tilde{z}_1}\right)^{\frac{1-\alpha_i}{1-\alpha_1}} - 1\right]\right) - 1$$

the root of which defines the equilibrium mixing strategy, μ. Taking derivatives with respect to $\tilde{z}_k$ and μ_1 gives[11]

$$\begin{aligned} \frac{\partial G}{\partial \tilde{z}_k} &= \left(1+\frac{\mu_1}{\tilde{z}_1}\right)^{\frac{1-\alpha_k}{1-\alpha_1}} - 1 \\ \frac{\partial G}{\partial \mu_1} &= \sum_{i=1}^{K} \frac{(1-\alpha_i)}{(1-\alpha_1)} \frac{\tilde{z}_i}{\tilde{z}_1} \left(1+\frac{\mu_1}{\tilde{z}_1}\right)^{\frac{1-\alpha_i}{1-\alpha_1}-1} \end{aligned}$$

The implicit function theorem gives

$$\begin{aligned} \frac{\partial \mu_1(\tilde{z})}{\partial \tilde{z}_k} &= -\left[\sum_{i=1}^{K} \frac{(1-\alpha_i)}{(1-\alpha_1)} \frac{\tilde{z}_i}{\tilde{z}_1} \left(1+\frac{\mu_1}{\tilde{z}_1}\right)^{\frac{1-\alpha_i}{1-\alpha_1}-1}\right]^{-1} \\ &\quad \times \left[\left(1+\frac{\mu_1}{\tilde{z}_1}\right)^{\frac{1-\alpha_k}{1-\alpha_1}} - 1\right] \end{aligned}$$

To go from μ_1 to μ_k, the following function is used

$$\begin{aligned} \mu_k &= \tilde{z}_k \left[\left(1+\frac{\mu_1(\tilde{z})}{\tilde{z}_1}\right)^{\frac{1-\alpha_k}{1-\alpha_1}} - 1\right] \\ &\Longrightarrow \\ \frac{\partial \mu_k}{\partial \tilde{z}_k} &= \left[\left(1+\frac{\mu_1}{\tilde{z}_1}\right)^{\frac{1-\alpha_k}{1-\alpha_1}} - 1\right] \\ &\quad + \left[\frac{(1-\alpha_k)}{(1-\alpha_1)} \frac{\tilde{z}_k}{\tilde{z}_1} \left(1+\frac{\mu_1}{\tilde{z}_1}\right)^{\frac{1-\alpha_k}{1-\alpha_1}-1}\right] \times \frac{\partial \mu_1}{\partial \tilde{z}_k} \end{aligned}$$

---

and for all $l \neq k$

$$\begin{aligned} pr\,(\text{being behavioral l-type}) &= pr\,(\text{not a computer AND being l-type}) \\ &= pr\,(\text{not a computer})\,pr\,(l-\text{type}) \\ &= \frac{13}{15} z_l \end{aligned}$$

Thus $\tilde{z}_l$ does not change for all $l \neq k$.

[11] It is assumed that $k > 1$. This simplifies the algebra and is without loss of generality since the choice of μ_1 as the "anchor" is completely arbitrary.

Substituting the expression for $\frac{\partial \mu_1}{\partial \tilde{z}_k}$ calculated earlier into the above expression gives

$$\begin{aligned}
\frac{\partial \mu_k}{\partial \tilde{z}_k} &= \left[\left(1+\frac{\mu_1}{\tilde{z}_1}\right)^{\frac{1-\alpha_k}{1-\alpha_1}} - 1\right] \\
&\quad \times \left[1 - \frac{\left(\frac{(1-\alpha_k)}{(1-\alpha_1)}\frac{\tilde{z}_k}{\tilde{z}_1}\left(1+\frac{\mu_1}{\tilde{z}_1}\right)^{\frac{1-\alpha_k}{1-\alpha_1}-1}\right)}{\sum_{i=1}^{K}\left(\frac{(1-\alpha_i)}{(1-\alpha_1)}\frac{\tilde{z}_i}{\tilde{z}_1}\left(1+\frac{\mu_1}{\tilde{z}_1}\right)^{\frac{1-\alpha_i}{1-\alpha_1}-1}\right)}\right] \\
&\geq 0
\end{aligned}$$

1.4 Key Equilibrium Predictions: Announcements in the First Stage

Five properties of first-stage announcement behavior are explicitly mentioned in the main text. These are:

1. Rational players will only make announcements that mimic some behavioral type.
2. If the announcement of a behavioral type is mimicked in equilibrium, then the announcements of all more demanding behavioral types are also mimicked.
3. Aggressive types are always mimicked in equilibrium.
4. If the set C contains only aggressive types, then

$$\alpha_k > \alpha_l \iff \mu_k < \frac{z_k}{z_l} \cdot \mu_l$$

   That is, if the less demanding type is at least as probable as the more demanding type, then rational players will mimic the more demanding announcement less often.

5. If the set C contains a type that is compatible with all other types in C (i.e., $\alpha + \alpha_i \leq 1$, for all $\alpha_i \in C$), then this type is never mimicked by rational players.

For a proof of property 1, the reader is referred to, for example, Section 2 of *Abreu and Sethi* (2003). However, the consequences of making an announcement that does not mimic some behavioral type can be seen by plugging in $\overline{\pi}_k = 0$ into the expression for $q_{kl} = (1 - c_{kl})$ from Section 1.1. A rational player making such an announcement would concede instantly to any demand by their opponent that mimics some behavioral type. Since the set C contains at least one demand of at least half of the pie, rational players would prefer to mimic this type, rather than announce a type that does not correspond to some behavioral type.

For proofs of properties 2 and 3, the reader is referred to Section 2 of *Abreu and Sethi* (2003). Property 4 is simply a restatement of Observation 1 from Subsection 1.3. Property 5 is immediate since announcing the always-compatible strategy is strictly dominated by making the most demanding announcement and conceding instantly should a second-stage be reached, so this demand cannot be a member of the support of the equilibrium mixing strategy.

1.5 Key Equilibrium Predictions: Concession in the Second Stage

1.5.1 Initial Concession

The first point to note is that, considering only the implications of second-stage equilibrium play, initial concession in the second stage between a player that announced α_i and a player that announced α_j is determined by which is smaller $\overline{\pi}_i^{\frac{1}{1-\alpha_i}}$ or $\overline{\pi}_j^{\frac{1}{1-\alpha_j}}$. This follows from the expression for T_0, the time by which rational players who announced either α_i or α_j must have conceded by. Suppose $T_{ji} < T_{ij}$, i.e. the α_i-announcer must initially concede to the

α_j-announcer with strictly positive probability. Then,

$$
\begin{aligned}
T_0 &= -\frac{\ln(\overline{\pi}_j)}{\lambda_{ji}} \\
&= -\frac{\ln\left(\frac{\overline{\pi}_i}{c_{ij}}\right)}{\lambda_{ij}} \\
\Longleftrightarrow & \\
c_{ij} &= \frac{\overline{\pi}_i}{\overline{\pi}_j^{\left(\frac{1-\alpha_i}{1-\alpha_j}\right)}}
\end{aligned}
$$

Since in this case we have $c_{ij} < 1$, the above implies that

$$\overline{\pi}_i^{\frac{1}{1-\alpha_i}} < \overline{\pi}_j^{\frac{1}{1-\alpha_j}}$$

Note that the above inequality only considers the restrictions imposed by second-stage equilibrium play. Adding the restriction that the symmetric equilibrium is in mixed strategies, and therefore all choices in the mixed strategy must give the same expected payoff, gives a stronger result. Namely,

Claim 1 *Suppose $\alpha_i + \alpha_j > 1$. If $\alpha_i < \alpha_j$, then $\overline{\pi}_i^{\frac{1}{1-\alpha_i}} \geq \overline{\pi}_j^{\frac{1}{1-\alpha_j}}$.*

Proof. Suppose the contrary. That is there exits $\alpha_i, \alpha_j \in C$ such that $\alpha_i + \alpha_j > 1$, $\alpha_i < \alpha_j$ and $\overline{\pi}_i^{\frac{1}{1-\alpha_i}} < \overline{\pi}_j^{\frac{1}{1-\alpha_j}}$. The proof contains two steps. The first is to show that for any announcement, α_k-announcement that concedes initially to an α_i-announcement with strictly positive probability, this α_k-announcer will concede initially to an α_j-announcer with even greater probability. The second step is to work through all possible subgames an α_i-announcer could face and show that the expected payoff is strictly less than that which could be obtained by announcing α_j. This provides a contradiction to both α_i and α_j being part of the same mixed equilibrium strategy.

Step 1: The aim here is to show that for any $\alpha_k \in \{\alpha \in C | \alpha + \alpha_i > 1\}$ such that $\overline{\pi}_k^{\frac{1}{1-\alpha_k}} < \overline{\pi}_i^{\frac{1}{1-\alpha_i}}$, it is the case that $c_{kj} < c_{ki}$. Since, $\overline{\pi}_k^{\frac{1}{1-\alpha_k}} < \overline{\pi}_i^{\frac{1}{1-\alpha_i}} < \overline{\pi}_j^{\frac{1}{1-\alpha_j}}$, both $c_{kj} < 1$ and $c_{ki} < 1$. Using the expression for c derived above gives

$$
\begin{aligned}
c_{ki} = \quad \frac{\overline{\pi}_k}{\overline{\pi}_i^{\left(\frac{1-\alpha_k}{1-\alpha_i}\right)}} \quad &= \left(\frac{\overline{\pi}_k^{\left(\frac{1}{1-\alpha_k}\right)}}{\overline{\pi}_i^{\left(\frac{1}{1-\alpha_i}\right)}}\right)^{1-\alpha_k} \\
c_{kj} = \quad \frac{\overline{\pi}_k}{\overline{\pi}_j^{\left(\frac{1-\alpha_k}{1-\alpha_j}\right)}} \quad &= \left(\frac{\overline{\pi}_k^{\left(\frac{1}{1-\alpha_k}\right)}}{\overline{\pi}_j^{\left(\frac{1}{1-\alpha_j}\right)}}\right)^{1-\alpha_k}
\end{aligned}
$$

By assumption $\overline{\pi}_i^{\frac{1}{1-\alpha_i}} < \overline{\pi}_j^{\frac{1}{1-\alpha_j}}$, thus $c_{kj} < c_{ki}$ and $(1 - c_{kj}) > (1 - c_{ki})$; the latter inequality being the probability of α_k-announcer initially conceding to an α_j-announcer is strictly larger than the probability of an α_k-announcer initially conceding to an α_i-announcer.

Step 2: For each of the following cases, the expected payoff of announcing α_i when facing an α_k-announcer, denoted by $EP\left[\alpha_i|\alpha_k\right]$, is compared to the expected payoff to announcing α_j, $EP\left[\alpha_j|\alpha_k\right]$:

- for $\alpha_k \in C$ such that $\alpha_k + \alpha_j \leq 1$:

$$
\begin{aligned}
EP\left[\alpha_i|\alpha_k\right] &= \frac{1}{2} + \frac{\alpha_i - \alpha_k}{2} \\
EP\left[\alpha_j|\alpha_k\right] &= \frac{1}{2} + \frac{\alpha_j - \alpha_k}{2}
\end{aligned}
$$

  Since, $\alpha_i < \alpha_j$, this implies $EP\left[\alpha_i|\alpha_k\right] \leq EP\left[\alpha_j|\alpha_k\right]$.

- for $\alpha_k \in C$ such that $\alpha_k + \alpha_j > 1$ and $\alpha_k + \alpha_i \leq 1$:

$$
\begin{aligned}
EP\left[\alpha_i|\alpha_k\right] &= \frac{1}{2} + \frac{\alpha_i - \alpha_k}{2} \\
EP\left[\alpha_j|\alpha_k\right] &= (1 - \alpha_k) + c_{kj}\left(\alpha_j + \alpha_k - 1\right) \\
&\geq 1 - \alpha_k
\end{aligned}
$$

  Since $\alpha_k + \alpha_i \leq 1$ implies $\alpha_i \leq 1 - \alpha_k$, it is the case that $EP\left[\alpha_i|\alpha_k\right] \leq 1 - \alpha_k \leq EP\left[\alpha_j|\alpha_k\right]$.

- for $\alpha_k \in C$ such that $\alpha_k + \alpha_i > 1$ and $\overline{\pi}_k^{\frac{1}{1-\alpha_k}} \geq \overline{\pi}_i^{\frac{1}{1-\alpha_i}}$: In this case, the α_k-announcer will *not* concede initially to the α_i-announcer (this could or could not be the case for the α_j-announcer). Consequently,

$$\begin{aligned} EP\left[\alpha_i|\alpha_k\right] &= 1-\alpha_k \\ EP\left[\alpha_j|\alpha_k\right] &\geq 1-\alpha_k \end{aligned}$$

which again implies that $EP\left[\alpha_i|\alpha_k\right] \leq EP\left[\alpha_j|\alpha_k\right]$.

- for $\alpha_k \in C$ such that $\alpha_k + \alpha_i > 1$ and $\overline{\pi}_k^{\frac{1}{1-\alpha_k}} < \overline{\pi}_i^{\frac{1}{1-\alpha_i}}$: In this case, the α_k-announcer will concede initially to both the α_i- and the α_j-announcers with strictly positive probability. Consequently the payoffs are

$$\begin{aligned} EP\left[\alpha_i|\alpha_k\right] &= \left(1-\alpha_k\right) + \left(1-c_{ki}\right)\left(\alpha_i+\alpha_k-1\right) \\ EP\left[\alpha_j|\alpha_k\right] &= \left(1-\alpha_k\right) + \left(1-c_{kj}\right)\left(\alpha_j+\alpha_k-1\right) \end{aligned}$$

Since $\alpha_i < \alpha_j$ and, by step one, $(1-c_{ki}) < (1-c_{kj})$, again it is the case that $EP\left[\alpha_i|\alpha_k\right] \leq EP\left[\alpha_j|\alpha_k\right]$.

Consequently, for all possible announcements by the opposing player, the expected payoff from announcing α_j is at least as large as that from announcing α_i. Given that there is a strictly positive probability of the other player being rational and announcing α_i themselves, and the expected payoffs in this case being

$$\begin{aligned} EP\left[\alpha_i|\alpha_i\right] &= \left(1-\alpha_i\right) \\ EP\left[\alpha_j|\alpha_i\right] &= \left(1-\alpha_i\right) + \left(1-c_{ij}\right)\left(\alpha_j+\alpha_i-1\right) \end{aligned}$$

announcing α_j gives a strictly higher payoff than announcing α_i. This is a contradiction to α_i and α_j being a part of the same mixing strategy. ■

The above claim implies that, in subgames following announcements of α_L and α_H with $\alpha_L + \alpha_H > 1$ and $\alpha_L < \alpha_H$, it is the player that made the lower announcement, α_L, that does not concede initially. That is $c_{LH} = 1$ and $c_{HL} \leq 1$. Therefore, the probability that a player concedes initially (without knowing whether or not they are rational) is given by:

$$\begin{aligned} p\left(\alpha_H \text{ concedes at } t=0\right) &= \left(1-c_{HL}\right) \\ &\geq 0 \\ &= p\left(\alpha_L \text{ concedes at } t=0\right) \end{aligned}$$

In addition, the above claim enables a simplification in setting up the system of equations used to solve numerically for the equilibrium mixing strategy of rational players. Now the ordering of $\left\{ \overline{\pi}_k^{\frac{1}{1-\alpha_k}} \right\}_{k=1}^{K}$ is known in advance, which means the rational announcers that need to concede initially in any given subgame is known prior to solving for μ.

1.5.2 Interior Concession

If one conditions on the second stage not ending instantly (i.e. no initial concession), then it is possible to calculate the probability of the α_L announcer and the α_H announcer conceding in a subgame following announcements α_L and α_H, such that $\alpha_L + \alpha_H > 1$ and $\alpha_L < \alpha_H$. Using the result of Section 1.5.1, only the α_H announcer has the possibility of making an initial concession. The equilibrium distribution for concession by α_H announcers, unconditional on rationality, is given by

$$1 - c_{HL} e^{-\lambda_{HL} t} = (1 - c_{HL}) + c_{HL} \left(1 - e^{-\lambda_{HL} t}\right)$$

This is a convex combination of the distribution that puts full weight on instant concession and the distribution that ensures a rational α_L-announcer is indifferent between conceding or not at time t, for $t \in (0, T_0)$. The probability of conceding by a time $t > 0$ is then $c_{HL}\left(1 - e^{-\lambda_{HL} t}\right)$, while, as before, the probability of *not* conceding by t is $c_{HL} e^{-\lambda_{HL} t}$. An analogous calculation to those made for observation 2 can be made to show that, once initial concession has been ruled out, it is the α_L announcer that is more likely to concede in this subgame. Specifically,

$$\begin{aligned} p\left(\alpha_L \text{ concedes at } t > 0\right) &= \left(\frac{\lambda_{LH}}{\lambda_{LH} + \lambda_{HL}}\right)\left(1 - e^{-(\lambda_{LH} + \lambda_{HL}) T_0}\right) c_{HL} \\ p\left(\alpha_H \text{ concedes at } t > 0\right) &= \left(\frac{\lambda_{HL}}{\lambda_{LH} + \lambda_{HL}}\right)\left(1 - e^{-(\lambda_{LH} + \lambda_{HL}) T_0}\right) c_{HL} \end{aligned}$$

Thus, $\lambda_{LH} > \lambda_{HL}$ implies:[12]

$$p\left(\alpha_L \text{ concedes at } t > 0\right) > p\left(\alpha_H \text{ concedes at } t > 0\right)$$

[12] Note that the expression $\left(1 - e^{-(\lambda_{LH} + \lambda_{HL}) T_0}\right) c_{HL}$ can be rewritten as $c_{HL} - \overline{\pi}_L \overline{\pi}_H$. This expression is the probability of interior concession by either player, that is the probability of concession at $t > 0$ minus the probability of disagreement.

1.6 Key Equilibrium Predictions: Amount of Delay in the Second Stage

1.6.1 Upper Bound on Average Delay

The stochastic process that governs the time until either player concedes (given at least one eventually does) is first order stochastically dominated by the process defined as conceding over the interval $[0, \infty)$ with a constant hazard rate $\lambda_{kl} + \lambda_{lk}$. This is because the latter rules out initial concession by the player making the greater first-stage demand, and extends the support from $[0, T_0]$ to $[0, \infty)$. This results in both the mean and the median of the latter being larger than the true stochastic process, thus providing upper bounds. Specifically, in the subgame following announcements α_k and α_l, the mean upper bound is given by

$$\frac{1}{\lambda_{kl} + \lambda_{lk}}$$

and the median upper bound is given by

$$\frac{\ln(2)}{\lambda_{kl} + \lambda_{lk}}$$

where $\alpha_k + \alpha_l > 1$. Given values for the parameters of the game (that is the underlying distribution of types, z), the actual averages can be calculated for any given subgame since the distribution for concession time (by either player) conditional on eventual agreement is given by

$$\hat{F}_{min}(t) = \frac{1 - c_{HL} e^{-(\lambda_{LH} + \lambda_{HL})t}}{1 - \overline{\pi}_L \overline{\pi}_H}$$

where $\alpha_L + \alpha_H > 1$ and $\alpha_L \leq \alpha_H$. In particular for the median, this gives

$$t_{med} = \begin{cases} 0 & , \text{ if } \frac{1 - c_{HL}}{1 - \overline{\pi}_L \overline{\pi}_H} > \frac{1}{2} \\ \frac{\ln\left(\frac{2c_{HL}}{1 + \overline{\pi}_L \overline{\pi}_H}\right)}{\lambda_{LH} + \lambda_{HL}} & , \text{ if } \frac{1 - c_{HL}}{1 - \overline{\pi}_L \overline{\pi}_H} \leq \frac{1}{2} \end{cases}$$

1.6.2 Bounds on z Derived from Observed Delay

Consider the subgame following the announcement of α_k and α_l where $\alpha_k + \alpha_l > 1$ for which there has been at least one observed concession (by either player). Denote by α_k the weakly larger of the two announcements (i.e. $\alpha_k \geq \alpha_l$). Let t_{kl}^{max} be the largest such concession time for this subgame. Since only rational players concede, and rational players concede over the support $[0, T_0]$, it must be the case that $t_{kl}^{max} \leq T_0$. Inserting t_{kl}^{max} into the expression for second stage behavior by rational players, given in equation 3, results in:

$$\begin{aligned}
1 - c_{k,l} e^{-\lambda_{kl} t_{kl}^{max}} &\leq 1 - \overline{\pi}_k \\
1 - e^{-\lambda_{lk} t_{kl}^{max}} &\leq 1 - \overline{\pi}_l \\
&\Longrightarrow \\
\overline{\pi}_k &\leq e^{-\lambda_{kl} t_{k,l}^{max}} \\
\overline{\pi}_l &\leq e^{-\lambda_{lk} t_{k,l}^{max}}
\end{aligned}$$

Since $\overline{\pi}_i = \frac{z_i}{z_i + z_0 \mu_i}$ is bounded below by z_i, for all $i = 1, ..., K$ (the denominator is bounded from above by $z_k + (1 - z_k) \times 1 = 1$), this gives the following bounds on z_k and z_l:

$$\begin{aligned}
z_k &\leq e^{-\lambda_{kl} t_{kl}^{max}} \\
z_l &\leq e^{-\lambda_{lk} t_{kl}^{max}}
\end{aligned}$$

While these bounds are not explicitly used in the main text, they are referred to in section 3.3.4 of this online appendix, where they are used to provide further evidence of excessive delays in the second stage.

1.7 Numerical Solution for a General Set of Behavioral Types

For a given support for the equilibrium mixed strategy in first stage announcements, $\{\alpha_R, ..., \alpha_p, ..., \alpha_K\}$, using the system of equations defined in 1.2, including the simplifications contained in 1.5, it is possible to obtain a numerical solution for μ. However, for a general set of behavioral types C that include types that are non-aggressive, it is not possible to know in advance what the support of the mixing strategy will be. Nonetheless, the support μ is unique.

To see why the support must be unique, consider a bargaining game with obstinate types (C, z). The maximum a rational type could ever expect to earn from an announcement of $\alpha \in C$ is achieved if this rational type concedes instantly to any announcement by irrational types that demand more than $1-\alpha$, but at the same time is *conceded to* instantly by rational types that demand more than $1-\alpha$. Denote this value by $vmax\,(\alpha)$. Suppose there is an equilibrium in first-stage announcements of this bargaining game, μ, in which at least one element $\alpha \in C$ that is not mimicked – i.e. $\mu\,(\alpha) = 0$. The expected value to a rational type from deviating from μ and announcing α instead is arbitrarily close to this $vmax\,(\alpha)$.[13] For μ to be an equilibrium distribution of first-stage announcements, the expected value to rational types from playing the equilibrium must be larger than this deviation payoff:

$$v\,(\mu) \geq vmax\,(\alpha)$$

for all $\alpha \in C$ such that $\mu\,(\alpha) = 0$. Now suppose that there is a second equilibrium distribution of first-stage announcements, μ', that has a strictly larger support. That is, there exists an $\alpha \in C$ such that $\mu\,(\alpha) = 0$ and $\mu'\,(\alpha) > 0$. Since there is a strictly positive probability of delay in second-stages that follow from announcing α, the expected value to a rational type from announcing this α must be strictly smaller than $vmax\,(\alpha)$. Since α is part of the equilibrium mixed strategy, the expected value to rational players of playing the equilibrium μ' must also be strictly smaller than this bound:

$$v\,(\mu') < vmax\,(\alpha)$$

for all $\alpha \in C$ such that $\mu'\,(\alpha) > 0$. Putting these two inequalities together gives

$$v\,(\mu') < v\,(\mu)$$

That is, the equilibrium with the larger support has a higher expected payoff for rational types. This is a contradiction to a known property of the game, implicit in *Abreu and Gul* (2000) and explicitly stated in Sections 2 and 3 of *Abreu and Sethi* (2003), that equilibrium expected payoffs are uniquely determined by the parameters (C, z).

In addition to uniqueness, the following features of the equilibrium are used to develop an algorithm to find a full solution for μ:

- Aggressive types are always replicated in equilibrium.

[13] Consider any second stage after a rational type deviates from μ and announces α. If the other player is also rational, then the other player concedes instantly. If the other player is irrational, the deviating rational player waits an $\epsilon > 0$, but arbitrarily small, for this other player to reveal their irrationality by not conceding instantly and then concedes themself.

- If a type α is replicated in equilibrium, then all larger types are also replicated.
- If a type α is compatible with all other types in C, i.e. $\alpha + \alpha_k \leq 1$ for all $k = 1, ..., K$, then this demand is *never* replicated by rational players.

For step $s = 0, ..., (p - m)$, where m denotes the smallest index such that α_m is incompatible with some other type:

1. Assume the support of μ is $S = \{\alpha_{p-s}, ..., \alpha_p, ..., \alpha_K\}$.
2. Using the assumption on the support, set up and solve the system equations.
3. For each $\alpha \in \{\alpha_m, ..., \alpha_{p-s-1}\}$, calculate the expected value to a rational player from deviating by choosing α, assuming other rational players continue to mix according to μ.
4. If there is no incentive to deviate to an α outside of the assumed set S, end the algorithm. Otherwise, continue to the next step, which will add the largest demand outside of S to the set S and repeat the process.

1.8 Numerical Examples

This subsection provides numerical examples of the equilibrium predictions of the game. The examples are chosen with the experimental design in mind. Accordingly, all types are now reported as integers demands out of a total pie of 30. In the first example, all subjects are assumed to correspond to a rational type of the model. The second example considers the possibility that there may be 50-50 behavioral types in the subject population.

1.8.1 All subjects are Rational Types

Table 1 gives the equilibrium predictions for first stage behavior in the four treatments *U1*, *U2*, *R3* and *R4* under the assumption that all subjects correspond to rational types. Since this example assumes all subjects to be rational, the *C0* treatment is a game of complete information. This does not have a unique equilibrium prediction. Consequently, it is omitted from the tables. The first sets of columns gives the distribution over types implied by the addition of the computer players. The second set of columns gives the equilibrium mixing strategy for rational players (i.e. subject announcements in this example).

	z					μ				
Treatment	8	12	15	18	20	8	12	15	18	20
All subjects rational										
U1	0	0	0	0	13.3	0	0	0	0	100.0
U2	0	6.7	0	0	6.7	0	30.9	0	0	69.1
R3	0	0	6.7	6.7	6.7	0	0	48.2	30.2	21.5
R4	6.7	0	6.7	6.7	6.7	0	0	44.9	31.4	23.6

Table 1: Equilibrium prediction for first-stage behavior assuming all subjects rational

		Median	Delay Upper Bounds		Prob. of Concession§ Initial		Interior	
Treatment	Subgame		Median	T_0	α_L	α_H	α_L	α_H
All subjects rational								
U1	20-20	33.8	34.7	201.5	0	0	0.49	0.49
U2	12-20	0	5.0	17.9	0	0.75	0.15	0.08
	20-20	34.2	34.7	230.1	0	0	0.49	0.49
R3	15-18	7.4	7.7	38.3	0	0	0.54	0.43
	15-20	13.1	13.9	63.8	0	0	0.58	0.38
	18-18	16.2	17.3	76.6	0	0	0.48	0.48
	18-20	23.1	25.2	102.1	0	0	0.51	0.43
	20-20	30.9	34.7	127.7	0	0	0.46	0.46
R4	15-18	6.5	7.7	35.6	0	0.07	0.50	0.40
	15-20	11.1	13.9	59.4	0	0.09	0.52	0.35
	18-18	16.1	17.3	74.7	0	0	0.47	0.47
	18-20	21.7	25.2	99.7	0	0.03	0.49	0.41
	20-20	30.9	34.7	128.1	0	0	0.46	0.46

Table 2: Equilibrium prediction for second-stage behavior assuming all subjects rational

§ Columns do not sum to one since there is a positive chance of no concession by either player.

Table 2 provides a summary of predicted second stage behavior for the relevant second stage subgames in the treatments *U1*, *U2*, *R3* and *R4*. Again *C0* is omitted. The first set of columns refers to the predicted delay behavior, giving the median, the theoretical upper bound on the median and the maximum possible delay. The last set of columns gives the probability of concession by the player making the lower (α_L) and higher announcements (α_H) for both the case of initial concession (at time zero) and interior concession (strictly after time zero).

1.8.2 Adding 50-50 Types to the Subject Population

This numerical example considers the possibility that the subject population may contain 50-50 behavioral types. In the context of the experimental design, this corresponds to the inclusion of 15-types in the subject population. Three assumptions on the probability of being matched with a subject 15-type are considered: $\frac{1}{13}$, $\frac{5}{13}$ and $\frac{12}{13}$, referred to as scenario A12, A8 and A1, respectively. In the first case, the probability of being matched to a subject 15-type is assumed to be such that, in *U1* and *U2*, on average 1 out of the 13 other subjects (i.e. human rather than computer players) will be a 15-type. Note that, since there are also two computer players in the pool of potential matches, this gives a 1 out of 15 chance of being matched to a subject that is a 15-type in treatments *U1* and *U2*. This is chosen to be an example of a small probability of being matched to a behavioral-type subject. The other two cases are chosen to be examples of a medium and larger probability of being matched to a behavioral-type subject. They correspond to assuming in the control treatment that the probability of being a rational type, z_0, is $\frac{8}{13}$ and $\frac{1}{13}$. These numbers are comparable to scenarios B and C reported in the experimental design section of the paper, which estimate z from the control data assuming z_0 is equal to $\frac{1}{13}$ and $\frac{8}{13}$, respectively.[14]

The equilibrium predictions for first-stage behavior in the treatments *U1*, *U2*, *R3* and *R4* are given in Table 3. Table 4 provides a summary of the predicted second stage behavior for the relevant second stage subgames in the treatments *U1*, *U2*, *R3* and *R4*. The table focusses on the subgames 15-20 and 20-20, which can occur in all four treatments.

In addition to the fact that tables 1 through 4 provide numerical illustrations of the highlighted key equilibrium predictions, two further points can be raised with regard to second-stage delay. The first, when z_0 is not too small, as in the case where all subjects rational and the A12 scenario, the actual median is numerically very close to the upper bound. That is the

[14] Although the *C0* is now a game of incomplete information, the predictions of the model for this treatment are again excluded from the table. This is because the predictions are immediate: the only behavioral type is the 15 type; thus all subjects, whether rational or behavioral, will announce 15 and the game will end without moving to a second stage.

Treatment	z 8	12	15	18	20	μ 8	12	15	18	20
Scenario A1: $z_0 = \frac{1}{13}$ *in C0*										
U1	0	0	80.	0	13.3	0	0	90.1	0	9.9
U2	0	6.7	80.	0	6.7	0	0	66.3	0	33.7
R3	0	0	80.5	6.7	6.7	0	0	89.3	5.9	4.9
R4	6.7	0	74.4	6.7	6.7	0	0	25.1	33.1	41.8
Scenario A8: $z_0 = \frac{8}{13}$ *in C0*										
U1	0	0	33.3	0	13.3	0	0	81.4	0	18.6
U2	0	6.7	33.3	0	6.7	0	0	86.4	0	13.6
R3	0	0	37.4	6.7	6.7	0	0	80.9	10.6	8.4
R4	6.7	0	34.9	6.7	6.7	0	0	72.5	14.4	13.1
Scenario A12: $z_0 = \frac{12}{13}$ *in C0* (scenario A in the paper)										
U1	0	0	6.7	0	13.3	0	0	53.4	0	46.6
U2	0	6.7	6.7	0	6.7	0	0	69.	0	31.
R3	0	0	12.8	6.7	6.7	0	0	62.5	21.6	15.9
R4	6.7	0	12.3	6.7	6.7	0	0	57.4	23.8	18.8

Table 3: Equilibrium prediction for first-stage behavior adding subject behavioral types

bound is reasonably tight when z_0 is not too small. Secondly, as z_0 gets small, second-stage delay, conditional on eventual agreement, collapses. That is, if the players are to agree, then it should happen almost instantly.

Treatment	Subgame	Median	Delay Upper Bounds: Median	Delay Upper Bounds: T_0	Prob. of Concession Initial: α_L	Initial: α_H	Interior: α_L	Interior: α_H
Scenario A1: $z_0 = \frac{1}{13}$ in C0								
U1	15-20	1.2	13.9	2.4	0	0	0.07	0.05
	20-20	2.4	34.7	4.8	0	0	0.05	0.05
U2	15-20	0	13.9	1.8	0	0.22	0.04	0.03
	20-20	12.4	34.7	29.	0	0	0.22	0.22
R3	15-20	1.1	13.9	2.2	0	0	0.06	0.04
	20-20	2.2	34.7	4.4	0	0	0.04	0.04
R4	15-20	0	13.9	0.6	0	0.25	0.01	0.01
	20-20	12.9	34.7	30.3	0	0	0.23	0.23
Scenario A8: $z_0 = \frac{8}{13}$ in C0								
U1	15-20	9.4	13.9	27.8	0	0	0.45	0.3
	20-20	20.4	34.7	55.6	0	0	0.34	0.34
U2	15-20	7.1	13.9	28.9	0	0.14	0.39	0.26
	20-20	24.3	34.7	73.4	0	0	0.38	0.38
R3	15-20	8.6	13.9	24.2	0	0	0.42	0.28
	20-20	18.5	34.7	48.3	0	0	0.31	0.31
R4	15-20	5.1	13.9	22.1	0	0.18	0.33	0.22
	20-20	22.3	34.7	63.6	0	0	0.36	0.36
Scenario A12: $z_0 = \frac{12}{13}$ in C0 (scenario A in the paper)								
U1	15-20	13.2	13.9	66.7	0	0	0.58	0.39
	20-20	31.3	34.7	133.5	0	0	0.47	0.47
U2	15-20	12.1	13.9	74.3	0	0.06	0.55	0.37
	20-20	32.5	34.7	155.1	0	0	0.48	0.48
R3	15-20	12.3	13.9	50.9	0	0	0.55	0.37
	20-20	28.5	34.7	101.7	0	0	0.43	0.43
R4	15-20	9.9	13.9	47.5	0	0.11	0.48	0.32
	20-20	29.1	34.7	106.8	0	0	0.44	0.44

Table 4: Equilibrium prediction for second-stage behavior adding subject behavioral types

2 Experimental Design

2.1 Sample instructions

2.1.1 *C0* treatment

Welcome

You are about to participate in a session on decision-making, and you will be paid for your participation in cash, privately at the end of the session. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance.

Please turn off pagers and cellular phones now. Please close any programs you may have open on the computer. The entire session will take place through computer terminals, and all interaction between you and other session participants will take place through the computers. Please do not talk directly to or attempt to communicate with other participants during the session.

We will start with a brief instruction period. During the instruction period you will be given a description of the main features of the session and will be shown how to use the computers. If you have any questions during this period, raise your hand and your question will be answered so everyone can hear.

Instructions

In this experiment you will be asked to make decisions in 15 periods. At the beginning of each period you will be matched at random to another player. In the room there are 16 players. During the period your task is to divide 30 points between yourself and the other player you are matched with.

Each period has up to two stages:

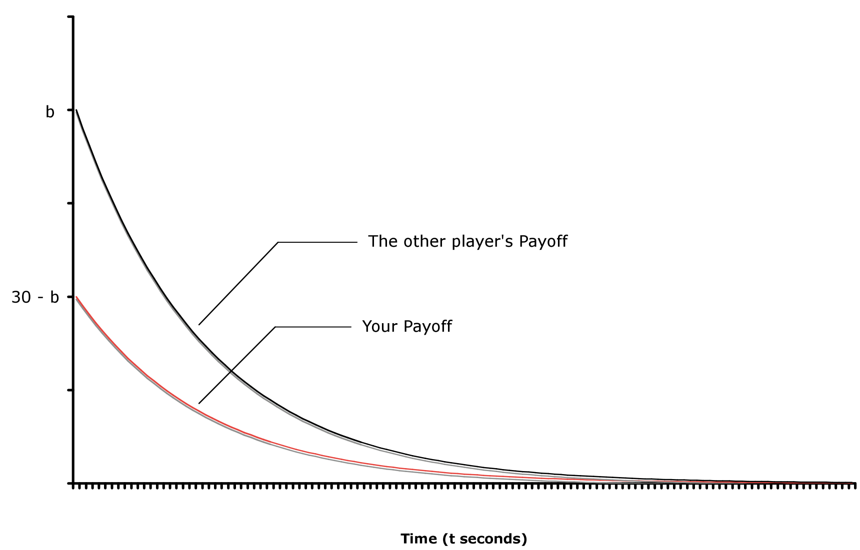
- <u>First Stage</u>: You place an announcement for the number of points that you want for yourself out of the 30 (denote this by a). Simultaneously, the other player will make an announcement for the number of points they want for themselves (denote this by b).

  – If the two announcements sum to 30 or less, then you will receive your announcement plus half of what is left over (30 minus the sum of the two announcements) and the period will end. In other words, you will receive $a + \frac{(30-a-b)}{2}$ points and the other player receives $b + \frac{(30-a-b)}{2}$.

  - If the two announcements sum to more than 30, then you move on to the second stage.

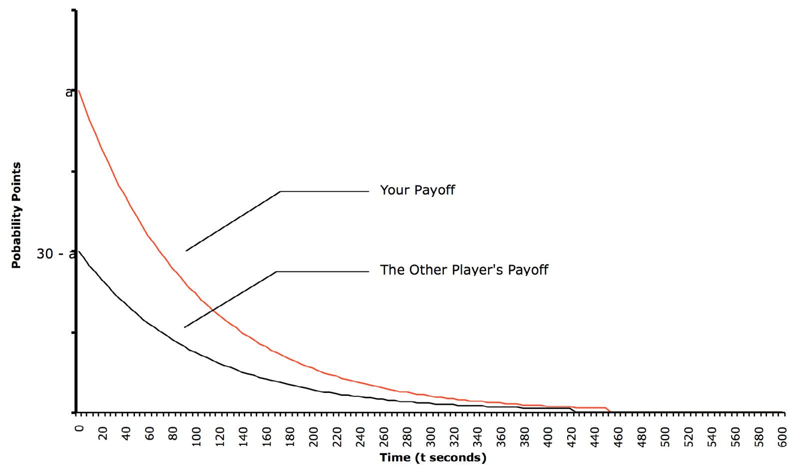
- Second Stage: You can now either accept the other player's announcement or wait until they accept your announcement. Accepting their announcement immediately means that you receive $30 - b$ points for that period. However, the longer you wait the less your points are worth. Approximately, points decrease at a rate of 1% per second. More precisely, if you accept the other player's announcement after t seconds, you will receive $(30 - b) \times (0.99)^t$ and the other player will receive $b \times (0.99)^t$. Figure 1 illustrates this.

Figure 1



If on the other hand, the other player accepts your offer after t seconds, you will receive $a \times (0.99)^t$ and the other player will receive $(30 - a) \times (0.99)^t$. Figure 2 illustrates this.

Figure 2



Your computer screen will display the points you and the other player would receive if you were to accept, or if they were to accept your announcement at different points in time. Once either you or the other player has accepted, or the value of the points have reached zero, the period is over.

A few examples might help your understanding. These are not meant to be realistic:

1. In the first stage, you announce 1.5 and the other player announces 3.5. Since $1.5 + 3.5 = 5$, which is smaller than 30, the period ends and you receive $1.5 + (30 - 5)/2 = 14$ points. If instead the other player had announced 23.5, then you would have received $1.5 + (30 - 25)/2 = 4$ points.

2. In the first stage, you announce 15 and the other player announces 23. Since $15 + 23 = 38$, which is greater than 30, you go to the second stage. In the second stage, the other accepts your announcement after 1 second. You get $15 \times (0.99)^1 = 14.85$ points. If instead, the other player does not accept immediately and you accept after 10 seconds, then you obtain $(30 - 23) \times (0.99)^{10} = 6.33$ points.

3. In the first stage, you announce 25 and the other player announces 5. Since $25 + 5 = 30$, the period ends and you obtain 25 points.

As you can see there are many possibilities.

When every pair has finished this task, the next period begins. You will be randomly re-assigned to a player in the next period. The task in the next period is exactly the same as the one just described (but with the randomly re-matched player). The session consists of 15 such periods.

Once the 15 periods have been completed, the total number of points you have earned will be displayed (denote this by P). This determines the odds of winning a prize in your lottery. Your lottery has the following structure:

- The odds of winning are given by the number of points you earned throughout the experiment divided by the total number of points available. Since there are 15 periods and there are 30 points available in each period, the total number of points available is given by $15 \times 30 = 450$. Thus the odds of winning are $\frac{P}{450}$.
- The prize is \$20.
- That is, you have $\frac{P}{450}$ chance of winning the prize and $1 - \frac{P}{450}$ chance of receiving \$0.

In summary, your earning from this session is comprised of a \$15 participation fee and the outcome of your lottery. The probabilities associated with your lottery depend on the number of points you have earned throughout the session. You can earn either \$0 or \$20 from the lottery.

Are there any questions?

Summary

Before we start, let me remind you that:

- After a period is finished, you will be randomly re-matched to a player for the next period.
- In each period, you and another player will make announcements to divide 30 points between both of you. If the sum of your two announcements is less than 30 the period ends. If the sum of the two

announcements is 30 or more you move to a second stage. In the second stage, the points decrease in value until either you or the other player accepts the announcement made by the other party, at which point the period ends.

- At the end of the session, your earnings are determined by a lottery with probabilities that depend on the number of points you have earned throughout the experiment. You can earn either \$0 or \$20 from the lottery. In addition you will receive a \$15 show-up fee.

Good Luck.

2.1.2 *U1* treatment

Welcome

You are about to participate in a session on decision-making, and you will be paid for your participation in cash, privately at the end of the session. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance.

Please turn off pagers and cellular phones now. Please close any programs you may have open on the computer. The entire session will take place through computer terminals, and all interaction between you and other session participants will take place through the computers. Please do not talk directly to or attempt to communicate with other participants during the session.

We will start with a brief instruction period. During the instruction period you will be given a description of the main features of the session and will be shown how to use the computers. If you have any questions during this period, raise your hand and your question will be answered so everyone can hear.

Instructions

In this experiment you will be asked to make decisions in 15 periods. At the beginning of each period you will be matched at random to another player. That player will be either another subject in the room or a computer player (more on this later). In the room there are 14 human players and 2 computer players. During the period your task is to divide 30 points between yourself and the other player you are matched with.

Each period has up to two stages:

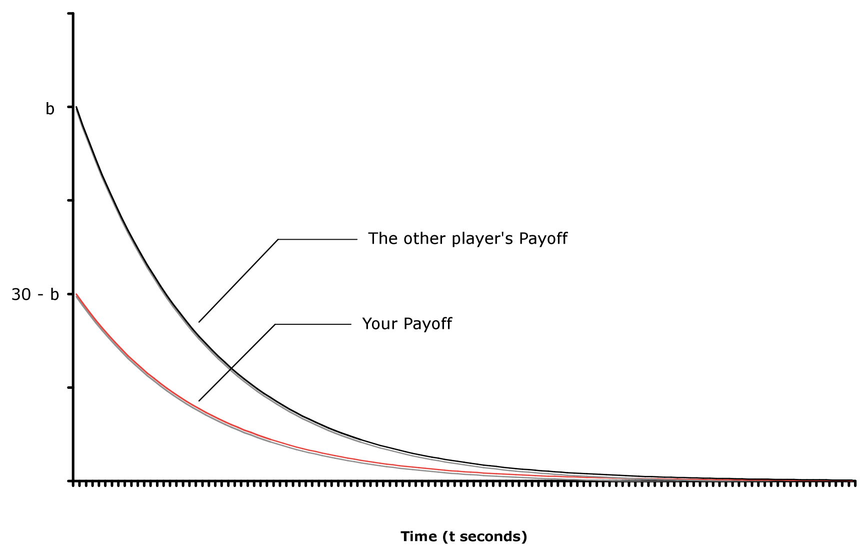
- First Stage: You place an announcement for the number of points that you want for yourself out of the 30 (denote this by a). Simultaneously, the other player will make an announcement for the number of points they want for themselves (denote this by b).

  - If the two announcements sum to 30 or less, then you will receive your announcement plus half of what is left over (30 minus the sum of the two announcements) and the period will end. In other words, you will receive $a + \frac{(30-a-b)}{2}$ points and the other player receives $b + \frac{(30-a-b)}{2}$.

  - If the two announcements sum to more than 30, then you move on to the second stage.

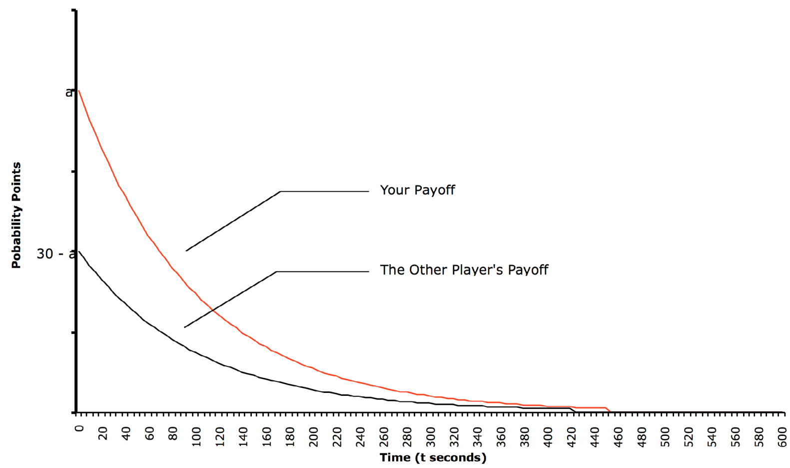
- Second Stage: You can now either accept the other player's announcement or wait until they accept your announcement. Accepting their announcement immediately means that you receive $30 - b$ points for that period. However, the longer you wait the less your points are worth. Approximately, points decrease at a rate of 1% per second. More precisely, if you accept the other player's announcement after t seconds, you will receive $(30 - b) \times (0.99)^t$ and the other player will receive $b \times (0.99)^t$. Figure 1 illustrates this.

Figure 1



If on the other hand, the other player accepts your offer after t seconds, you will receive $a \times (0.99)^t$ and the other player will receive $(30 - a) \times (0.99)^t$. Figure 2 illustrates this.

Figure 2



Your computer screen will display the points you and the other player would receive if you were to accept, or if they were to accept your announcement at different points in time. Once either you or the other player has accepted, or the value of the points have reached zero, the period is over.

A few examples might help your understanding. These are not meant to be realistic:

1. In the first stage, you announce 1.5 and the other player announces 3.5. Since $1.5 + 3.5 = 5$, which is smaller than 30, the period ends and you receive $1.5 + (30 - 5)/2 = 14$ points. If instead the other player had announced 23.5, then you would have received $1.5 + (30 - 25)/2 = 4$ points.

2. In the first stage, you announce 15 and the other player announces 23. Since $15 + 23 = 38$, which is greater than 30, you go to the second stage. In the second stage, the other accepts your announcement after 1 second. You get $15 \times (0.99)^1 = 14.85$ points. If instead, the other player does not accept immediately and you accept after 10 seconds, then you obtain $(30 - 23) \times (0.99)^{10} = 6.33$ points.

3. In the first stage, you announce 25 and the other player announces 5. Since $25 + 5 = 30$, the period ends and you obtain 25 points.

As you can see there are many possibilities.

When every pair has finished this task, the next period begins. You will be randomly re-assigned to a player in the next period. The task in the next period is exactly the same as the one just described (but with the randomly re-matched player). The session consists of 15 such periods.

Computer players do the same thing every period. In the first stage, the computer player will always announce that they want 20 points. If the period goes to the second stage (that is the announcements are incompatible), the computer player will never accept your offer. At the beginning of each period, you have a 2/15 chance of being matched to a computer player.

Once the 15 periods have been completed, the total number of points you have earned will be displayed (denote this by P). This determines the odds of winning a prize in your lottery. Your lottery has the following structure:

- The odds of winning are given by the number of points you earned throughout the experiment divided by the total number of points available. Since there are 15 periods and there are 30 points available in each period, the total number of points available is given by $15 \times 30 = 450$. Thus the odds of winning are $\frac{P}{450}$.
- The prize is \$20.
- That is, you have $\frac{P}{450}$ chance of winning the prize and $1 - \frac{P}{450}$ chance of receiving \$0.

In summary, your earning from this session is comprised of a \$15 participation fee and the outcome of your lottery. The probabilities associated with your lottery depend on the number of points you have earned throughout the session. You can earn either \$0 or \$20 from the lottery.

Are there any questions?

Summary

Before we start, let me remind you that:

- After a period is finished, you will be randomly re-matched to a player for the next period.

- In each period, you and another player will make announcements to divide 30 points between both of you. If the sum of your two announcements is less than 30 the period ends. If the sum of the two announcements is 30 or more you move to a second stage. In the second stage, the points decrease in value until either you or the other player accepts the announcement made by the other party, at which point the period ends.

- At the end of the session, your earnings are determined by a lottery with probabilities that depend on the number of points you have earned throughout the experiment. You can earn either \$0 or \$20 from the lottery. In addition you will receive a \$15 show-up fee.

Good Luck.

2.1.3 *R3* treatment

Welcome

You are about to participate in a session on decision-making, and you will be paid for your participation in cash, privately at the end of the session. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance.

Please turn off pagers and cellular phones now. Please close any programs you may have open on the computer. The entire session will take place through computer terminals, and all interaction between you and other session participants will take place through the computers. Please do not talk directly to or attempt to communicate with other participants during the session.

We will start with a brief instruction period. During the instruction period you will be given a description of the main features of the session and will be shown how to use the computers. If you have any questions during this period, raise your hand and your question will be answered so everyone can hear.

Instructions

In this experiment you will be asked to make decisions in 15 periods. At the beginning of each period you will be matched at random to another player. That player will be either another subject in the room or a computer player (more on this later). In the room there are 13 human players and 3 computer players. During the period your task is to divide 30 points between yourself and the other player you are matched with.

Each period has up to two stages:

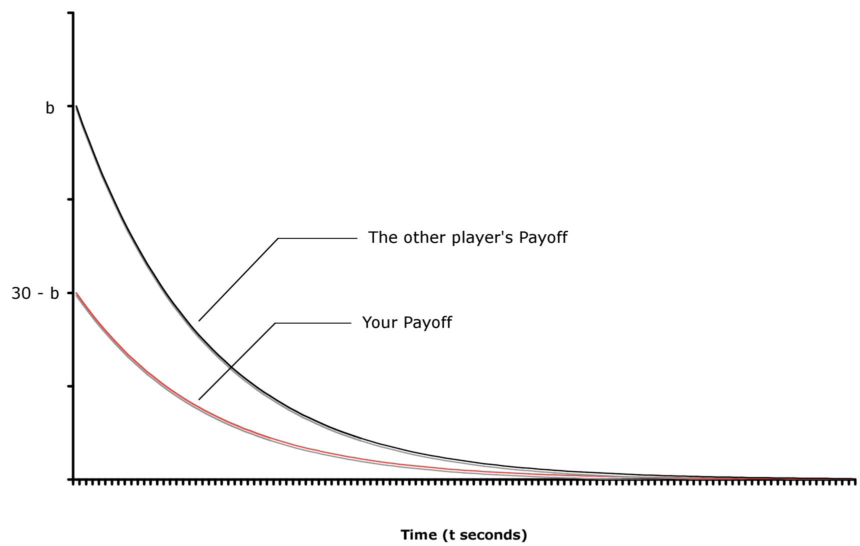
- First Stage: You place an announcement for the number of points that you want for yourself out of the 30 (denote this by a). This announcement can either be $15, 18$, or 20. These are the only three options that are available to you. Simultaneously, the other player will make an announcement for the number of points they want for themselves (denote this by b). He too can only ask for $15, 18$, or 20.

    - If the two announcements sum to 30 or less, then you will receive your announcement and the period will end.

- If the two announcements sum to more than 30, then you move on to the second stage.

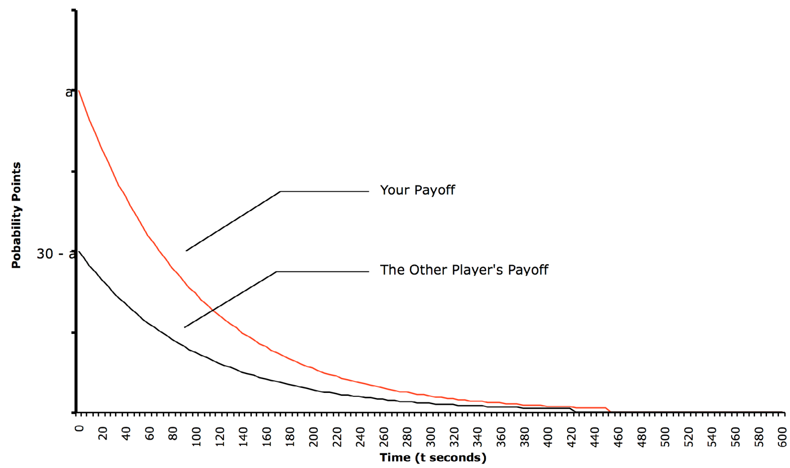
- Second Stage: You can now either accept the other player's announcement or wait until they accept your announcement. Accepting their announcement immediately means that you receive $30 - b$ points for that period. However, the longer you wait the less your points are worth. Points decrease at a rate of approximately 1% per second. More precisely, if you accept the other player's announcement after t seconds, you will receive $(30 - b) \times (0.99)^t$ and the other player will receive $b \times (0.99)^t$. Figure 1 illustrates this.

Figure 1



If on the other hand, the other player accepts your offer after t seconds, you will receive $a \times (0.99)^t$ and the other player will receive $(30 - a) \times (0.99)^t$. Figure 2 illustrates this.

Figure 2



Your computer screen will display the points you and the other player would receive if you were to accept, or if they were to accept your announcement at different points in time. Once either you or the other player has accepted, or the value of the points have reached zero, the period is over.

A few examples might help your understanding. These are not meant to be realistic:

1. In the first stage, you announce 15 and the other player announces 15. Since $15 + 15 = 30$ the period ends and you receive 15 points.

2. In the first stage, you announce 15 and the other player announces 20. Since $15 + 20 = 35$, which is greater than 30, you go to the second stage. In the second stage, the other accepts your announcement after 1 second. You get $15 \times (0.99)^1 = 14.85$ points. If instead, the other player does not accept immediately and you accept after 10 seconds, then you obtain $(30 - 20) \times (0.99)^{10} = 9.04$ points.

3. In the first stage, you announce 20 and the other player announces 18. Since $20 + 18 = 38$, you would move on to the second stage...

As you can see there are many possibilities.

When every pair has finished this task, the next period begins. You will be randomly re-assigned to a player in the next period. The task in the next period is exactly the same as the one just described (but with the randomly re-matched player). The session consists of 15 such periods.

Computer players do the same thing every period. The first computer player, "Computer I", acts as follows: In the first stage, the "Computer I" will always announce that they want 20 points. If the period goes to the second stage (that is the announcements are incompatible), "Computer I" will never accept your offer. The second computer player, "Computer II", acts as follows: In the first stage, "Computer II" will always announce that they want 18 points. If the period goes to the second stage (that is the announcements are incompatible), "Computer II" will never accept your offer. The third computer player, "Computer III", acts as follows: In the first stage, "Computer III" will always announce that they want 15 points. If the period goes to the second stage (that is the announcements are incompatible), "Computer III" will never accept your offer. At the beginning of each period, you have a $\frac{1}{15}$ chance of being matched to "Computer I", a $\frac{1}{15}$ chance of being matched to "Computer II", and a $\frac{1}{15}$ chance of being matched to "Computer III".

Once the 15 periods have been completed, the total number of points you have earned will be displayed (denote this by P). This determines the odds of winning a prize in your lottery. Your lottery has the following structure:

- The odds of winning are given by the number of points you earned throughout the experiment divided by the total number of points available. Since there are 15 periods and there are 30 points available in each period, the total number of points available is given by $15 \times 30 = 450$. Thus the odds of winning are $\frac{P}{450}$.
- The prize is \$20.
- That is, you have $\frac{P}{450}$ chance of winning the prize and $1 - \frac{P}{450}$ chance of receiving \$0.

In summary, your earning from this session is comprised of a \$15 participation fee and the outcome of your lottery. The probabilities associated with your lottery depend on the number of points you have earned throughout the session. You can earn either \$0 or \$20 from the lottery.

Are there any questions?

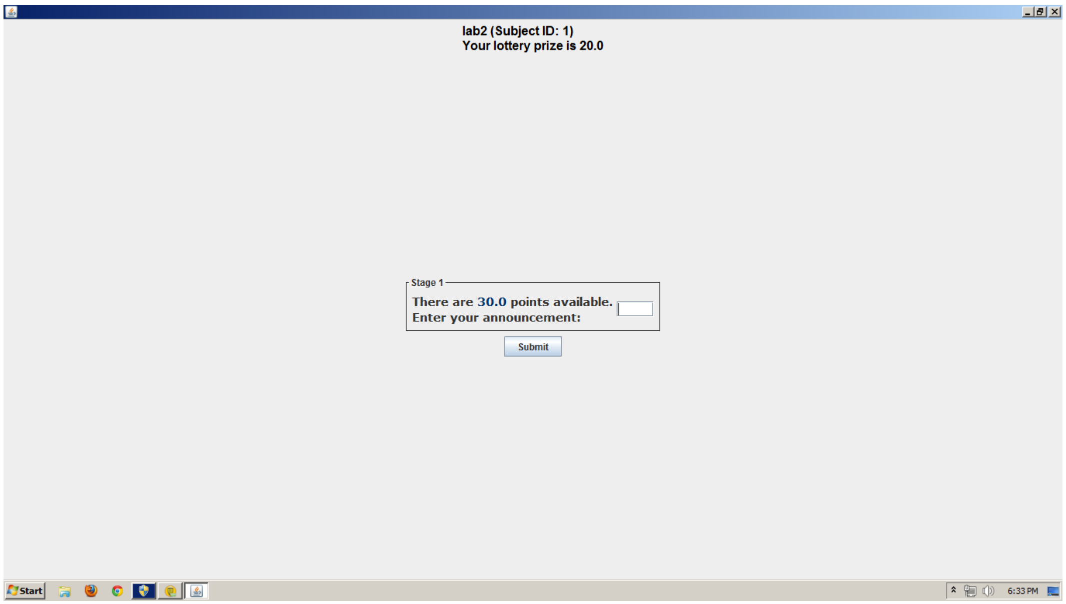
Summary

Before we start, let me remind you that:

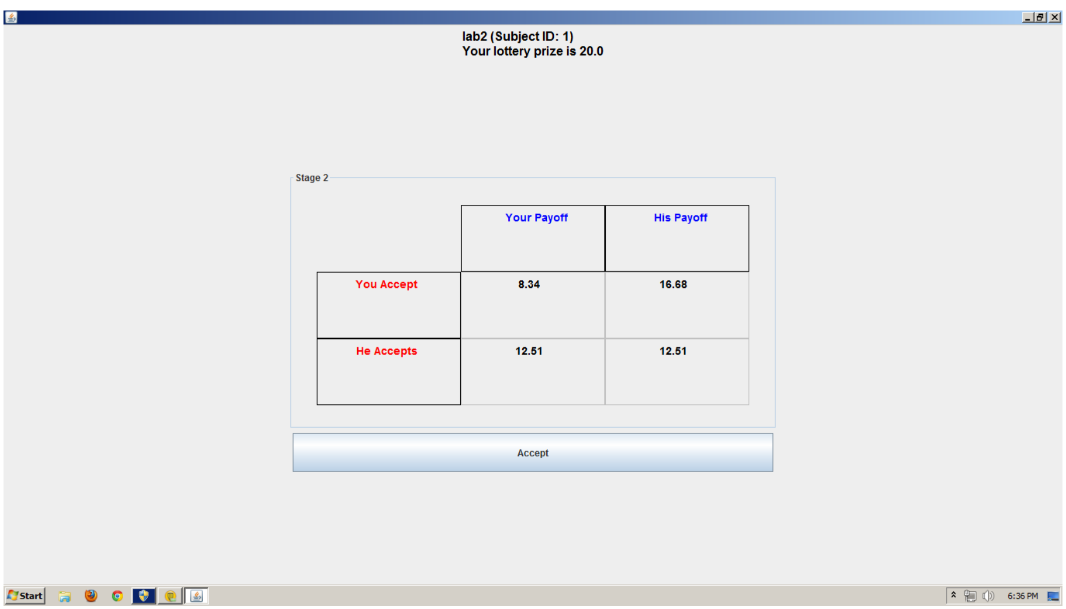
- After a period is finished, you will be randomly re-matched to a player for the next period.
- In each period, you and another player will make announcements to divide 30 points between both of you. If the sum of your two announcements is less than 30 the period ends. If the sum of the two announcements is 30 or more you move to a second stage. In the second stage, the points decrease in value until either you or the other player accepts the announcement made by the other party, at which point the period ends.
- You can make one of three possible announcements: $15, 18$, and 20.
- At the end of the session, your earnings are determined by a lottery with probabilities that depend on the number of points you have earned throughout the experiment. You can earn either \$0 or \$20 from the lottery. In addition you will receive a \$15 show-up fee.

Good Luck.

2.2 Screenshots



Stage 1



Stage 2

2.3 Unrestricted Design Comparative Static: ***Non-Estimated*** Rationality Scenarios

This subsection considers the comparative static predictions for the unrestricted design for some non-estimated rationality scenarios. Table 5 gives the predictions for the assumption that all subjects correspond to a rational player. The first set of columns gives the distribution over types, the second set gives the resulting equilibrium mixing strategy for rational players. The last set of columns gives the probability a subject will announce a particular demand (that is unconditional on knowing if the subject is rational or not, but excluding announcements made by computer players). Note, moving from *C0* to *U1* results in

Treatment		z_0	z 12	15	20	μ 12	15	20	Prob. Observing[§] 12	15	20
All Subjects Rational											
C0	Assumed	100.	0	0	0	.	.	.	.	.	.
U1	Predicted	86.7	0	0	13.3	0	0	100.	0.	0.	100.
U2	Predicted	86.7	6.7	0	66.7	30.9	0	69.1	30.9	0.	69.1

Table 5: Unrestricted design comparative static assuming all subjects rational

§ Probability of observing an announcement, excluding those made by computer players. All values in the table are probabilities represented as percentages (for reference, $\frac{1}{15} \approx 6.7\%$ and $\frac{2}{15} \approx 13.3\%$).

Given the saliency of the 50-50 norm, it is reasonable to consider the possibility that there might be 50-50 behavioral types in the subject population. Table 6 gives the predictions for the case that includes this possibility – in the experimental design, this corresponds to including a probability that a subject might be a 15-type. Three values for this probability are considered, which correspond to the numerical examples given in section 1.8.2: $\frac{1}{13}$, $\frac{5}{13}$ and $\frac{12}{13}$, referred to as scenarios A1, A8 and A12, respectively. In the first, the probability of being matched to a subject 15-type is assumed to be such that, in *U1* and *U2*, on average 1 out of the 13 other human subjects (i.e. not a computer player) will be a 15-type. Since there are also two computer players in the pool of potential matches, this would give a 1 out of 15 chance of being matched to a subject that is a 15-type. This example assumes there is a small probability of being matched to a subject behavioral type. The other two cases are chosen to be examples of a medium and a large probability of being matched to a subject behavioral type. They correspond to assuming in the control treatment that the probability of being a rational type, z_0, is $\frac{8}{13}$ and $\frac{1}{13}$. These numbers are comparable to scenarios B and C reported in the experimental design section of the paper, which estimate z from the control data assuming z_0 is equal to $\frac{1}{13}$ and $\frac{8}{13}$, respectively.

As Table 6 shows, the basic comparative static discussed in the experimental design section of the paper – namely, an increase in announcements of 20 in both *U1* and *U2*, but no increase in announcements of 12 in *U2* – is predicted for all three scenarios that include the possibility of 50-50 types subjects. Furthermore, the analysis shows that, as the probability of meeting a 50-50 type subject is increased, announcements of 15 quickly dominate the predictions for *U1* and *U2* – i.e. the probability of observing subjects announcing 15 approaches 1. The reason for this is two fold. First, increasing the probability of 50-50 types subject decreases the probability of rational types subject, and it is only rational types subject that would switch their demands to 20. Second, as the probability of 50-50 types subject increases, it is also generally the case that the probability a rational type subject mimics the 50-50 demand also increases (that is μ_{15} generally tends to increase). This is monotonically the case for *U1*, where in these scenarios all types are aggressive; in *U2*, μ_{15} eventually decreases but only after the probability of observing 15 announcements is already over 90%. Consequently, in these scenarios a large move away from demands of 15 in the control to demands of 20 in *U1* and *U2* can only be consistent with a small probability of being matched to a 50-50 type subject.

2.4 Unrestricted Design Comparative Static: ***Estimated*** Rationality Scenarios

This subsection considers general comparative static predictions for the unrestricted design where the set of behavioral types and its distribution are estimated using data from the control sessions. Two approaches are considered. The first uses only first-stage announcement data and is reported in the main text; the second attempts to use both first and second-stage data.

Treatment		z_0	z 12	15	20	μ 12	15	20	Prob. Observing§ 12	15	20
Scenario A1: $z_0 = \frac{1}{13}$ *in C0*											
C0	Assumed	7.7	0	92.3	0	0	100	0	0	100	0
U1	Predicted	6.7	0	80.	13.3	0	90.1	9.9	0.	99.2	0.8
U2	Predicted	6.7	6.7	80.	6.7	0.	66.3	33.7	0.	97.4	2.6
Scenario A8: $z_0 = \frac{8}{13}$ *in C0*											
C0	Assumed	61.5	0	38.5	0	0	100	0	0	100	0
U1	Predicted	53.3	0	33.3	13.3	0	81.4	18.6	0.	88.6	11.4
U2	Predicted	53.3	6.7	33.3	6.7	0.	86.4	13.6	0.	91.6	8.4
Scenario A12: $z_0 = \frac{12}{13}$ *in C0* (scenario A in the paper)											
C0	Assumed	92.3	0.0	7.7	0.0	0.0	100.0	0.0	0.0	100.0	0.0
U1	Predicted	80.0	0.0	6.7	13.3	0.0	53.4	46.6	0.0	57.0	43.0
U2	Predicted	80.0	6.7	6.7	6.7	0.0	69.0	31.0	0.0	71.4	28.6

Table 6: Unrestricted design comparative static adding behavioral types subject

§ Probability of observing an announcement, excluding those made by computer players. All values in the table are probabilities represented as percentages (for reference, $\frac{1}{13} \approx 7.7\%$ and $\frac{1}{15} \approx 6.7\%$).

2.4.1 Estimation Strategy: Only Announcement Data

Using the numerical solution algorithm described above any distribution, z, over a given set of behavioral types, C, can be mapped into an equilibrium announcement strategy, $\mu(z)$. Consequently, for a series of n observed announcement pairs, $\{\alpha_i\}_{i=1}^{2n}$, the likelihood of z (given C) is given by

$$L\left(z, C; \{\alpha_i\}_{i=1}^{2n}\right) = \prod_{i=1}^{2n} \mathbb{I}(\alpha_i \in C)\left(z_i + z_0\mu_i(z)\right)$$

where $\mathbb{I}(\alpha_i \in C) = 1$ if $\alpha_i \in C$ and 0 otherwise, and z_i and $\mu_i(z)$ are the elements of the vectors z and $\mu(z)$ that correspond to the type α_i.[15]

Since choosing an estimate of the set C such that there exists an observed announcement that is *not* an element of C results in a zero likelihood for any z over C, the estimated set of behavioral types is taken to be the union of all observed announcements, $\hat{C} := \cup\{\alpha_i | i = 1, ..., 2n\}$.[16] Given this choice of

[15]Note that $(z_i + z_0\mu_i(z))$ is the probability of observing an announcement α_i without knowledge of the rationality of the announcer.

[16]Note that adding elements to $\hat{C}$ that are not observed would result in these announce-

$\hat{C}$, the log likelihood is given by

$$\ln L\left(z;\{\alpha_i\}_{i=1}^{2n}\right)=\sum_{i=1}^{2n}\ln\left(z_i+z_0\mu_i\left(z\right)\right)$$

Thus, the estimated z is given by

$$\hat{z}:=\arg\max\left\{\sum_{i=1}^{2n}\ln\left(z_i+z_0\mu_i\left(z\right)\right)|z\in\Delta^{\left|\hat{C}\right|}\right\}$$

where $\Delta^{\left|\hat{C}\right|}$ is the unit simplex in $\mathbb{R}^{\left|\hat{C}\right|}$.

A limitation of this strategy is that the above function does not identify the probability of being rational, z_0. This is most simply illustrated by considering any estimate $\hat{z}$ that maximizes the above likelihood function and results in $\hat{z}_0=1-\sum_{k=1}^{K}\hat{z}_k>0$. In this case, an alternative maximizer for the log likelihood, $\tilde{z}$, can be constructed by setting $\tilde{z}_0=0$ and $\tilde{z}_k=\hat{z}_k+\hat{z}_0\mu\left(\hat{z}\right)$. Consequently, the maximization must be made conditional on a specified value for z_0. We will consider various values in our analysis.

2.4.2 Results: Only Announcement Data

Given the wide variety of announcements made under the unrestricted design, and that a parameter needs to be estimated for each unique announcement observed, only data from the last 10 periods was used. This eliminated demands that were only made during the first five periods, when subjects may have been unfamiliar with the game. Tables 7 and 8 give the comparative static predictions for the unrestricted design that follow from using the above procedure to estimate the distribution of types in the subject population. A range of values for the constraint z_0 is considered, namely $z_0=\left\{\frac{1}{13},\frac{2}{13}...,\frac{12}{13}\right\}$. $z_0=\frac{1}{13},\frac{8}{13}$ and $\frac{12}{13}$ are used for scenarios B, C and D, respectively, in the example rationality scenarios presented in the paper For all values of the constraint z_0, the basic comparative static discussed in the experimental design section of the paper is seen. In particular, an increase in announcements of 20 in both *U1* and *U2* is predicted. However, as the constraint z_0 gets close to zero, the predicted increase in demands of 20 becomes very small for both *U1* and *U2*. This is solely driven by the fact that only rational subjects

ments having strictly positive probability of being observed, and some of the other elements of $\hat{C}$ having strictly smaller probability of being observed.

would switch their demand to 20. Not surprisingly, this means that a large move away from demands of 15 in the control to demands of 20 in *U1* and *U2* can only be consistent with a larger probability of being matched to a 50-50 type subject.[17] For announcements of 12, there is no change predicted for most values of z_0. Rational types start making demands of 12 in *U2* once z_0 is $\frac{10}{13}$, however there is only an appreciable increase if z_0 is a least $\frac{11}{13}$.

2.4.3 Using First and Second Stage Data

We also considered a second approach that would use all the data from the experiment; that is, both first and second stage data. Similar to Section 2.4.1, the numerical solution algorithm can be used to calculate the equilibrium announcement strategy, $\mu(z)$, for any distribution, z, over a given set of behavioral types, C. The implied *ex-post* probabilities of being rational, $\overline{\pi}(z)$, can then be used to calculate the equilibrium distribution of concession by either player, without knowing whether they are rational or not, in any second-stage subgame. Together these give the likelihood of an observation $\{\alpha_1^i, \alpha_2^i, t^i, d^i\}$ for $i = 1, ..., n$, where t^i is the second-stage delay and d^i the identity of the player conceding should there have been a second stage in which an agreement was eventually reached.

Again, the union of all observed announcements

$$\hat{C} := \left\{\cup\left\{\alpha_1^i | i = 1, ..., n\right\}\right\} \cup \left\{\cup\left\{\alpha_2^i | i = 1, ..., n\right\}\right\}$$

can be used as the estimate for the set of behavioral types. Given this choice of $\hat{C}$, an estimate for z is constructed by finding the vector $\hat{z} \in \Delta^{|\hat{C}|}$ that maximizes the log likelihood of the control data, $\{\alpha_1^i, \alpha_2^i, t^i, d^i\}_{i=1}^n$.

This approach also confirms the basic comparative static reported in the main text. However, the results of this approach are implausible and hence not reported here.[18] The estimated probability of observing an announcement of 15 is almost trivial at 0.1% (a numerical minimum set for probabilities that are small, but strictly larger than zero). This is a consequence of the impact of the excessive delays observed in the second-stage of the

[17] Note that, in contrast to the non-estimated rationality scenario with the possibility of 50-50 types subject, rational subjects do not mimic the 50-50 demand more often as the probability of being matched to a rational type decreases.

[18] Results using data from the first three sessions of *C0* are available upon request. All the data and scripts to run this estimation are included in the supplementary materials of the main text.

Treatment		z_0	z: 12	15	20	μ: 12	15	20	Prob. Observing[§]: 12	15	20
Scenario B1: $z_0 = \frac{1}{13}$ *in C0* (scenario B in the paper)											
C0	Estimated	7.7	1.1	30.4	9.7	0.0	9.9	23.2	1.1	31.2	11.5
U1	Predicted	6.7	0.9	26.4	21.7	0.0	8.9	41.3	1.1	31.1	12.9
U2	Predicted	6.7	7.6	26.4	15.1	0.0	0.0	38.9	1.1	30.4	12.7
Scenario B2: $z_0 = \frac{2}{13}$ *in C0*											
C0	Estimated	15.4	1.1	26.7	8.8	0.	30.6	17.1	1.1	31.4	11.4
U1	Predicted	13.3	0.9	23.1	21.	0.	25.9	34.	1.1	30.7	14.
U2	Predicted	13.3	7.6	23.1	14.3	0.	20.4	30.4	1.1	29.8	13.5
Scenario B3: $z_0 = \frac{3}{13}$ *in C0*											
C0	Estimated	23.1	1.1	22.8	7.9	0.	37.2	15.2	1.1	31.4	11.4
U1	Predicted	20.	0.9	19.8	20.2	0.	30.2	32.5	1.1	29.8	15.4
U2	Predicted	20.	7.6	19.8	13.5	0.	28.6	27.6	1.1	29.4	14.3
Scenario B4: $z_0 = \frac{4}{13}$ *in C0*											
C0	Estimated	30.8	1.1	19.3	7.1	0.	39.2	14.1	1.1	31.4	11.4
U1	Predicted	26.7	0.9	16.8	19.5	0.	30.8	32.2	1.1	28.8	17.
U2	Predicted	26.7	7.6	16.8	12.8	0.	31.6	26.6	1.1	29.1	15.3
Scenario B5: $z_0 = \frac{5}{13}$ *in C0*											
C0	Estimated	38.5	1.1	16.1	6.3	0.	39.8	13.4	1.1	31.4	11.4
U1	Predicted	33.3	0.9	13.9	18.8	0.	30.6	33.1	1.1	27.8	19.
U2	Predicted	33.3	7.6	13.9	12.1	0.	32.3	26.5	1.1	28.5	16.5
Scenario B6: $z_0 = \frac{6}{13}$ *in C0*											
C0	Estimated	46.2	1.1	12.9	5.4	0.	40.1	13.	1.1	31.4	11.4
U1	Predicted	40.	0.9	11.2	18.	0.	29.3	34.8	1.1	26.4	21.5
U2	Predicted	40.	7.6	11.2	11.4	0.	32.2	27.3	1.1	27.8	18.
Scenario B7: $z_0 = \frac{7}{13}$ *in C0*											
C0	Estimated	53.8	1.1	9.8	4.5	0.	40.	12.8	1.1	31.4	11.4
U1	Predicted	46.7	0.9	8.5	17.3	0.	27.	37.3	1.1	24.4	24.6
U2	Predicted	46.7	7.6	8.5	10.6	0.	31.4	29.	1.1	26.7	20.1
Scenario B8: $z_0 = \frac{8}{13}$ *in C0 (scenario C in the paper)*											
C0	Estimated	61.5	1.1	7.1	3.6	0.0	39.4	12.7	1.1	31.4	11.4
U1	Predicted	53.3	0.9	6.2	16.5	0.0	24.2	40.9	1.1	22.	28.8
U2	Predicted	53.3	7.6	6.2	9.8	0.0	29.3	31.3	1.1	25.2	22.9

Table 7: Unrestricted design comparative static with types estimated using announcement data only

[§] Probability of observing an announcement, excluding those made by computer players. All values in the table are probabilities represented as percentages (for reference, $\frac{1}{13} \approx 7.7\%$ and $\frac{1}{15} \approx 6.7\%$). The μ columns do not sum to one since $\{12, 15, 20\}$ does not include the entire support of μ.

Treatment		z_0	z 12	z 15	z 20	μ 12	μ 15	μ 20	Prob. Observing[§] 12	Prob. Observing[§] 15	Prob. Observing[§] 20
Scenario B9: $z_0 = \frac{9}{13}$ in C0											
C0	Estimated	69.2	1.1	4.7	2.8	0.	38.6	12.5	1.1	31.4	11.4
U1	Predicted	60.	0.9	4.1	15.7	0.5	20.8	46.4	1.4	19.1	34.9
U2	Predicted	60.	7.6	4.1	9.1	0.	26.5	35.2	1.1	23.	27.1
Scenario B10: $z_0 = \frac{10}{13}$ in C0											
C0	Estimated	76.9	1.1	2.5	1.8	0.	37.6	12.5	1.1	31.4	11.4
U1	Predicted	66.7	0.9	2.1	14.9	2.5	15.3	55.3	3.	14.2	44.3
U2	Predicted	66.7	7.6	2.1	8.2	0.8	21.7	42.8	1.7	19.2	34.7
Scenario B11: $z_0 = \frac{11}{13}$ in C0											
C0	Estimated	84.6	1.	0.8	0.8	0.	36.5	12.6	1.	31.6	11.5
U1	Predicted	73.3	0.9	0.7	14.1	6.8	7.4	69.5	6.8	7.	59.7
U2	Predicted	73.3	7.5	0.7	7.4	13.6	10.8	53.9	12.5	9.9	46.5
Scenario B12: $z_0 = \frac{12}{13}$ in C0 (scenario D in the paper)											
C0	Estimated	92.3	0.5	1.2	1.0	0.0	69.2	15.4	0.5	65.1	15.2
U1	Predicted	80.0	0.4	1.0	14.2	5.5	13.4	75.4	5.5	13.6	70.6
U2	Predicted	80.0	7.1	1.0	7.5	17.2	18.5	57.1	16.3	18.3	53.7

Table 8: Unrestricted design comparative static with types estimated using announcement data only (cont.)

[§] Probability of observing an announcement, excluding those made by computer players. All values in the table are probabilities represented as percentages (for reference, $\frac{1}{13} \approx 7.7\%$ and $\frac{1}{15} \approx 6.7\%$). The μ columns do not sum to one since $\{12, 15, 20\}$ does not include the entire support of μ.

control sessions, in particular for subgames involving announcements of 15. Table 9 illustrates the restrictions the observed maximum second-stage delay places on z_{15} (see Section 1.6.2 for further details of these theoretical upper bounds). Note that values of zero represent numbers that are only rounded to zero after five decimal places (they are strictly larger than zero). Since it is not subject to these strong implications of second-stage delay, we only report estimates of the distribution over behavioral types from the strategy using only announcement data in the paper.

Subgame		Largest Observed Delay	Implied Upper Bound on z_{15}
15	16	207	0.00000
15	17.5	5.8	70.60990
15	18.5	25	34.25190
15	19	845	0.00000
15	20	920.6	0.00000
15	21	69.7	17.50820
15	22	468	0.00441
15	23	4662.5	0.00000
15	24	68.5	31.92860
15	28	35.4	66.46720

Table 9: Upper bounds on z_{15} implied by T_0

z_{15} probabilities represented as percentages (rounded to 5 d.p).

3 Results

This section provides further details of the data analysis reported and discussed in the results section of the main text. The section is organized to match the order of the results section of the paper: summary statistics on bargaining outcomes, announcement behavior, delay and concession behavior.

3.1 Bargaining Outcomes

3.1.1 All Data Analysis

Table 10 replicates the "summary of bargaining outcomes" analysis presented in Table 3 of the paper using all 15 periods of data. The overall conclusions are unaffected by the inclusion of the first five periods: on average the difference between subject earnings across treatments was small and statistically insignificant, with matches in the unrestricted design wasting more of the pie. In all treatments, the majority of matches involved a second stage more often than not.

	Treatment					
	C0	*U1*	*U2*	*R0*	*R3*	*R4*
Mean points per period	13.7	13.8	14.0	13.9	14.4	14.5
Percent of second stages	69.2	63.5	71.2	70.3	72.8	71.5
Percent of pie wasted	8.7	8.2	6.7	7.0	4.2	3.3
... conditional on second stage	12.5	13.0	9.4	10.0	5.8	4.6

Table 10: Summary of bargaining outcomes - all data

3.2 Announcement Behavior

3.2.1 Details of Statistical Tests

A Does the introduction of induced types change announcement behavior? For each of the key announcements, 8, 10, 12, 15, 18 and 20, and for each of the unrestricted design treatment variations, *U1* and *U2*, a dummy variable, indicating whether that announcement was made or not, is regressed on a treatment indicator as well as a subject-level random effect. To conduct inference, robust standard errors allowing for arbitrary correlation between observations in the same session are calculated. A t-test on the coefficient for the treatment indicator (the baseline being the control) is used to test the null hypothesis that there is no change in the proportion of a given announcement, versus the two-sided alternate hypothesis. An analogous test is used for the restricted design, except data from *R0* is used as the reference group.

P1 Are payoffs for announcements of 18 lower than those for announcements of 15 or 20 in *R3* and *R4*? Using data from announcements of 18 in the restricted design (i.e. treatments *R3* and *R4*), the payoff in points is regressed on a constant. Robust standard errors are used, allowing for arbitrary correlation between observations in the same session, as well as a subject-level random effect. A similar procedure is used for announcements of 15 or 20. These regressions give an estimate for the mean payoff in points, and an associated cluster-robust estimate of the standard error, for announcing 18 and for announcing either 15 or 20. A t-test is used to test the null hypothesis that these means are the same, versus the one-sided alternative hypothesis that the mean for announcing 15 or 20 is higher. The test has a p-value < 0.001.

P2 Are payoffs for announcements of 20 larger than for announcements of 15 in *U1*, *U2*, *R3* and *R4*? Using data from announcements of 20 in treatments *U1*, *U2*, *R3* and *R4*, the payoff in points is regressed on a constant as well as a subject-level random effect. To conduct inference, robust standard errors allowing for arbitrary correlation between observations in the same session are calculated. A similar procedure is used for announcements of 15. These regressions give an estimate for the mean payoff in points, and an associated cluster-robust estimate of the standard error, for announcing 20 and for announcing 15. A t-test is used to test the null hypothesis that these means are the same, versus the one-sided alternative hypothesis that the mean for announcing 20 is higher. The test has a p-value < 0.001.

P3 Are payoffs for concessionary announcements smaller than for non-concessionary announcements? Using data from concessionary announcements (i.e. strictly smaller than 15) in all treatments except *R0* and *R3*, where concessionary announcements were not possible, the payoff in points is regressed on a constant as well as a subject-level random effect. To conduct inference, robust standard errors allowing for arbitrary correlation between observations in the same session are calculated. A similar procedure is used for non-concessionary announcements (i.e. greater than or equal to 15). These regressions give an estimate for the mean payoff in points, and an associated cluster-robust estimate of the standard error, for making a concessionary announcement and for making a non-concessionary announcement. A t-test is used to test the null hypothesis that these means are the same, versus the one-sided alternative hypothesis that the mean for non-concessionary announcements is higher. The test has a p-value < 0.001.

3.2.2 All Data Analysis

Figure 2 and Table 11 replicate the "subject announcements in the first stage" and "summary of key announcements" analysis of Figure 1 and Table 4 from the paper using all 15 periods of data. The pattern of subject announcements in the unrestricted design, illustrated in Figure 2, is similar to that illustrated in the corresponding figure in the paper. Similarly, Table 11 leads to the same main conclusions as those drawn from its counterpart in the paper: the introduction of an induced 20 type in the unrestricted design leads to a significant increase in demands for 20; a complementary type emerges in the *U1* treatment; demands of 15 and 20 dominate the restricted design sessions with much fewer announcements of 18 than 20, and, finally, demands of 8 in *R4* are infrequent.

Finally, table 12 replicates the "average payoffs in points" analysis from the paper using all fifteen periods of data. Including the first five periods does not change the main conclusion of this analysis: in the restricted design,

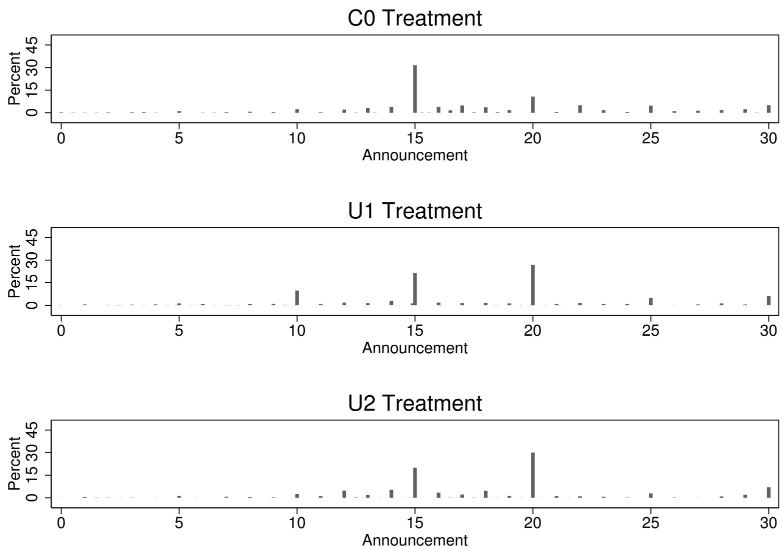


Figure 2: Subject announcements in the unrestricted design - all data

Treatment	Proportion of Announcements of 8		10		12		15		18		20	
C0	0.7	(6)	2.3	(20)	2.1	(18)	31.6	(275)	3.7	(32)	10.7	(93)
U1	0.8	(8)	9.8	(103)	1.8	(19)	21.6	(227)	1.6	(17)	27.0	(283)
p-value		0.852		0.000		0.738		0.224		0.047		0.033
U2	0.6	(6)	2.6	(27)	4.8	(50)	20.0	(210)	4.7	(49)	30.1	(316)
p-value		0.783		0.786		0.108		0.139		0.579		0.000
R0	.	(.)	.	(.)	.	(.)	54.7	(574)	9.2	(97)	36.1	(379)
R3	.	(.)	.	(.)	.	(.)	50.6	(493)	7.7	(75)	41.7	(407)
p-value		.		.		.		0.522		0.448		0.314
R4	5.3	(48)	.	(.)	.	(.)	40.0	(360)	11.1	(100)	43.6	(392)
p-value		.		.		.		0.003		0.481		0.165

Table 11: Summary of key announcements - all data

Number of observations are given in parentheses.
P-values give the probability of a type I error in a two-sided t-test of the difference between the control and the treatment listed in the prior row (using robust standard errors clustered at the session level). See statistical test A of section 3.2.1.

	Average Payoff to Announcements of					
Treatment	8	10	12	15	18	20
C0	4.2	10.8	12.6	14.0	14.4	13.1
	(2.85)	(1.03)	(0.95)	(0.52)	(1.02)	(0.35)
U1	8.2	11.0	11.4	12.6	12.8	13.7
	(0.25)	(0.50)	(1.07)	(0.52)	(0.63)	(0.86)
U2	11.0	12.5	13.2	13.3	13.8	14.8
	(1.64)	(0.68)	(0.46)	(0.24)	(0.36)	(0.30)
R0				13.9	14.0	13.4
				(0.21)	(0.44)	(0.44)
R3				12.8	11.5	14.4
				(0.36)	(0.55)	(0.27)
R4	9.9			13.9	13.0	14.5
	(0.28)			(0.28)	(0.39)	(0.24)

Table 12: Average payoffs in points - all data

Parentheses give robust standard deviations, clustered at the session level.

demands of 18 lead on average to lower payoffs relative to demands of 15 and 20 (test P1 has p-value < 0.001); in sessions with induced types, announcements of 20 lead on average to greater payoffs than 15 (test P2 has p-value < 0.001); making concessionary demands leads on average to lower payoffs (test P3 has p-value < 0.001), and there are few demands that on average earn more than half the pie.

3.2.3 Non-Parametric Analysis on Session Averages

Table 13 replicates the "summary of key announcements" analysis from the paper except using a non-parametric test on session averages.[19] This leads to the same main conclusions as those drawn from its counterpart in the paper: the introduction of an induced 20 type in the unrestricted design leads to a significant increase in demands for 20 (p-value 0.076 in *U1* and 0.028 in *U2*); a complementary type emerges in the *U1* treatment (p-value 0.009); demands of 15 and 20 dominate the restricted design sessions with much fewer announcements of 18 than 20, and, finally, demands of 8 in *R4* are infrequent.

[19] Note that the reported p-values for all rank-sum and signed-rank tests are calculated using an approximated distribution. For the highlighted results in the main text, the approximation does not result in a different categorization of significance – that is, whenever the exact value is larger than the approximate value and it does not lie on the other side of a 10%, 5% or 1% cut-off.

Treatment	Proportion of Announcements of 8		10		12		15		18		20	
C0	0.5	(3)	1.7	(10)	1.0	(6)	29.8	(173)	4.5	(26)	10.9	(63)
U1	0.7	(5)	9.3	(65)	2.0	(14)	20.7	(145)	1.4	(10)	25.1	(176)
p-value		0.572		0.009		0.595		0.347		0.209		0.076
U2	0.4	(3)	1.9	(13)	4.0	(28)	17.4	(122)	4.7	(33)	31.9	(223)
p-value		0.906		0.916		0.172		0.251		0.675		0.028
R0	.	(.)	.	(.)	.	(.)	55.3	(387)	9.6	(67)	35.1	(246)
R3	.	(.)	.	(.)	.	(.)	49.4	(321)	6.2	(40)	44.5	(289)
p-value		.		.		.		0.465		0.172		0.347
R4	4.5	(27)	.	(.)	.	(.)	39.0	(234)	9.8	(59)	46.7	(280)
p-value		.		.		.		0.076		0.917		0.346

Table 13: Summary of key announcements - non-parametric tests using data from the last ten periods

Number of observations are given in parentheses.
P-values give the significance of the difference between the control and the treatment listed in the prior row using a Wilcoxon rank-sum test on session averages.

Tests P1, P2 and P3 can also be replicated using a signed-rank test on sessions averages. All three tests remaining statistically significant at the usual levels: P1 has a p-value of 0.009, P2 a p-value < 0.001 and P3 a p-value < 0.001. Thus, all the conclusions presented in the main text are robust to carrying out the analysis with session-level, rather than individual-level, data.

3.3 Delay in the Second Stage

3.3.1 Details of Statistical Tests

D1 Are observed average (mean) delays significantly above their theoretical upper bounds? For each observation that leads to a second stage, the ratio $\left(\frac{t - \text{mean upper bound}}{\text{mean upper bound}}\right)$ is calculated, where t is the time spent in the second stage. This ratio is regressed on a complete set of treatment indicator variables. To conduct inference, robust standard errors allowing for arbitrary correlation between observations in the same session are calculated. A Wald test is used to test the null hypothesis that all the coefficients are jointly equal to zero. The null hypothesis is rejected if the coefficients are significantly larger than zero. The test has a p-value < 0.001.

D2 Are observed delays significantly closer to their theoretical upper bounds in the restricted design than in the unrestricted design? The same ratio, defined in D1 but only using sessions including induced types, is regressed on a complete set of design indicator variables. To conduct inference, robust standard errors allowing for arbitrary correlation between observations in the same session are calculated. A Wald test is used to test the null hypothesis that the two coefficients are equal against the two-sided alternate hypothesis. The test has a p-value of 0.014. The test is also carried out on a subset of the unrestricted design data referred to as explicit subgames. These only include announcement combinations 15-20 and 20-20 from treatment *U1*, and announcement combinations 12-20, 15-20 and 20-20 from treatment *U2*. The test, using only the explicit subgames from the unrestricted design, has a p-value of 0.109. Finally, an analogous test is run comparing delay in *R0* to delay in both *R3* and *R4*, and has a p-value < 0.001.

3.3.2 All Data Analysis

Table 14 replicates the "second-stage delay" analysis from the paper for all 15 periods of data. Including the first five periods of data does not change the main conclusions of this analysis. Mean delay values are generally above their respective upper bounds, with test D1 having a p-value of 0.000 for all treatments. The move from the unrestricted to restricted design continues to result in overall delay statistics that are closer to the upper bound: the average ratio of mean delay to upper bound is 5.53 in the unrestricted design and 4.53 in the restricted design, and test D2 has a p-value of 0.019; restricting attention to explicit subgames in the unrestricted design continues to result in a more favorable comparison with the restricted design (test D2 using this sub-sample has a p-value of 0.111), while there continues to be a significant difference between treatments with explicit types in the restricted design (p-value < 0.001).

3.3.3 Non-Parametric Analysis on Session Averages

Tests D1 and D2 above can be carried out using non-parametric tests on session averages. For the D1 test, a sign-test is used to test whether the ratio is different from zero, and has a p-value < 0.001. For the D2 tests, a

Treatment	Subgame[§] α_L	α_H	Obs Freq	%	Mean	Delay Bound	Ratio[§§]
C0	*All*		259		204.2		4.73
	15	17	16	6.2	32.8	7.1	
	15	20	25	9.7	117.9	20.0	
U1	*All*		234		192.6		6.06
	15	20	51	21.8	74.6	20.0	
	20	20	49	20.9	313.5	50.0	
U2	*All*		260		104.1		5.85
	15	20	46	17.7	79.4	20.0	
	20	20	38	14.6	77.4	50.0	
R0	*All*		369		139.2		5.16
	15	18	56	15.2	49.9	11.1	
	15	20	206	55.8	99.9	20.0	
	18	20	39	10.6	199.2	36.4	
	20	20	67	18.2	297.4	50.0	
R3	*All*		283		63.3		4.74
	15	18	22	7.8	49.7	11.1	
	15	20	164	58.0	49.2	20.0	
	18	20	25	8.8	112.5	36.4	
	20	20	70	24.7	84.3	50.0	
R4	*All*		233		49.6		3.28
	15	18	27	11.6	23.5	11.1	
	15	20	109	46.8	23.3	20.0	
	18	20	25	10.7	69.6	36.4	
	20	20	64	27.5	98.7	50.0	

Table 14: Second-stage delay - all data

[§] Only subgames with at least 10 observations reported.
[§§] Weighted mean of median delay to upper bound.

rank-sum test is used to compare treatments and designs. For all sessions with induced types, this has a p-value of 0.016; the subsample that only uses explicit subgames has a p-value of 0.650. The test that compares *R0* to both *R3* and *R4* has a p-value of 0.003. Thus, the conclusions presented in the main text are robust to carrying out the analysis with session-level, rather than individual-level, data.

3.3.4 Further Evidence of Excessive Delay

As shown in section 1.6.1, the longest delay observed in a given subgame has implications for the distribution over behavioral types, since it is an estimate for T_0. Table 15 shows the upper bounds on z that are implied by observed delay for subgames with at least 8 observations, for the four treatments that include induced types. This analysis provides further evidence of excessive delay, since many of the implied upper bounds are smaller than the probability of being matched to the induced types for that treatment.[20] Since this latter probability is a lower bound, the observed delays in these subgames are too long to be consistent with this quantitative prediction of the model.

Subgame		Obs		Delay	Upper Bounds§	
α_L	α_H	Freq	%	T_{max}	z_L	z_H
U1						
15	20	32	10.5	437.0	0.0	0.0
20	20	24	7.9	1,713.0	0.0	0.0
U2						
15	20	29	9.6	697.7	0.0	0.0
20	20	27	8.9	306.6	4.7	4.7
R3						
15	20	119	45.9	411.5	0.0	0.0
18	20	16	6.2	572.5	0.0	0.1
20	20	49	18.9	356.2	2.8	2.8
R4						
15	18	15	6.9	160.9	0.0	0.2
15	20	73	33.5	201.4	0.2	1.8
18	20	17	7.8	384.6	0.3	0.8
20	20	51	23.4	691.8	0.1	0.1

Table 15: Further evidence of excess delay in the second stage

Only subgames with at least 8 observations reported.

§ Upper bounds on z (in %) implied by T_{max}.

[20] Note that $\frac{1}{15} \approx 6.7\%$.

3.4 Concession in the Second Stage

3.4.1 Details of Statistical Tests

C1 Is interior concession by the lower announcer significantly more likely than by the higher announcer? To address this question, data from asymmetric subgames that lead to a second stage and ended with interior concession is used. An indicator variable, indicating whether the lower announcer conceded, is regressed on a complete set of treatment indicator variables. To conduct inference, robust standard errors allowing for arbitrary correlation between observations in the same session are calculated. Wald tests are used to test the null hypotheses that the coefficient for the treatment is equal to 0.5.[21] The null hypothesis is rejected if the coefficient is significantly larger. The tests have p-values of 0.999, 0.003 and 0.364, for treatments *C0*, *U1* and *U2*, respectively. For the join test for treatments *U1* and *U2*, the p-value is 0.582. A similar test, except using an indicator for whether the treatment included induced types or not, is used to compare the outcomes from *U1* and *U2* with those from *C0*, and has a p-value of 0.010. The test is also run on a subset of the unrestricted design data, referred to as explicit subgames. These only include announcement combinations 15-20 from treatment *U1*, and announcement combinations 12-20 and 15-20 from treatment *U2*. No data from the control treatment is included. The test, using only the explicit subgames from the unrestricted design, has a p-value of 0.004. Analogous tests are also run for the restricted design. The p-value for the restricted design as a whole is 0.129, while the comparison of *R0* versus *R3* and *R4* is 0.008.

C2 Is initial concession significantly different in R4 compared to R3? To address this question, data from asymmetric subgames in the restricted design that lead to a second stage and ended with initial concession is used. A dummy variable, indicating whether the higher announcer conceded, is regressed on a complete set of treatment indicator variables. To conduct inference, robust standard errors allowing for arbitrary correlation between observations in the same session are calculated. A Wald test is used to test the null hypothesis that the

[21]Note that only data for interior concession is included. Thus, the proportion of lower announcers conceding being larger than 0.5 is equivalent to this proportion being larger than the corresponding proportion for higher announcers.

coefficient for *R4* is equal to the coefficient for *R3*. The test has a p-value of 0.001.

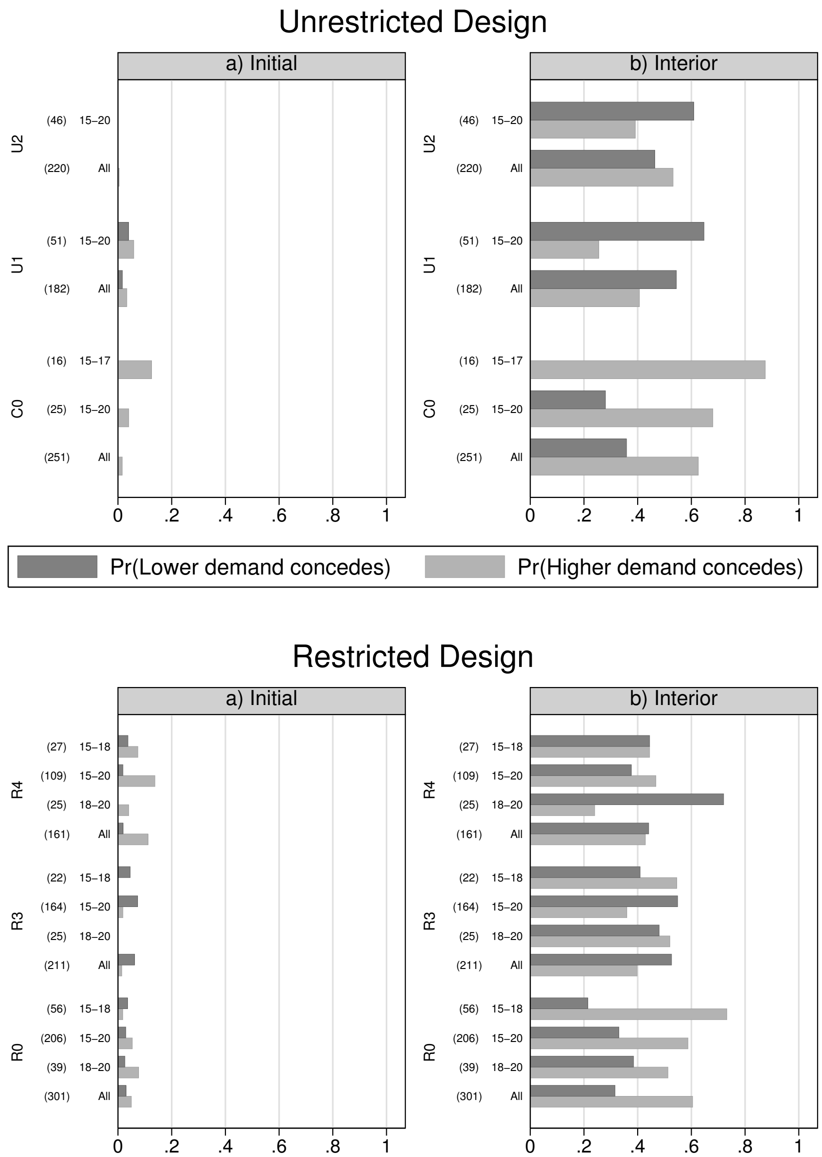
3.4.2 All Data Analysis

Figure 3 replicates the "concession behavior in the asymmetric subgames" analysis of Figure 2 from the paper using all 15 periods of data. Including the first five periods of data does not change the main conclusions of this analysis. For the unrestricted design, overall interior concession does not appear consistent with the predicted pattern. Test C1 has p-values 0.980, 0.010 and 0.897, for treatments *C0*, *U1* and *U2*, while the joint test for the latter two is 0.582, which is significant improvement on *C0* (p-value 0.010). Again, the explicit subgames in *U1* and *U2* are consistent with the predicted pattern (p-value 0.000). For all three treatments there is little evidence for initial concession.

For the restricted design, interior concession in *R3* and *R4* is more often by the lower announcer, as predicted, but the opposite is true in *R0* (test C1 on all restricted sessions has p-value 0.150, while the comparison between *R0* with *R3* and *R4* has p-value 0.062). In *R4* there is evidence of initial concession by the higher announcer, while there is no such evidence in *R3*, as is predicted; test C2 has a p-value < 0.001.

3.4.3 Non-Parametric Analysis on Session Averages

Tests C1 and C2 can be carried out using non-parametric tests on session averages. For test C1, a sign-test is used to test whether the interior concession by the lower announcer is equal to that by the higher announcer and has p-values 0.063, 0.375 and 0.375 for treatments *C0*, *U1* and *U2*, respectively. For the join test for treatments *U1* and *U2*, the p-value is 1.000. A ranksum test is used to compare the outcomes from *U1* and *U2* with those from *C0*, and has a p-value of 0.037. Analogous tests are also run for the restricted design. The p-value for the restricted design as a whole is 1.000, while the comparison of *R0* versus *R3* and *R4* is 0.007. Thus, all the conclusions presented in the main text are robust to carrying out the analysis with session-level, rather than individual-level, data. For the C2 test, a rank-sum test is used to compare initial concession in *R3* with *R4*, but has a p-value of 0.332.



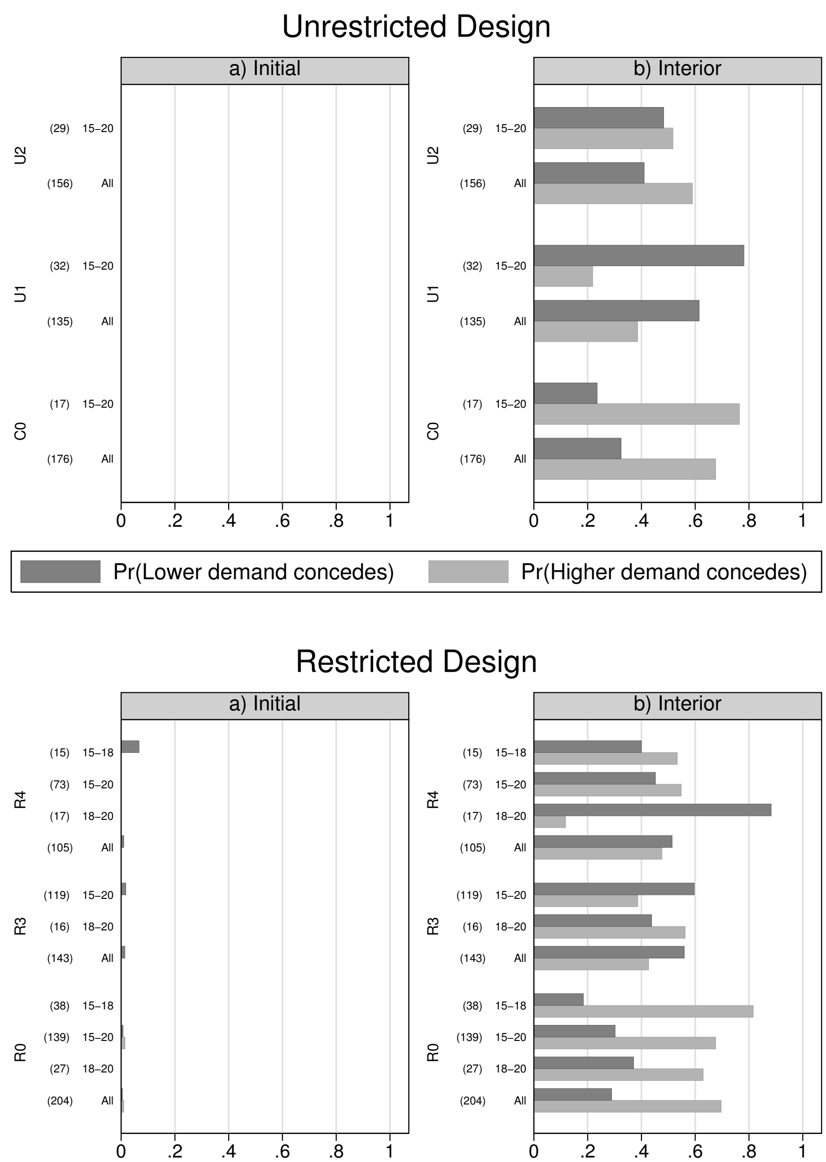
Asymmetric subgames with at least 15 observations. Parentheses give number of observations.
Initial concession is represented by concession within the first 2 seconds.
Interior concession is represented by concession after the first 2 seconds.

Figure 3: Concession Behavior in Asymmetric Subgames - All Data

3.4.4 Robustness Check on the Threshold Used to Define Initial Concession

Figures 4, 5 and 6 replicate the "concession behavior in the asymmetric subgames" analysis of Figure 2 from the paper using different thresholds for defining initial concession. The thresholds are 1 second, 5 seconds and 10 seconds, respectively. Note that this analysis only uses data from the last 10 periods. Changing the initial concession threshold does not change the main conclusions of this analysis. For the unrestricted design, interior concession in the control treatment is still mostly made by the higher announcer, and in the *U1* and *U2* treatments the 15-20 subgames are more consistent with the predicted pattern. For all three treatments, there is more evidence for initial concession when the threshold is set to 5 or 10, as would be expected, but with the sole exception of 15-20 subgame in *U2* using the 5 second threshold, the predicted pattern for initial/interior concession does *not* emerge (that is higher announcer concession initially followed by lower announcer concession in the interior).

For the restricted design, interior concession in both the *R3* and *R4* is more likely by the lower announcer, as predicted, although concession by the higher announcer eventually catches up in *R3* if the threshold is set large enough. This evidence is in stark contrast to *R0* where interior concession is always by the higher announcer. Decreasing the threshold to 1 second eliminates the evidence for initial concession in *R4*, while increasing the threshold to 5 or 10 seconds increases initial concession for all treatments, as would be expected. However, when there is evidence for initial concession, it is more likely to be by the higher announcer in *R4* than in *R3*.



Asymmetric subgames with at least 15 observations. Parentheses give number of observations.
Initial concession is represented by concession within the first 2 seconds.
Interior concession is represented by concession after the first 2 seconds.

Figure 4: Concession Behavior in Asymmetric Subgames - Initial Concession ≤ 1 Second, Last 10 Data

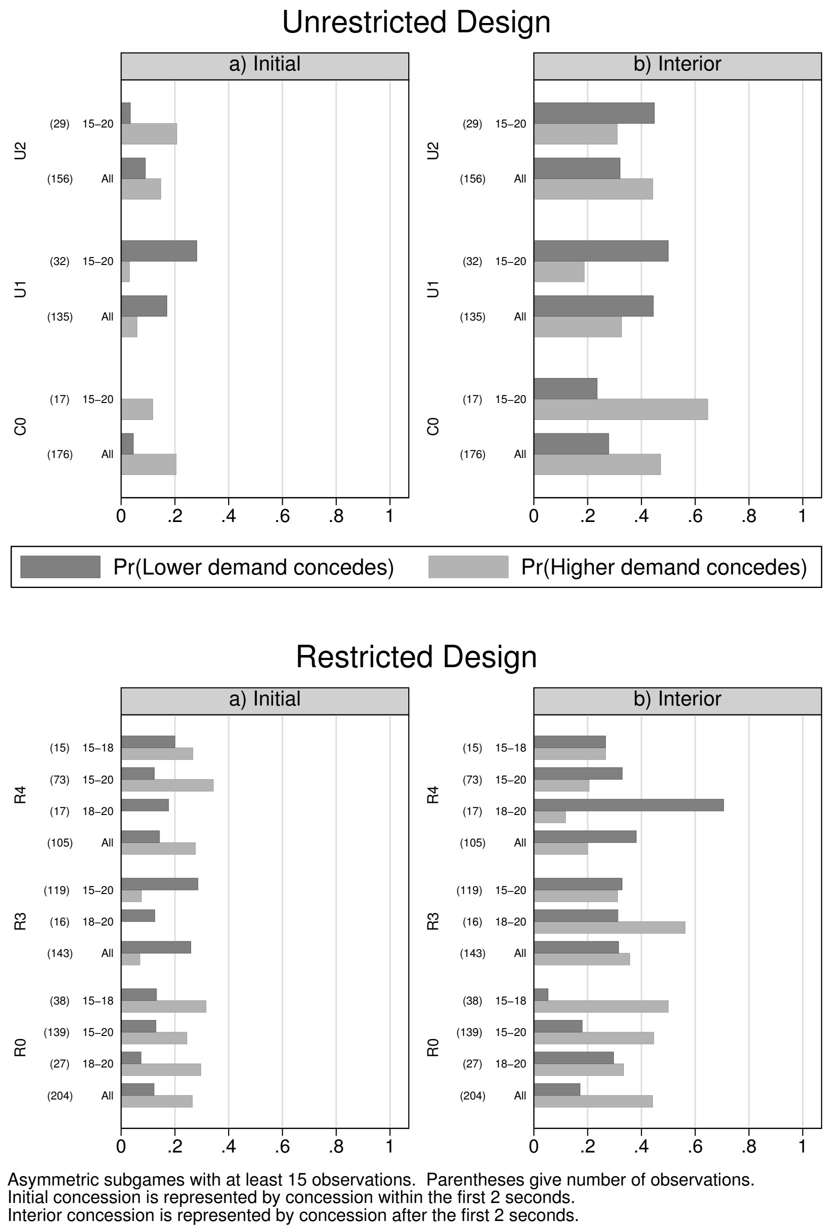


Figure 5: Concession Behavior in Asymmetric Subgames - Initial Concession ≤ 5 Seconds, Last 10 Data

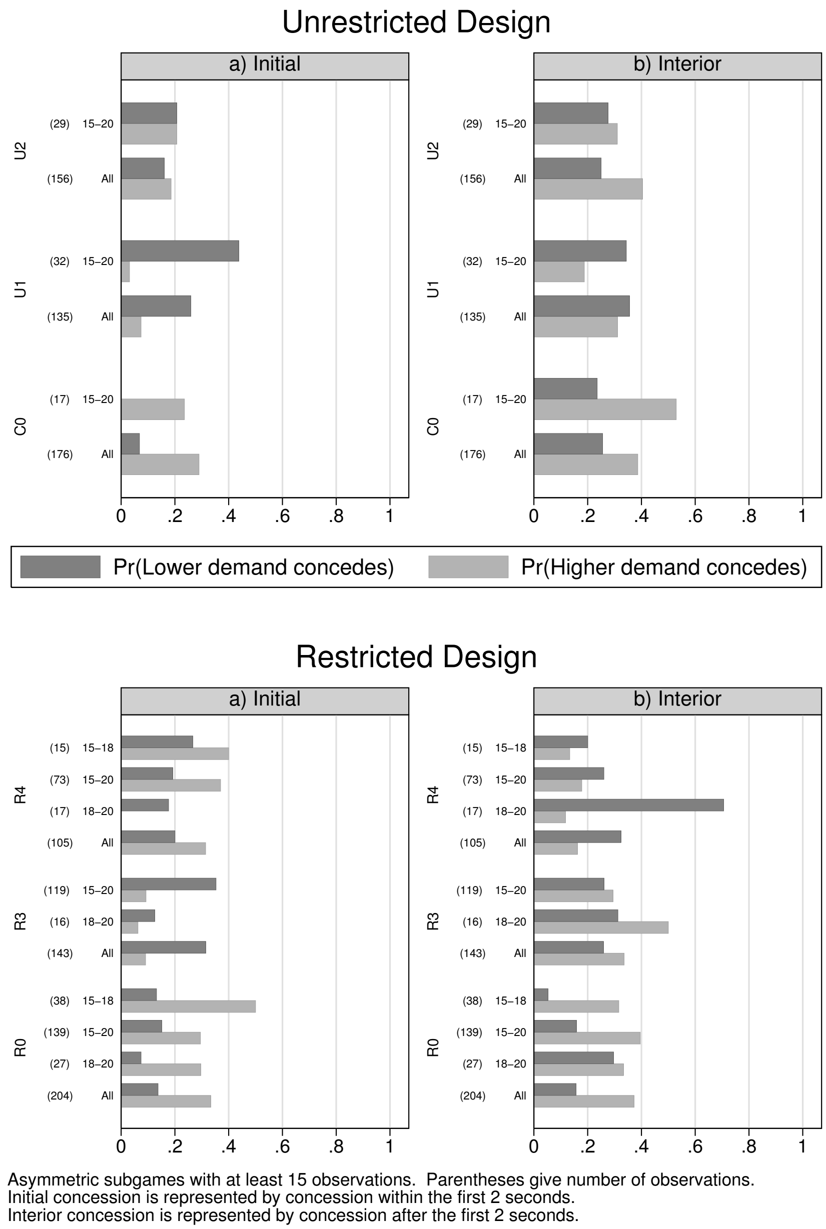


Figure 6: Concession Behavior in Asymmetric Subgames - Initial Concession $\leq$ 10 Seconds, Last 10 Data

4 Relaxing the Behavior of Obstinate Types

This section provides a simple extension of the stylized model of *Abreu and Gul* (2000) that allows for strategically unresponsive types that are more flexible in their second-stage behavior. In the experiment, the computer types were programmed to be inflexible in the manner assumed in *Abreu and Gul* (2000). However, should a rational player believe that they might be matched with subjects that are strategically unresponsive – for example, the subject population could contain fair, 50-50 behavioral types – then it seems unreasonable to suppose that such subjects would be so inflexible as to never concede no matter how much the pie has already shrunk. In this sense, this modification is in the spirit of the perturbation of the standard alternating-offers bargaining given in *Binmore and Swierzbinski* (2006), and is consistent with the observation from the experiments that outright disagreement – or something approximating this – is not observed in matches containing two subject players.

First, consider the situation where obstinate types can concede in the concession game, but this happens with a given, constant hazard rate that does not differ across subgames. The hazard rate is assumed to be small so that the nature of equilibrium behavior is not fundamentally changed. In particular, the hazard rate is small enough so that there is no possible subgame – with a strictly positive probability that the other player is an obstinate type – where a rational-type player would find it optimal to never concede, waiting instead for eventual concession by a possibly obstinate type.[22] This minimal adjustment to the behavior of obstinate types ensures that these types maintain their commitment to a bargaining norm and their lack of strategic responsiveness, while allowing them to exhibit less than complete inflexibility in the concession game.

The result is that rational players, in equilibrium, concede at a slower rate during the interior of the concession stage, exactly compensating for the introduced probability that an obstinate type might themselves concede. However, rational players that make higher demands will generally find themselves in an even weaker position in subgames in which they meet

[22] The motivation for relaxing the complete-inflexibility-of-obstinate-types assumption is that it seems unreasonable to suppose that a player would under no circumstances concede in a concession game. We rule out situations where the probability that an obstinate type might concede is large enough that rational types would choose to engage in exactly the behavior considered to be unreasonable.

incompatible demands that ask for less than them. As a consequence, the predictions for overall delay and mimicking of more demanding types, when compared to the standard case, will be ambiguous.

However, it seems most likely, and is a consistent interpretation of our results and prior experiments, that the most common behavioral type among subjects is the 50-50 type. If we consider the intermediate case where there are, in addition to the obstinate types of the *Abreu and Gul* (2000) variety implemented using computer players, also the possibility of subjects being 50-50 obstinate types, but of the less inflexible variety, then the results align with what is observed in the experiment. The possibility that the 50-50 obstinate type might concede reduces the "weakness" against the 50-50 demand of rational types that make larger demands. Consequently, the probability of an initial concession is reduced, while the probability of mimicking more demanding types is increased. In summary, this intermediate case results in both greater delay, conditional on eventual agreement, and an increase in more demanding announcements.

The details of the set-up with the modified obstinate types and subsequent equilibrium behavior is detailed in the next subsection. A separate subsection compares the two extreme cases – namely the only-modified-types case and the standard-types case. A third subsection considers the intermediate case in which 50-50 obstinate types are of the modified form, whilst the others are of the standard form.

4.1 Set Up and Equilibrium Behavior - Modified Types Only

Two agents bargain over a pie of size one in two stages exactly as described in section 1.1. With probability z_k, a player is an α_k-type that always demands $\alpha_k \in (0,1)$ in the first stage. The set of behavioral types is finite and denoted by $C := \{\alpha_1, \ldots, \alpha_K\}$, with $\alpha_i < \alpha_{i+1}$, for $i = 1, ..., K-1$, and $\alpha_K \geq \frac{1}{2}$. If such a behavioral type finds themselves in the second-stage concession game then, irrespective of the demand of the other player, they concede at time t with probability given by the cumulative distribution functions

$$F_\gamma(t) = \ 1 - e^{-\gamma \cdot t}$$

where $\gamma > 0$ is such that

$$\gamma \leq \ \lambda_{LH} := \ \frac{r(1-\alpha_L)}{(\alpha_L + \alpha_H - 1)} \tag{5}$$

for any $\alpha_L, \alpha_H \in C$ such that $\alpha_L \leq \alpha_H$ and $\alpha_L + \alpha_H > 1$.

The assumptions underlying the inequalities in equation 5 ensure that the probability an obstinate type would concede is small enough so that, in equilibrium, a rational player would concede instantly rather than wait, once it was evident that their opponent was an obstinate type.[23] It is no surprise that having obstinate types concede at a slower rate than the hazard rate λ_{LH} is sufficient: the latter is calculated as exactly the rate that would make a rational types indifferent between conceding or not.

As in the standard case, a key property of the equilibrium is that rational players only choose demands that mimic some behavioral type. Consequently, players can be identified by the element of C that they announced in the first stage. In a symmetric equilibrium, define μ'_k to be the probability that a rational player announces demand α_k. Given this equilibrium, the probability that a player is irrational given an announcement α_k is given by

$$\overline{\pi}'_k = \frac{z_k}{z_k + z_0 \mu'_k} \tag{6}$$

Now, suppose a rational player announced α_k and faces an opponent who has announced α_l, where $\alpha_l + \alpha_k > 1$ and $\mu'_l > 0$. The unique equilibrium play in this incomplete information war of attrition game is given by a mixed strategy over the concession. The α_k player concedes with a constant hazard rate, λ'_{kl}, given by

$$\lambda'_{kl} = \frac{r\,(1-\alpha_k)}{\alpha_k + \alpha_l - 1} - \overline{\pi}'_k \gamma \tag{7}$$

over the interval $[0, T'_0]$, where $T'_0 = \min\left(T'_{kl}, T'_{lk}\right)$ and $T'_{kl} = \frac{-\ln\left(\overline{\pi}'_k\right)}{\lambda'_{kl}}$ and $T'_{lk} = \frac{-\ln\left(\overline{\pi}'_l\right)}{\lambda'_{lk}}$.[24] The notational convention of section 1 is maintained so that

[23]It is only necessary to consider the case of rational players that demand more than their opponent since, in equilibrium, these rational types concede at a slower rate than the rational types that made the smaller demand. Consequently, the only case that needs to be ruled out is the one where the higher-demand rational players do not concede initially, reach the point in the concession game where almost surely a lower-demand rational type would have conceded and yet still prefer to wait it out for the obstinate type to concede. Equation 5 rules this scenario out for all subgames that could happen with strictly positive probability in equilibrium.

[24]So long as it remains possible that their opponent is a rational-type, a rational player who announced α_k is indifferent between conceding and not conceding at a time t if $r\,(1-\alpha_l) = [\alpha_k - (1-\alpha_l)]\,\lambda_{lk}$, where λ_{lk} is the hazard rate for concession by the opponent

the distribution of concession by rational α_k-announcers is given by $\frac{\hat{F}'_{kl}}{1-\overline{\pi}'_k}$, where

$$\hat{F}'_{kl}(t) = \begin{cases} 1 - c'_{kl} e^{-\lambda'_{kl} t}, & \text{for } t \in [0, T'_0] \\ 1 - \overline{\pi}'_k, & \text{for } t > T'_0 \end{cases} \tag{8}$$

and $c'_{kl} = \overline{\pi}'_k e^{\lambda'_{kl} T'_0}$ and $(1 - c'_{kl})(1 - c'_{lk}) = 0$. The notation is maintained to facilitate comparison, despite the presence of $\overline{\pi}'_k$ in the equation 7 making the notation less convenient.

As in the standard case, the value of T'_{kl} is a measure of the α_k rational player's "strategic" weakness when facing an α_l player: if $T'_{kl} > T'_{lk}$, then the α_k rational player will have to concede at time $t = 0$ with strictly positive probability (mass), given by $q'_{kl} := (1 - c'_{kl})$. In a symmetric equilibrium, it will always be the rational player that made the higher announcement that will concede with positive probability at time $t = 0$, if at all.

4.2 Comparison: Modified Versus Standard Types

This subsection considers how equilibrium behavior by rational types differs in the modified case to the standard case when the same set of parameters are applied. The primary concern will be whether, in a subgame following announcements α_L and α_H, in which $\alpha_H \geq \alpha_L$ and $\alpha_L + \alpha_H > 1$, the "weakness" of a rational α_H-announcer is more or less in the modified case compared to the standard case. Intuitively, weakness is the amount by which rational types overshoot their rational counterparts, who made a less demanding offer, by needing to concede at a slower rate.[25] This overshoot determines the amount of initial concession that is then necessary; the larger the overshoot, the more initial concession needed. Noting that the equation for T'_{kl} implies $\overline{\pi}'_H = e^{-\lambda'_{HL} T'_{HL}}$ and plugging this into the expression for c'_{HL} gives

$$q'_{HL} := \; 1 - c'_{HL} = \; 1 - e^{\lambda'_{HL}\left(T'_{LH} - T'_{HL}\right)}$$

Thus, a player's weakness in a subgame is given by $\lambda'_{HL}(T'_{LH} - T'_{HL})$. The closer to zero this is, the closer c'_{HL} is to one and the smaller the initial concession is. If the modified-types case improves the position of the higher

unconditional on knowing whether the opponent is rational or not. In this modified setting λ_{lk} is a convex combination of λ'_{lk} and γ, with weights $(1 - \overline{\pi}'_k)$ and $\overline{\pi}'_k$, respectively.

[25] Note that this is "converted" into probability units by multiplying by the hazard rate that the higher announcer concedes at.

announcer, then it should be that

$$\begin{array}{rcl} \lambda'_{HL}\left(T'_{LH}-T'_{HL}\right) & < & \lambda_{HL}\left(T_{LH}-T_{HL}\right) \\ & \Longleftrightarrow & \\ \ln\left(\overline{\pi}'_{H}\right)-\ln\left(\overline{\pi}'_{L}\right)\dfrac{\lambda'_{HL}}{\lambda'_{LH}} & > & \ln\left(\overline{\pi}_{H}\right)-\ln\left(\overline{\pi}_{L}\right)\dfrac{\lambda_{HL}}{\lambda_{LH}} \end{array}$$

Consider the following comparative static: starting from $\gamma = 0$ (i.e. the standard-type case) what is the effect of marginally increasing γ on second-stage behavior? Plugging in $\mu' = \mu$ results in comparing $\frac{\lambda'_{HL}}{\lambda'_{LH}}$ to $\frac{\lambda_{HL}}{\lambda_{LH}}$. If the former is larger than the latter, then marginally increasing γ results in a less weak position for the higher demander. In words, we are asking whether there is a reduction in the proportional difference between the concession rates of the higher and lower-announcers. Re-arranging and using the definition of λ'_{kl} gives

$$\begin{array}{rcl} \dfrac{\lambda'_{HL}}{\lambda'_{LH}} & > & \dfrac{\lambda_{HL}}{\lambda_{LH}} \\ & \Longleftrightarrow & \\ \dfrac{\lambda'_{HL}}{\lambda_{HL}} & > & \dfrac{\lambda'_{LH}}{\lambda_{LH}} \\ & \Longleftrightarrow & \\ \dfrac{\lambda_{HL}-\overline{\pi}'_{H}\gamma}{\lambda'_{Hl}} & > & \dfrac{\lambda_{LH}-\overline{\pi}'_{L}\gamma}{\lambda_{LH}} \\ & \Longleftrightarrow & \\ \dfrac{\overline{\pi}'_{H}}{\overline{\pi}'_{L}} & < & \dfrac{\lambda_{HL}}{\lambda_{LH}}=\dfrac{1-\alpha_{H}}{1-\alpha_{L}}<1 \end{array}$$

However, in general this inequality cannot be expected to hold. For example, when all types are aggressive, $\overline{\pi}'_{H} = (\overline{\pi}'_{L})^{\left(\frac{1-\alpha_{H}}{1-\alpha_{L}}\right)}$, meaning that $\overline{\pi}'_{H} > \overline{\pi}'_{L}$.[26] While equilibrium behavior in the more general setting, which includes non-aggressive types, does not exclude the possibility that $\overline{\pi}'_{H} \leq \overline{\pi}'_{L}$, the difference would need to be large enough to overcome the fact that λ_{HL} is already smaller than λ_{LH} – and thus subtracting a fixed number has a larger proportional effect on λ_{HL} than λ_{LH}.

[26] If all types are aggressive there is no initial concession. Instead, the proportion by which a demand is chosen must be adjusted so that $T_{HL} = T_{LH}$. Hence, just ensuring second-stage behavior is in equilibrium provides more structure on how $\overline{\pi}'_{H}$ and $\overline{\pi}'_{L}$ must be related.

In summary, moving to modified types will generally lead to rational players making higher demands finding themselves in an even weaker position in subgames in which they meet a lower incompatible demand. If this is the case, despite hazard rates being slower in the modified case, initial concession will be more likely, making the overall impact on delay in a given subgame ambiguous. Furthermore, more demanding announcements will be made less often.

4.3 An Intermediate Case

This subsection considers an intermediate case where all types are of the standard, completely inflexible variety except for the 50-50 obstinate type, which might also be of the modified form. This assumption is somewhat tailored to our experimental design: In addition to the obstinate types induced using the computer players, we include the possibility that subjects might themselves be a 50-50 obstinate type. While the computer types are programmed to be completely inflexible, if a subject is an obstinate type, then they are of the less inflexible variety.

Now consider second-stage concession games that involve one player making the 50-50 demand, denoted by α_L, and another player making a strictly larger demand, denoted α_H. As detailed in subsection 4.1, a rational α_L announcer will need to concede at a slower rate to take into account the fact that the obstinate type they are mimicking might now concede. However, this is not the case for the rational α_H announcer. For them, the required rate of concession is unchanged. It is clear this will reduce the weakness of the α_H announcer: $T'_{LH} > T_{LH}$, while $T'_{HL} = T_{HL}$, if it were the case that $\mu' = \mu$. As a consequence, the new equilibrium would require a smaller probability of initial concession and a smaller probability that the α_H announcer is irrational, while the probability of the α_L announcer being irrational must increase.

Since the $50 - 50$ announcers are now conceding at a slower rate and are also being conceded to initially less often, there will be more delay, conditional on eventual agreement, in second-stages involving a 50-50 announcement. Finally, to ensure that the ex-post probability of a higher announcer being irrational decreases and the ex-post probability of the 50-50 announcer being irrational increases, rational types must choose the higher announcements more often and the 50-50 announcement less often.

4.4 Other Approaches to Extending the Theory

The above approach to extending the theory seeks a minimal adjustment to the behavior of the obstinate, "irrational", types in order to accommodate the observed deviations from the baseline theory. The intermediate case that results from this exercise also fits nicely with the features of the experimental design. In particular, while the computer players are programmed to never concede in the second-stage, it is more difficult for a obstinate type subject to achieve this, since they must actually wait until the other player has conceded. Furthermore, the evidence from the control treatments (*C0* and *R0*) suggest that demands for strictly more than half the pie, when there are no induced types that make such a demand, prove to be non-credible when faced with an equal splits demand; we observe the player making the larger demand making interior concession more often, something that should not happen if the demands were credible. This evidence supports the assumption that obstinate types from the subject population are of the 50-50 variety.

However, adjusting the behavior of the obstinate types is not the only direction in which the theory can be extended. An alternative would be to adjust the preferences of the rational types, for example to incorporate social preferences or fairness goals. The analysis conducted above is informative for how such an extension might work. Assuming the changes in preferences do not drastically alter the behavior of rational agents in the second stage, the following two points should be noted:

- Any change that results in the rational player who announced the lower of the two demands, which subsequently enter a second-stage, conceding at a slower rate, while not changing the rate at which the higher-announcer concedes, will result in an unambiguous increase in delay in this subgame. The difficulty when the concession rates of both decrease is that initial concession by the higher-announcer might also increase, resulting in an ambiguous effect on delay in the subgame. This was also seen with the only-modified-types comparison of subsection 4.2.

- Any change that reduces the "weakness" of the rational player that makes the higher announcement – i.e., less initial concessions by the higher-announcer – will result in more equilibrium announcements of this higher type. A slower concession rate by the lower-announcer, without changing the higher-announcer's concession rate, will reduce the weakness of the higher-announcer.

In summary, to be consistent with what is observed in the experiment, the small adjustment to the baseline model should result in longer delays in a given subgame and a greater probability of a rational type mimicking the more demanding type. This can be achieved, without depending on the specifics of the parameters, by adjusting the baseline model in a way that slows down the concession rate for only rational players who make the lower of two incompatible demands.

As an illustration, consider changing the preferences of rational types for the outcomes of the concession stage, while maintaining the usual exponential discounting. Suppose these preferences can be represented by a utility function. Ignoring the time discount, let $U(c, \alpha_i, \alpha_j)$ denote the utility to a rational player who announced α_i and conceded in the second stage to a player that announced α_j (with $\alpha_i + \alpha_j > 1$); let $U(nc, \alpha_i, \alpha_j)$ denote the utility to same player except in the case the other player concedes, and $U(disagree)$ the utility from disagreement. In the baseline model these would be $(1 - \alpha_j)$, α_i and 0, respectively.

To ensure that these changes do not drastically change the equilibrium behavior of rational types in the second stage, it is sufficient to assume:

- The (non-discounted) utility function U does not depend on time.
- U is common knowledge amongst rational agents; rational players commonly know each other's preferences up to the uncertainty about whether the other is an obstinate type or not.[27]
- The rational players' preference ordering for the outcomes is unchanged; That is,
$$U(nc, \alpha_i, \alpha_j) > U(c, \alpha_i, \alpha_j) > U(disagree)$$

[27]This assumption is clearly strong, and thus the analysis should be seen as a first pass at what might be possible with changing rational players' preferences. Aside from simplifying the analysis, the assumption mimics the implicit assumption in the modification of Subsections 4.1 through 4.3 that rational types commonly know the exogenous rate at which the modified obstinate types will concede, and thus provides a better comparison between the approaches. The assumption is easier to justify in the examples below, which consider a simple utility boost or cost to various actions, than for more general utility specifications that allow for a broad spectrum of trade-offs between self-regarding motives and other goals as in, for example, the models of *Fehr and Schmidt* (1999), *Bolton and Ockenfels* (2000) and *Charness and Rabin* (2002).

for all subgames that lead to a second stage.[28]

Given this, the structure of equilibrium behavior in a given second stage is very similar to the baseline model. Rational players will adopt a mixed-strategy over the time to concession that seeks to keep the other rational-type player indifferent between conceding or not conceding. This is achieved by conceding with a constant hazard rate

$$\lambda_{ij} = \frac{r \cdot U\left(c, \alpha_j, \alpha_i\right)}{U\left(nc, \alpha_j, \alpha_i\right) - U\left(c, \alpha_j, \alpha_i\right)}$$

until it is no longer possible that the opponent is a rational type.

To illustrate this framework, consider the following two simple examples of modifying the preferences of rational players, where $\alpha_L, \alpha_H \in C$ are such that $\alpha_L \leq \alpha_H$ and $\alpha_L + \alpha_H > 1$:

1. Simple inequality aversion or spite: The lower-announcer receives a utility cost to accepting the demand of a higher-announcer.[29] That is,

$$U\left(c, \alpha_L, \alpha_H\right) = \left(1 - \alpha_H\right) - c$$

for a given $c > 0$, while all other utilities are as in the baseline model. Or, the lower-announcer receives a utility boost for not accepting the demand of a higher-announcer.[30] That is,

$$U\left(nc, \alpha_L, \alpha_H\right) = \alpha_L + c$$

for a given $c > 0$, while all other utilities are as in the baseline model. Since both these modifications reduce the value for the *lower-announcer* of concession compared to waiting for the other to concede, the *higher-announcer* would need to reduce their rate of concessions to keep them

[28]If $U(disagree) \geq U\left(c, \alpha_i, \alpha_j\right)$ then this rational type would in fact behave in an analogous manner to an obstinate α_i-type, at least for the subgame in which the other player chose α_j.

[29]Such a model could be viewed as a reduced-form way of capturing inequality aversion, where the player's aversion to inequality is not so severe that it makes conceding unacceptable compared to outright disagreement.

[30]Such a model could be interpreted as spite: the lower-announcer who, for example, announces the 50-50 split, gets an extra utility kick from not conceding to the player that demanded an unfair share.

in equilibrium. Consequently, this would result in an even weaker position for the higher-announcer, leading to an ambiguous effect on delay (more initial concessions versus slower interior concessions) and less demanding first-stage announcements.

2. Simple shame or competitiveness: The higher-announcer receives a utility cost to accepting the demand of a lower-announcer.[31] That is,

$$U(c, \alpha_H, \alpha_L) = (1 - \alpha_L) - c$$

for a given $c > 0$, while all other utilities are as in the baseline model. Or, the higher-announcer receives a utility boost for not accepting the demand of a lower-announcer.[32] That is,

$$U(nc, \alpha_H, \alpha_L) = \alpha_H + c$$

for a given $c > 0$, while all other utilities are as in the baseline model. Since both these modifications reduce the value for the *higher-announcer* of concession compared to waiting for the other to concede, the *lower-announcer* would need to reduce their rate of concessions to keep them in equilibrium. Consequently, such a modification would achieve the desired result in a comparable manner to the intermediate case outlined in Subsection 4.3.

To conclude, if incorporating other-regarding preferences results in making concession less appealing for players making more demanding announcements, then the predictions will move in the direction suggested by the experiment. If instead, the main effect is to make concession less appealing to players making lower, or fairer, announcements, then the modification is likely to result in ambiguous predictions for delay and first-stage announcements, making it potentially more difficult to produce a modification that can be reconciled with the data.

[31]Such a model could be interpreted as a cost of shame: Having pretended to be a computer player and demanded a more unfair share when, for example, the other player demanded the 50-50 split, the higher-announcer might feel embarrassed or suffer from shame for having tried it on.

[32]Such a model could be interpreted as competitiveness: By not conceding to the other when, for example, the other demands the 50-50 split, the higher-announcer guarantees they receive a strictly larger share of the pie, thus “winning” the game.

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