The Efficiency of Voluntary Pollution Abatement when Countries can Commit

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Abstract

In this paper, we characterize a mechanism for reducing pollution emissions in which countries, acting non-cooperatively, commit to match each others' abatement levels and subsequently engage in emissions quota trading. The analysis shows that the mechanism leads to a fully efficient outcome. The level of emissions is efficient, as well as the allocation of emissions across countries. Given the equilibrium matching rates, the initial allocation of emission quotas (before trading) reflects each country's marginal valuation for lower pollution relative to its marginal benefit from emissions. These results hold for any number of countries, and in an environment where countries have different abatement technologies and different benefits from emissions. In a dynamic two-period setting, the mechanism achieves both intra-temporal and inter-temporal efficiency. We extend the model by assuming that countries are voluntarily contributing to an international public good, in addition to undertaking pollution abatements, and find that the level of emissions and its allocation across countries may be efficient even without any matching abatement commitments and emissions quota trading.

Keywords: Voluntary pollution abatement, matching commitments, emissions quota trading

JEL classification: H23, H41, H87

1 Introduction

International agreements on pollution reduction targets are difficult to achieve and sustain in the absence of a central authority with the ability to enforce the abatement objectives of national governments. Cooperative initiatives also require that countries be able to agree on the overall objectives of emissions reduction and on how abatement efforts should be distributed across countries. This is particularly difficult to achieve given that the costs and benefits of pollution abatement vary considerably across countries. Without cooperative agreements, emission reductions rely essentially on the voluntary contributions of countries.

In this paper, we show that voluntary pollution abatement by countries behaving non-cooperatively can lead to efficient outcomes provided that countries can commit to matching the abatement efforts of each other at some announced rates. The efficiency of voluntary contributions to international public goods when countries can commit has been established by Guttman (1978), Danziger and Schnytzer (1991), Varian (1994) and Boadway, Song and Tremblay (2007). We show how similar reasoning can be adapted to the case of international pollution abatement when countries have different abatement technologies and may be able to engage in emissions quota trading. Remarkably, we also find that efficiency can occur even in the absence of commitment provided that countries are also contributing to an international public good.

Recently, a number of papers have proposed mechanisms for implementing efficient contributions by countries to international public goods, such as pollution abatement. In particular, Gersbach and Winkler (2007) and Gerber and Wichardt (2008) have proposed schemes in which countries make up-front payments to a neutral institution as a way of pre-committing to contributions. The payments are eventually refunded, at least in part, if countries provide their intended contributions. The neutral institution's ability to deny refunds induces countries to act according to prior commitments. In principle, these mechanisms can be designed to implement any desired emission reduction objectives, although they require some prior cooperative agreement to establish such objectives, as well as the distribution of the surplus across countries. In contrast, we take the commitment ability of countries as given, but focus on a non-cooperative mechanism that can emerge and induce full efficiency in emission abatement when countries are making commitments voluntarily and are acting in their own self-interest.

Altemeyer-Bartscher, Rübbelke and Sheshinski (2008) consider another form of commitment mechanism whereby each of two countries voluntarily makes a take-it-or-leave-it offer of a payment to the other country conditional on the tax rate that the latter imposes on a polluting good, and show that such a mechanism can induce the efficient level of pollution. While their mechanism is based on side-payments between countries, the mechanism we characterize relies on matching abatement commitments and, crucially, allows emissions quota trading. Both mechanisms can lead to efficient allocations, although they do not generally result in the same distribution of net benefits across countries. Moreover, it is not clear if their mechanism easily generalizes to more than two countries, since any given country would receive take-it-or-leave-it offers from all other countries simultaneously.

Our analysis resembles that of Guttman and Schnytzer (1992) who consider a mechanism where players voluntarily offer to match each others' externality-producing activities. However, the mechanism that we characterize is explicitly applied to the problem of pollution reduction. Countries have access to different pollution abatement technologies and the mechanism includes emissions quota trading.

Specifically, the pollution abatement process we consider works as follows. Each country simultaneously (and non-cooperatively) announces a rate at which it will match the abatement efforts of the other countries. Countries then choose their direct abatement efforts simultaneously, taking the previously announced matching rates as given. After these two stages of decisions, countries are committed to achieving a total emissions quota equal to their initial emissions minus the sum of their direct and matching abatement efforts. However, these commitments are contingent in the sense that once they are determined, countries can trade emissions quotas at the competitively determined price.

The analysis shows that the unique subgame perfect equilibrium of this emission abatement process is fully efficient. The efficient level of pollution abatement is achieved, as well as the efficient allocation of emissions across countries. The equilibrium displays other interesting properties. In particular, the effective cost at which any country can induce an increase in world abatements, either through its own direct abatements or by matching the abatements of other countries, is equal to that country's marginal valuation for reduced pollution relative to its marginal valuation of the benefits of emissions. Thus, the countries' effective costs of abatement are the analogs of Lindahl prices in the context of this model.

As mentioned, the matching rate mechanism that we consider is easily applicable to a setup with any number of countries, in contrast to mechanisms where the voluntary offers made by countries are conditional on the contributions of other countries being set at specific levels, such as the mechanism proposed by Altemeyer-Bartscher, Rübbelke and Sheshinski (2008). Under a matching rate mechanism, the simultaneous offers of several countries readily add-up to an aggregate matching rate applying to the abatement effort of an individual country.

We consider a dynamic two-period extension and find that the mechanism achieves intra-temporal and inter-temporal efficiency: the total level of emissions is efficiency as well as its allocation across countries and across periods. We also extend the model by adding an international public good provided by the voluntary contributions of countries. If contributions to the public good are made after the pollution abatement process, we find that the efficient level of emissions occur even in the absence of matching abatement commitments. Remarkably, we also find that emissions are efficiently allocated across countries even in the absence of quota trading.

In the next section, we start by describing the main features of the model. We then characterize the abatement process equilibrium in a simple two-country case. Various extensions of the basic model are considered in Section 4, while contributions to an international public good are added in Section 5. Concluding remarks are provided in the last section.

2 The Basic Two-Country Model

There are two countries denoted by $i, j = 1, 2$. In the absence of any abatement effort, the fixed level of emissions by country i is equal to $\overline{e}_i$. Both countries can commit to undertaking costly abatement which will reduce actual emissions. In the basic model, country i can commit to a given level of abatement a_i, as well as to matching the abatement commitment of country j at a rate m_i. Therefore, country i commits to achieving a total level of abatement equal to $A_i = a_i + m_i a_j$. Equivalently, it is committed to an emissions quota corresponding to $\overline{e}_i - A_i$. However, these are contingent commitments since countries can then trade emission quotas at market price p. The number of emission quotas purchased by country i is denoted by q_i, where $q_1 = -q_2$. Given the number of quotas traded, the actual emissions of country i are $e_i = \overline{e}_i - A_i + q_i$. Note that aggregate emissions by both countries are equal to the sum of their initial commitments before quota trading. The latter simply reallocates emissions from one country to another.

The benefits of emissions to country i as a function of actual emissions is $B_i(e_i)$, with $B_i' > 0$ and $B_i'' < 0$.[1] The damage to country i is a function of the total emissions of both countries, $D_i(e_1 + e_2)$, with $D_i' > 0$ and $D_i'' > 0$. Hence, the emissions of both countries are assumed to be perfect substitutes.

The analysis will characterize the equilibrium levels of abatement in a number of cases, starting with the basic case where both countries can commit to matching the abatement efforts of each other, and quota trading exists. We then consider various extensions of the basic model, starting with the case where there is no emissions quota trading. The analysis is then extended to the case where there are more than two countries, as well as to a dynamic setting with two-periods. Finally, we consider the case where countries also contribute voluntarily to an international public good after abatement efforts are determined. As we shall see, this has dramatic effects on the results: optimal emissions are obtained even in the absence of commitment.

Before turning to the basic two-country case with commitment to matching abatements, it is useful to characterize the social optimum. The socially optimal level of emissions maximizes the sum of benefits net of damages to both countries. It solves the following:

$$\max_{\{e_1, e_2\}} B_1(e_1) + B_2(e_2) - D_1(e_1 + e_2) - D_2(e_1 + e_2)$$

[1] The marginal benefits of emissions can be viewed as the negative of a marginal cost of abatement function, $B'(e) = -C'(A)$. A cost of abatement function has been used by Roberts and Spence (1976), for example.

The first-order conditions can be written:

$$B_1' = B_2', \quad \text{and} \quad \frac{D_1' + D_2'}{B_1'} = \frac{D_1' + D_2'}{B_2'} = 1$$

The first of these is the condition for the optimal allocation of emissions between countries. Emissions are allocated efficiently if the marginal benefit of emissions (or the marginal cost of abatement) is the same in both countries. The second determines the optimal level of emissions. It is the analog of the Samuelson condition for public goods, but in the context of a public bad. It says that emissions in each country should be taken to the point where the total marginal damages just equals the marginal benefit.

In the absence of international corrective action, country emissions would satisfy $B_1' = D_1'$ and $B_2' = D_2'$. Neither the level nor the allocation would be optimal. A world government could achieve the social optimum by imposing Pigouvian taxes on the emissions in each country at the tax rates $t_1 = D_2'$ and $t_2 = D_1'$. Our analysis explores commitment mechanisms as a way of achieving efficiency in the absence of a world government.

In what follows, we focus on the case where the socially optimal abatements of the two countries are both interior. That is, the levels of emissions e_1^* and e_2^* corresponding with the solution to the social optimum satisfy $e_1^* < \bar{e}_1$ and $e_2^* < \bar{e}_2$.

3 The Basic Case with Commitment and Quota Trading

In this section, we examine the basic case where two countries commit to matching the abatement efforts of each other, and where quota trading exists. The timing of decisions is the following. In Stage 1, both countries simultaneously choose the rate m_i at which they will match the direct abatement commitment of the other country. Countries commit to direct abatement levels a_i in Stage 2. Finally, in Stage 3, each country can buy or sell emissions quotas at the equilibrium price p. We characterize the subgame perfect equilibrium of this three-stage process by backward induction, starting with Stage 3.[2]

[2] Multi-stage processes of matching contributions to public goods have been analyzed in Guttman (1978), Danziger and Schnytzer (1991), Varian (1994) and Boadway, Song and Tremblay (2007), among others.

Stage 3: Emissions Quota Trading

The direct abatements (a_1, a_2) and matching rates (m_1, m_2) have been determined in the previous two stages. The demand for emission quotas by country 1 at price p solves (assuming an interior solution and assuming that both countries are price-takers):

$$\max_{\{q_1\}} \; B_1(\bar{e}_1 - a_1 - m_1 a_2 + q_1) - p q_1$$

At this stage, as mentioned, the total level of emission abatement for the two countries is fixed so the damage function can be left out of the problem. The first-order condition to this problem gives $B_1'(\bar{e}_1 - a_1 - m_1 a_2 + q_1) = p$. Its solution will be the demand for emissions quotas, $q_1(p, a_1, a_2, m_1)$. Differentiating the first-order condition $B_1'(\cdot) = p$ yields

$$\frac{\partial q_1}{\partial a_1} = \frac{\partial q_1}{\partial A_1} = 1, \qquad \frac{\partial q_1}{\partial m_1} = a_2, \qquad \frac{\partial q_1}{\partial a_2} = m_1 \tag{1}$$

Similarly, the demand for quotas by country 2 satisfies $B_2'(\bar{e}_2 - a_2 - m_2 a_1 + q_2) = p$ and is denoted by $q_2(p, a_1, a_2, m_2)$. In equilibrium, $q_1(\cdot) + q_2(\cdot) = 0$, and, using $A_i = a_i + m_i a_j$, the price satisfies

$$p(A_1, A_2) = B_1'(\bar{e}_1 - A_1 + q_1) = B_2'(\bar{e}_2 - A_2 + q_2)$$

Therefore, quota trading leads to an efficient allocation of emissions across countries.

Stage 2: Choosing Direct Abatements $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$

We assume that countries correctly anticipate the price of quotas in Stage 3 and take it as given when making their abatement commitments. Given (m_1, m_2) from Stage 1, the problem of country 1 is:

$$\max_{\{a_1\}} \; \mathit{\Pi}_1 = B_1\Big(\bar{e}_1 - a_1 - m_1 a_2 + q_1(p, a_1, a_2, m_1)\Big)$$
$$-D_1\Big(\bar{e}_1 - (1 + m_2) a_1 + \bar{e}_2 - (1 + m_1) a_2\Big) - p q_1(p, a_1, a_2, m_1)$$

The first-order condition, using $p = B_1'$, and assuming an interior solution, is:

$$F^1(a_1, a_2, m_1, m_2) \equiv -B_1'\Big(\bar{e}_1 - a_1 - m_1 a_2 + q_1(p, a_1, a_2, m_1)\Big)$$

$$+(1+m_2)D_1'\Big(\bar{e}_1-(1+m_2)a_1+\bar{e}_2-(1+m_1)a_2\Big)=0 \qquad (2)$$

or,

$$\frac{D_1'(\cdot)}{B_1'(\cdot)}=\frac{1}{1+m_2}$$

The solution to this first-order condition is country 1's reaction function $a_1(a_2;m_1,m_2)$. At any a_2, country 1 will choose its level of abatement such that the ratio of marginal damage to marginal benefit equals the effective cost at which it can increase world abatement by one unit, given the matching rate offered by country 2.

In an interior solution (including at the boundary), we have the following properties of country 1's reaction curve in $(a_1,a_2)-$space (differentiating $F^1(\cdot)$, using the expressions in (1) and noting that $F^1_{a_1}$ is just the second-order condition):

$$\frac{\partial a_1}{\partial a_2}=-\frac{F^1_{a_2}}{F^1_{a_1}}=-\frac{1+m_1}{1+m_2} \qquad \text{(slope)}$$

$$\frac{\partial a_1}{\partial m_1}=-\frac{F^1_{m_1}}{F^1_{a_1}}=-\frac{a_2}{1+m_2} \qquad (=0 \text{ at } a_2=0)$$

$$\frac{\partial a_1}{\partial m_2}=-\frac{F^1_{m_2}}{F^1_{a_1}}=-\frac{a_1}{1+m_2}-\frac{D_1'}{F^1_{a_1}} \qquad (=-D_1'/F^1_{a_1}>0 \text{ at } a_1=0)$$

Country 1's reaction curve is depicted in Figure 1. It is a straight line with a slope of $-(1+m_1)/(1+m_2)$ in the interior and coincides with the a_2-axis once it hits that axis. The intercept on the a_1-axis is labeled $\bar{a}_1$. An increase in m_1 rotates the reaction curve counterclockwise with the a_1-intercept remaining unchanged, as represented by the dashed line. An increase in m_2 steepens the reaction curve, and causes the a_2-intercept to move up (since $\partial a_1/\partial m_2>0$ at $a_1=0$), as shown by the dotted line. The a_1-intercept may move left or right. Figure 1 depicts the case where it moves right.

Analogous results hold for country 2:

$$\frac{\partial a_2}{\partial a_1}=-\frac{1+m_2}{1+m_1}, \quad \frac{\partial a_2}{\partial m_2}=-\frac{a_1}{1+m_1}, \quad \frac{\partial a_2}{\partial m_1}=-\frac{a_2}{1+m_1}-\frac{D_2'}{F^2_{a_2}}$$

Thus, the slopes of the two reaction curves in $(a_1,a_2)-$space are the same regardless of the values of m_1 and m_2. The fact that reaction curves are parallel implies that either there will be a corner solution in Stage 2, or the curves will overlap in the interior so the solution is indeterminate.

Stage 1: Choosing Matching Rates m_1 and m_2

In Stage 1, both countries simultaneously choose their matching rates, m_1 and m_2, anticipating the outcomes of Stages 2 and 3. We shall show that there are unique equilibrium values of m_1 and m_2, and that these are such that $m_1 m_2 = 1$ and $F^1 = F^2 = 0$. That is, the matching rates are reciprocals of each other and the Stage 2 reaction curves coincide. In order to derive this equilibrium, we proceed in three steps. First, we show that the equilibrium matching rates must be such that the Stage 2 reaction curves coincide. Second, we demonstrate that the equilibrium satisfies $m_1 m_2 = 1$. Finally, we show that the equilibrium values of m_1 and m_2 are unique.

1) Reaction Curves Coincide in Equilibrium

To begin, suppose that $F^1 = 0$ and $F^2 < 0$, so that $a_1 > 0$ and $a_2 = 0$. Thus, country 1's reaction curve lies outside country 2's reaction curve. (The other case where $a_2 > 0$ and $a_1 = 0$ is symmetric so we need not consider it.) Suppose as well that $m_1 m_2 \geqslant 1$ and consider country 2's net benefit:

$$\varPi_2 \equiv B_2\Big(\bar{e}_2 - m_2 a_1 + q_2(p, a_1, a_2, m_2)\Big) - D_2\Big(\bar{e}_1 - (1+m_2)a_1 + \bar{e}_2\Big) - pq_2(p, a_1, a_2, m_2).$$

Using the analog of (2) for country 2, $F^2 < 0$ implies $0 < D_2' < B_2'(1+m_1)$. Therefore,

$$\begin{aligned}
\frac{d\varPi_2}{dm_2} &= -B_2' \cdot \left(a_1 - m_2\left(\frac{a_1}{1+m_2} + \frac{D_1'}{F^1_{a_1}}\right)\right) - D_2' \frac{(1+m_2)D_1'}{F^1_{a_1}} \\
&< -B_2' \cdot \left(a_1 - m_2\left(\frac{a_1}{1+m_2} + \frac{D_1'}{F^1_{a_1}}\right)\right) - \frac{B_2'}{1+m_1}\frac{(1+m_2)D_1'}{F^1_{a_1}} \\
&= \frac{B_2' D_1'}{F^1_{a_1}}\left(m_2 - \frac{1+m_2}{1+m_1}\right) - B_2' \frac{a_1}{1+m_2}.
\end{aligned}$$

The second term is negative. The first term is non-positive if and only if $m_1 m_2 \geqslant 1$. So, country 2 would decrease m_2 if $m_1 m_2 \geqslant 1$. As country 2 decreases m_2, country 2's reaction curve would rotate upwards around the a_2-intercept. The a_2-intercept of country 1's reaction curve would fall. Thus, the reaction curves would move closer to each other (and remain parallel as we showed above).

In the case where $m_1 m_2 < 1$, the effect of changes in m_2 on country 2's net benefit is ambiguous, that is, the sign of $d\Pi_2/dm_2$ may be positive or negative. However, we can show that country 1 will have an incentive to change its matching rate in this case. Consider the effect of a unilateral increase in m_1, when $F^1 = 0$, $F^2 < 0$, and m_2 is held constant. Country 1's reaction curve rotates counterclockwise around the a_1-intercept, as shown in Figure 2. Country 2's reaction curve flattens and its a_1-intercept increases. The a_2-intercept of country 2's reaction curve could move up or down. Figure 2 illustrates the case where it moves up. The stage 2 outcome is unchanged until the two reaction curves coincide (dotted line in Figure 2). Thus, it is costless for country 1 to increase m_1 until the reaction curves coincide and country 2 is on the verge of contributing. Note that this increase does not affect country 2's net benefit.

Suppose now that the two reaction curves coincide. Note first that along the common reaction curve, total abatements are constant. To see this, write the formula for the common reaction curves in the interior as:

$$a_2 = \bar{a}_2 - \frac{1+m_2}{1+m_1} a_1 \tag{3}$$

where $\bar{a}_2$ is the a_2-intercept. Multiplying by $1+m_1$ and rearranging, we obtain:

$$A \equiv (1+m_2)a_1 + (1+m_1)a_2 = (1+m_1)\bar{a}_2$$

where $A = A_1 + A_2$ are total abatements.

Next, consider the effect on, say, country 1 of moving up the reaction curves. Country 1's total abatements are $A_1 = a_1 + m_1 a_2$, or, using expression (3) to substitute for a_1:

$$A_1 = \frac{1+m_1}{1+m_2}\bar{a}_2 - \frac{1+m_1}{1+m_2}a_2 + m_1 a_2 = \frac{1+m_1}{1+m_2}\bar{a}_2 + (1+m_1)\left(\frac{m_1}{1+m_1} - \frac{1}{1+m_2}\right) a_2$$

Therefore,

$$\frac{\partial A_1}{\partial a_2} < 0 \qquad \text{if} \qquad \frac{m_1}{1+m_1} < \frac{1}{1+m_2}$$

The left side of the second inequality above is the cost to country 1 of a one unit increase in world abatement induced by an increase in the direct abatement of country 2, given the matching rate m_1 offered by country 1. It can be seen as the effective cost of an indirect contribution to abatement by country 1. The right side of the inequality is the cost to country 1 of increasing world abatement by one unit through its own direct abatement, given the matching rate m_2 chosen by country 2. Country 1 would prefer to go up the reaction curve and force country 2 to contribute directly if the cost of its direct contributions exceeds the cost of its indirect contributions. Thus, country 1 would want to increase m_1 in this case since that would cause country 2's reaction curve to move outside country 1's so it becomes the sole contributor. Note that

$$\frac{m_1}{1+m_1} < \frac{1}{1+m_2} \quad \text{if} \quad m_1 m_2 < 1$$

Hence, matching rates for which $F^i = 0$ and $F^j < 0$ cannot hold in equilibrium. Starting with any values of m_1 and m_2, countries would have incentives to change their matching rates in ways that would cause the Stage 2 reaction curves to move closer together until they coincide.

2) In Equilibrium, $m_1 m_2 = 1$

To show this, note first that when the reaction curves overlap, country 2's total contributions would also fall by moving down the common reaction curve if

$$\frac{m_2}{1+m_2} < \frac{1}{1+m_1}, \quad \text{or} \quad m_1 m_2 < 1$$

Thus, when reaction curves coincide and $m_1 m_2 < 1$, both country 1 and country 2 would like to increase their matching rates. By the same token, we can readily verify that both countries would decrease their matching rates if reaction curves coincide and $m_1 m_2 > 1$. Therefore, matching rates for which $m_1 m_2 \neq 1$ cannot constitute an equilibrium.

Suppose that reaction curves coincide and $m_1 m_2 = 1$. Country 1's net benefits is

$$\Pi_1 = B_1\left(\bar{e}_1 - A_1 + q_1(a_1, a_2, m_1)\right) - D_1\left(\bar{e}_1 + \bar{e}_2 - A_1 - A_2\right) - pq_1(a_1, a_2, m_1) \tag{4}$$

Differentiating (4) with respect to m_1, we obtain, using $p = B_1'$ and the Stage 2 first-order condition (2):

$$\frac{d\Pi_1}{dm_1} = D_1'\left(-m_2\frac{\partial A_1}{\partial m_1} + \frac{\partial A_2}{\partial m_1}\right)$$

Since $A_1 = a_1 + m_1 a_2$, at $m_1 m_2 = 1$ we have $A_1 m_2 = m_2 a_1 + a_2 = A_2$. Therefore, at $m_1 m_2 = 1$,

$$m_2\frac{\partial A_1}{\partial m_1} = \frac{\partial A_2}{\partial m_1}$$

which implies that $d\Pi_1/dm_1 = 0$. Hence, country 1 would not want to change its matching rate when reaction curves coincide and $m_1 m_2 = 1$. Since the same argument applies to country 2, $F^1 = F^2 = 0$ and $m_1 m_2 = 1$ is an equilibrium in Stage 1.

3) Equilibrium Values of m_1 and m_2 are Unique

To see that there are unique equilibrium values of m_1 and m_2, consider an initial situation in which the reaction curves overlap and $m_1 m_2 = 1$. Now suppose we first increase m_1, holding m_2 constant. Using the Stage 2 first-order conditions, we can show that 1) the reaction curves become flatter in the (a_1-a_2)–space, 2) the intercept of country 1's reaction curve is unchanged along the a_1–axis, 3) the intercept of country 2's reaction curve along the a_1–axis moves right, so the reaction curves are unambiguously further apart, although the intercept of country 2's reaction curve along the a_2–axis can either go up or down. Next, starting with these new reaction curves, consider decreasing m_2 holding m_1 constant. 1) The reaction curves again become flatter, 2) the intercept of country 2's reaction curve is unchanged along the a_2–axis, 3) the intercept of country 1's reaction curve along the a_2–axis goes down so the reaction curves again go further apart unambiguously, although the intercept of country 1's reaction curve along the a_1–axis can increase or decrease. The opposite will occur if we increase m_2 and decrease m_1. These imply that there is only one pair of m_1 and m_2 such that $m_1 m_2 = 1$ and the two reaction curves overlap.

Properties of the Equilibrium

Some properties of the equilibrium are worth mentioning. First, since the Stage 2 reaction curves coincide in equilibrium, direct abatements a_1 and a_2 are indeterminate, although total abatements are uniquely determined. All combinations of a_1 and a_2 yield the same $A_1 \equiv a_1 + m_1 a_2$, $A_2 \equiv a_2 + m_2 a_1$, $A = A_1 + A_2$, and therefore the same level of net benefits for the two countries. Given that $m_1 m_2 = 1$, total abatements are such that

$$\frac{A_1}{A_2} = \frac{a_1 + m_1 a_2}{a_2 + m_2 a_1} = \frac{1}{m_2} \frac{m_2 a_1 + a_2}{a_2 + m_2 a_1} = \frac{1}{m_2} = m_1$$

Second, the equilibrium is fully efficient. Given that total contributions A_1, A_2, and A are the same for all combinations of a_1 or a_2 along the common reaction curve, the net benefit functions can be written

$$\Pi_1 = B_1\Big(\bar{e}_1 - A_1 + q_1(\cdot)\Big) - D_1\Big(\bar{e}_1 - A_1 + \bar{e}_2 - A_2\Big) - pq_1(\cdot)$$

$$\Pi_2 = B_2\Big(\bar{e}_2 - A_2 + q_2(\cdot)\Big) - D_2\Big(\bar{e}_1 - A_1 + \bar{e}_2 - A_2\Big) - pq_2(\cdot)$$

and the two Stage 2 first-order conditions together give us

$$\frac{D_1'\left(\bar{e}_1 - A_1 + \bar{e}_2 - A_2\right)}{B_1'\left(\bar{e}_1 - A_1 + q_1(\cdot)\right)} + \frac{D_2'\left(\bar{e}_1 - A_1 + \bar{e}_2 - A_2\right)}{B_2'\left(\bar{e}_2 - A_2 + q_2(\cdot)\right)} = \frac{1}{1 + m_2} + \frac{1}{1 + m_1} = 1 \qquad (5)$$

Since quota trading in Stage 3 ensures that $B_1'(\cdot) = B_2'(\cdot)$, the solution $\{A_1^*,\ A_2^*\}$ determined by (5) induces the the socially optimal allocation.

Third, the direct cost at which country 1 can abate emissions, $1/(1 + m_2)$, which is equal to D_1'/B_1' by the first-order condition in Stage 2, is the analog of a Lindahl price in the context considered here: it is the amount that country 1 would be willing to pay for the total abatements $A_1 + A_2$. To see this, simply note that the product of this price and total world abatements equals the total direct and matching abatement of country 1 (using $m_1 = 1/m_2$):

$$\frac{1}{1 + m_2}(A_1 + A_2) = \frac{(1 + m_2)a_1 + (1 + m_1)a_2}{1 + m_2} = a_1 + \frac{1 + m_1}{1 + m_2} a_2 = a_1 + m_1 a_2 = A_1$$

Thus, country 1's direct and matching abatement before quota trading, A_1, equals its marginal valuation for reduced pollution relative to its marginal valuation of the benefits of emissions, D_1'/B_1', applied to the world's total abatements, $(A_1 + A_2)$. The same applies for country 2. Thus, the total abatement each country makes (excluding the amounts related to quota trading) can be seen as quasi-Lindahl abatement efforts.[3]

Finally, in equilibrium, country 1 and country 2 are indifferent between making direct abatements and matching abatements. As explained earlier, the cost to country 1 of a direct contribution to abatement is $1/(1+m_2)$, whereas its cost of an indirect contribution is $m_1/(1+m_1)$. When $m_1 m_2 = 1$, $1/(1+m_2) = m_1/(1+m_1)$ and $1/(1+m_1) = m_2/(1+m_2)$. Thus, the cost to either country of reducing the world's pollution by one unit through direct abatement efforts or through matching abatement efforts are equal. If country 1 were to increase its matching rate, starting from an equilibrium with $m_1 m_2 = 1$, it would be reducing emissions indirectly at a cost higher than the cost at which it can reduce emissions directly. The same would apply for country 2. Therefore, neither country would want to increase their matching rate beyond $m_1 m_2 = 1$. By the same token, when $m_1 m_2 < 1$, $1/(1 + m_2) > m_1/(1 + m_1)$. It will be cheaper for country 1 to subsidize country 2 than to reduce emissions through direct contributions, so it will increase m_1. The same holds for country 2.

The main results of this section are summarized in the following proposition.

Proposition 1. *The subgame perfect equilibrium of the three-stage emissions abatement process with matching rate commitments and quota trading has the following properties:*

i. Direct contributions are indeterminate, but matching rates and total contributions before quota trading are uniquely determined;

ii. Matching rates satisfy $m_1 m_2 = 1$ and $A_1/A_2 = 1/m_2 = m_1$;

iii. The level of emissions and the allocation of emissions across countries are efficient; and

iv. The effective cost of abatement faced by each country is the analog of a Lindahl price.

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[3] Danziger and Schnytzer (1991) have shown that the Lindahl equilibrium in a public good contributions game can be implemented through a process where players can voluntarily subsidize the contributions of each other.

4 Extensions to the Basic Case

In this section, we consider three extensions to the basic case. First, we investigate the consequences of there being no emissions quota trading in the basic model. This might more closely correspond to the case where there is no central government with the authority to administer a quota trading system. Then, we extend our basic model to the case where there are more than two countries. Finally, we characterize the equilibrium of the abatement process in a dynamic two-period setting. In each case, the analysis is a straightforward extension of the basic case so detailed analysis is not necessary.

4.1 Full Commitment in Matching Rates without Quota Trading

In the absence of quota trading, there are only two stages of decisions. In Stage 1, both countries simultaneously choose the rates at which they commit to match the abatements of each other. Taking these matching rates as given, countries select their levels of direct abatements in Stage 2. Again, we consider Stage 2 first.

Stage 2: Choosing Direct Abatements $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$

Taking (m_1, m_2) as given, country 1 chooses a_1 to solve the following:

$$\max_{\{a_1\}} \ \mathit{\Pi}_1 = B_1\left(\bar{e}_1 - a_1 - m_1 a_2\right) - D_1\Big(\bar{e}_1 - (1+m_2)a_1 + \bar{e}_2 - (1+m_1)a_2\Big)$$

The first-order condition, assuming an interior solution, is:

$$F^1(a_1, a_2, m_1, m_2) \equiv -B_1'(\cdot) + (1+m_2)D_1'(\cdot) = 0 \quad \text{or} \quad \frac{D_1'(\cdot)}{B_1'(\cdot)} = \frac{1}{1+m_2} \tag{6}$$

Condition (6) has the same form as condition (2) characterizing the choice of direct abatement in the previous case with quota trading. Its solution gives country 1's reaction function, $a_1(a_2; m_1, m_2)$. Differentiating (6), we have:

$$F^1_{a_1} = B_1'' - (1+m_2)^2 D_1'' < 0, \quad F^1_{a_2} = m_1 B_1'' - (1+m_1)(1+m_2)D_1'' < 0 \tag{7}$$

$$F^1_{m_1} = a_2 B_1'' - a_2(1+m_2)D_1'' < 0, \quad F^1_{m_2} = -a_1(1+m_2)D_1'' + D_1' \gtreqless 0 \tag{8}$$

The problem of country 2 is analogous and its reaction function is $a_2(a_1; m_1, m_2)$. In contrast to the case where countries can trade abatement quotas, reaction curves are not linear and parallel for any (m_1, m_2) as can be seen by their slope $-F^1_{a_2}/F^1_{a_1}$ for country one and the analog for country 2, $-F^2_{a_2}/F^2_{a_1}$. However, as we will show below, reaction curves do coincide in equilibrium, as in the previous case.

The simultaneous solution to both reaction functions gives the Nash equilibrium abatements as functions of matching rates, $a_1(m_1, m_2)$ and $a_2(m_1, m_2)$. Differentiating $F^1(\cdot)$ and $F^2(\cdot)$, we have:

$$\begin{bmatrix} F^1_{a_1} & F^1_{a_2} \\ F^2_{a_1} & F^2_{a_2} \end{bmatrix} \begin{bmatrix} da_1 \\ da_2 \end{bmatrix} = \begin{bmatrix} -F^1_{m_1} & -F^1_{m_2} \\ -F^2_{m_1} & -F^2_{m_2} \end{bmatrix} \begin{bmatrix} dm_1 \\ dm_2 \end{bmatrix}$$

and,

$$\left.\frac{da_1}{dm_1}\right|_{m_2} = \frac{-F^1_{m_1}F^2_{a_2} + F^2_{m_1}F^1_{a_2}}{H}, \quad \text{and} \quad \left.\frac{da_2}{dm_1}\right|_{m_2} = \frac{-F^1_{a_1}F^2_{m_1} + F^2_{a_1}F^1_{m_1}}{H} \tag{9}$$

where $H \equiv F^1_{a_1}F^2_{a_2} - F^2_{a_1}F^1_{a_2}$. To have a stable interior Nash equilibrium in Stage 2, the slope of country 2's reaction curve in (a_1, a_2)–space has to be less than that of country 1 (i.e., more negative). That is, we must have $-F^2_{a_2}/F^2_{a_1} < -F^1_{a_2}/F^1_{a_1}$, which in turn implies that $H > 0$.

Stage 1: Choosing Matching Rates $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$

At this stage, both countries anticipate the subsequent Nash equilibrium choices of direct abatements. Country 1 chooses its matching rate m_1 to maximize the following:

$$\Pi_1 = B_1\Big(\bar{e}_1 - a_1(m_1, m_2) - m_1 a_2(m_1, m_2)\Big)$$

$$-D_1\Big(\bar{e}_1 - (1 + m_2)a_1(m_1, m_2) + \bar{e}_2 - (1 + m_1)a_2(m_1, m_2)\Big)$$

Differentiating this expression with respect to m_1 gives:

$$\frac{d\Pi_1}{dm_1} = -B_1'\left[\frac{\partial a_1}{\partial m_1} + a_2 + m_1\frac{\partial a_2}{\partial m_1}\right] + D_1'\left[(1 + m_2)\frac{\partial a_1}{\partial m_1} + a_2 + (1 + m_1)\frac{\partial a_2}{\partial m_1}\right] \tag{10}$$

Using (6), (9) and the expressions for $F^i_{a_i}$, $F^i_{a_j}$, $F^i_{m_i}$, $F^i_{m_j}$ and H, equation (10) can be written as (assuming an interior solution for Stage 2):

$$\frac{d\Pi_1}{dm_1} = -\frac{(1 - m_1 m_2) D_1' D_2' F^1_{a_1}}{H} \tag{11}$$

A similar expression holds for country 2. Using $F^i_{a_i}$ and $F^i_{a_j}$, we can derive

$$H = (1 - m_1 m_2)\Big[B_1'' B_2'' - (1 + m_1) B_1'' D_2'' - (1 + m_2) B_2'' D_1''\Big] \tag{12}$$

The expression in the square brackets in (12) is positive, and therefore

$$H \gtreqless 0 \quad \Longleftrightarrow \quad 1 - m_1 m_2 \gtreqless 0$$

As a result, the value of $d\Pi_1/dm_1$ in (11) is positive if $m_1 m_2 < 1$ or $m_1 m_2 > 1$. The same holds for country 2. However, $m_1 m_2 > 1$ implies that $H < 0$ which, as argued above, cannot hold in a stable Stage 2 equilibrium. If $m_1 m_2 < 1$, each country would want to increase its matching rate and induce a greater level of abatement from the other country. Then, ruling out unstable equilibria, the Stage 1 equilibrium must be such that $m_1 m_2 = 1$.[4] Using (7), the slope of country 1's reaction curve when $m_1 m_2 = 1$ becomes

$$\frac{\partial a_1}{\partial a_2} = -\frac{F^1_{a_2}}{F^1_{a_1}} = -\frac{m_1 B_1'' - (1 + m_1)(1 + m_2) D_1''}{B_1'' - (1 + m_2)^2 D_1''} = -\frac{1 + m_1}{1 + m_2}$$

An analogous calculation for country 2 reveals that the slope of its reaction curve is the same. Thus, when $m_1 m_2 = 1$, country reaction curves are linear and parallel. Moreover, using similar reasoning as in the previous case, the Stage 2 reaction curves coincide in equilibrium and the direct abatements of each country are indeterminate, although the matching rates and the total abatements are uniquely determined.

From the first order conditions of the countries' Stage 2 problem, and using $m_1 m_2 = 1$, we obtain $D_1'/B_1' + D_2'/B_2' = 1$. As in the optimum, abatements are such that the sum of the two countries' ratios of marginal damages to marginal benefits is equal to 1. However, the

[4] See Boadway, Song and Tremblay (2007) for more details about a similar demonstration in the context of international public goods.

actual level of emissions is higher than in the optimum because, without quota trading, nothing ensures that $B_1' = B_2'$. If abatements are inefficiently allocated across countries, so that $B_1' \neq B_2'$, the total cost of abating any given level of emissions will not be minimized, which implies that the level of emissions for which $D_1'/B_1' + D_2'/B_2' = 1$ will be higher than in the optimum.

Finally, as in the case with quota trading, when matching rates satisfy $m_1 m_2 = 1$, we can easily verify that the price at which each country can abate emissions directly, multiplied by total world abatements, is equal to its total direct and matching abatements. For country 1, for example,

$$\frac{1}{1+m_2}(A_1 + A_2) = a_1 + m_1 a_2 = A_1$$

Therefore, the total abatements of each country can again be interpreted as quasi-Lindahl abatement efforts.

These results lead to the following proposition.

Proposition 2. *Without emissions quota trading, the equilibrium of the abatement process when both countries can commit to a matching rate satisfies parts i, ii and iv of Proposition 1, but emissions are not allocated efficiently across countries and total emissions are higher than in the optimum.*

4.2 Full Commitment in Matching Rates with More than Two Countries

In this section, we show that all the results of the basic two-country model with full commitment in matching rates and quota trading can be generalized to the case where there are more than two countries. To do so, let us now assume that there are n countries denoted by $i, j = 1, ..., n$, and let m_{ij} be the matching rate offered by country i on the direct abatement commitment of country j. Thus, countries can commit to matching the direct abatements of all other countries at different rates. As in the two-country case, countries simultaneously choose their matching rates in Stage 1, then set their direct abatement commitment in Stage 2. Finally, countries trade emission quotas in Stage 3.

Stage 3: Emissions Quota Trading

At this stage, the total abatement commitment of country i is $A_i = a_i + \sum_{j=1}^{n} m_{ij} a_j$. The demand for emission quotas by country i maximizes $B_i(\bar{e}_i - A_i + q_i) - pq_i$. The first-order condition is $B_i'(\bar{e}_i - A_i + q_i) = p$, and the solution is country i's demand for emission quotas $q_i(p, a_1, ..., a_n, m_{i1}, ..., m_{in}) = q_i(p, A_i)$, which satisfies

$$\frac{\partial q_i}{\partial a_i} = \frac{\partial q_i}{\partial A_i} = 1, \qquad \frac{\partial q_i}{\partial m_{ij}} = a_j, \qquad \frac{\partial q_i}{\partial a_j} = m_{ij}$$

for $i, j = 1, ..., n$ and $i \neq j$. In equilibrium, $\sum_i q_i(\cdot) = 0$, and the price is such that $p(A_i, ..., A_n) = B_i'(\bar{e}_i - A_i + q_i)$ for all i. Quota trading leads to an efficient allocation of emissions across all n countries.

Stage 2: Choosing Direct Abatements $\boldsymbol{a}_i$

Matching rates are determined at this stage, and all countries take the price of quotas as given. The direct abatement commitment of country i solves the following:

$$\max_{\{a_i\}} \ \Pi_i = B_i\left(\bar{e}_i - a_i - \left(\sum_{j=1}^{n} m_{ij} a_j\right) + q_i(p, A_i)\right)$$

$$-D_i\left(\sum_{j=1}^{n}\left(\bar{e}_j - a_j - \sum_{k \neq j}^{n} m_{jk} a_k\right)\right) - pq_i(p, A_i)$$

The first-order condition, using $p = B_i'$ from the emissions quota trading equilibrium, is:

$$F^i(a_1, ..., a_n, m_{i1}, ..., m_{in}) \equiv -B_i'(\cdot) + \left(1 + \sum_{j \neq i}^{n} m_{ji}\right) D_i'(\cdot) = 0$$

or,

$$\frac{D_i'(\cdot)}{B_i'(\cdot)} = \frac{1}{1 + \sum_{j \neq i}^{n} m_{ji}} \tag{13}$$

The effective cost at which country i can increase world abatements by one unit depends on the total rate at which its direct abatement will be matched by all other countries. Country i chooses a_i to equalize this effective cost to the ratio of marginal damages and marginal benefits of emissions.

Stage 1: Choosing Matching Rates $\boldsymbol{m_{ij}}$

The equilibrium matching rates turn out to satisfy similar properties as in the two-country case. In fact, with n countries, matching rates are such that $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij}m_{jk} = 1$ (or equivalently $m_{ki}m_{ij} = m_{kj}$). Since the equilibrium is analog to that in the two-country case, we will not go through its full derivation. Rather, we will simply show that a set of matching rates satisfying these conditions constitute an equilibrium in Stage 1.

Start by considering country i's net benefit which is given by the following:

$$\Pi_i = B_i\left(\bar{e}_i - a_i - \left(\sum_{j=1}^{n} m_{ij}a_j\right) + q_i(\cdot)\right) - D_i\left(\sum_{j=1}^{n}\left(\bar{e}_j - a_j - \sum_{k\neq j}^{n} m_{jk}a_k\right)\right) - pq_i(\cdot)$$

Differentiating with respect to m_{ij} and using $p = B_i'$, as well as equation (13) characterizing the choices of a_i in Stage 2, we obtain:

$$\frac{d\Pi_i}{dm_{ij}} = D_i'\left(-\sum_{k\neq i}^{n} m_{ki}\frac{\partial A_i}{\partial m_{ij}} + \sum_{k\neq i}^{n}\frac{\partial A_k}{\partial m_{ij}}\right)$$

Noting that

$$\sum_{k\neq i}^{n} m_{ki}A_i = \sum_{k\neq i}^{n} m_{ki}\left(a_i + \sum_{j\neq i}^{n} m_{ij}a_j\right)$$

and using $m_{ki}m_{ij} = 1$ if $k = j$ and $m_{ki}m_{ij} = m_{kj}$ if $k \neq j$, we can show, after straightforward manipulations, that

$$\sum_{k\neq i}^{n}\frac{\partial A_k}{\partial m_{ij}} = \sum_{k\neq i}^{n} m_{ki}\frac{\partial A_i}{\partial m_{ij}}$$

which in turn implies that $d\Pi_i/dm_{ij} = 0$. Therefore, when matching rates satisfy $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij} = m_{kj}$, no country has any incentive to deviate. The equilibrium matching rates of any pair of countries are the reciprocals of each other, and as in the two-country case, direct contributions are indeterminate but matching rates and total contributions are uniquely determined.

The other properties of the equilibrium matching rates derived in the two-country case apply here as well. In particular, with matching rates satisfying $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij}m_{jk} = 1$, it is also the case that

$$\sum_{i=1}^{n}\frac{1}{1 + \sum_{j\neq i}^{n} m_{ji}} = \sum_{i=1}^{n}\frac{D_i'(\cdot)}{B_i'(\cdot)} = 1$$

Since emissions quota trading in Stage 3 ensures that $B_i' = p$ for all i, the equilibrium abatements are fully efficient.

The total abatement of each country are again quasi-Lindahl abatement efforts. To see this, note that country i's quasi-Lindahl price is D_i'/B_i', which is equal to $1/(1+\sum_{j\neq i}^{n} m_{ji})$ by (13), and that country i's quasi-Lindahl abatement effort is:

$$\frac{1}{1+\sum_{j\neq i}^{n} m_{ji}}(A_1^* + \cdots + A_n^*)$$

$$= \frac{1}{1+\sum_{j\neq i}^{n} m_{ji}}\left[\left(a_1 + \sum_{j\neq 1}^{n} m_{1j}a_j\right) + \cdots + \left(a_i + \sum_{j\neq i}^{n} m_{ij}a_j\right) + \cdots + \left(a_n + \sum_{j=1}^{n-1} m_{nj}a_j\right)\right]$$

$$= a_i + \sum_{j\neq i}^{n} m_{ij}a_j = A_i^*$$

using $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij} = m_{kj}$. Thus, country i's marginal rate of substitution, $1/(1+\sum_{j\neq i}^{n} m_{ji})$, multiplied by the world's total abatements, $\sum_{j=1}^{n} A_j^*$, equals its total abatement before quota trading, A_i^*.

Finally, when $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij} = m_{kj}$, $1/(1+\sum_{j\neq i}^{n} m_{ji}) = m_{ik}/(1+\sum_{j\neq k}^{n} m_{jk})$ for all i and k. Each country faces equal direct and indirect costs of reducing the world's emissions by one unit. Each country is therefore indifferent between making direct abatements or matching abatements.

The analysis of this section leads to the following:

Proposition 3. *When there are n countries that can commit to matching the abatement efforts of each other at country-specific rates, and emissions quota trading exists, the equilibrium matching rates satisfy $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij}m_{jk} = 1$ for $i, j, k = 1, ..., n$, and parts i, iii and iv of Proposition 1 hold.*

4.3 A Two-Period Model

In this section, we extend the analysis to a two-period setting and show that the three-stage abatement process can induce full efficiency even in a dynamic context where current emissions increase the stock of pollution that will exist in the future. For simplicity, we return to the two-country case. We assume that, in each period, countries can offer to match each other's abatement commitments in the current period before engaging in emissions quota trading. In each period, matching rates and direct abatement commitments determine the number of period-specific emission quotas that each country holds. Trading takes place in each period and countries are not permitted to transfer emission quotas across periods. Therefore, the three-stage process of the basic one-period model is undertaken in each period, and in the first period, both countries anticipate the impact of their decisions on the second period equilibrium.

In what follows, superscripts will denote time periods and subscripts will denote countries. We normalize the initial stock of pollution to S^0. In period 1, the actual emissions of country 1 and country 2 are e_1^1 and e_2^1, respectively, while the initial stock S^0 decays at the rate γ, with $0 < \gamma < 1$. Therefore, the stock of pollution at the end of period 1 is:

$$S^1 = (1-\gamma)S^0 + e_1^1 + e_2^1$$

Similarly, the stock of pollution at the end of period 2 is:

$$S^2 = (1-\gamma)S^1 + e_1^2 + e_2^2 = (1-\gamma)\left[(1-\gamma)S^0 + e_1^1 + e_2^1\right] + e_1^2 + e_2^2$$

The levels of emissions in the absence of any abatements are assumed to be constant in both periods and equal to $\overline{e}_1$ and $\overline{e}_2$. Before characterizing the equilibrium of the abatement process, let us briefly examine the social optimum in this two-period case.

The Social Optimum

The socially optimal levels of emissions of each country in each period maximize the discounted sum of the two countries' benefits net of damages over both periods. It solves the following problem:

$$\max_{\{e_1^1, e_2^1, e_1^2, e_2^2\}} \quad B_1(e_1^1) - D_1(S^1) + B_2(e_2^1) - D_2(S^1) + \delta\left[B_1(e_1^2) - D_1(S^2) + B_2(e_2^2) - D_2(S^2)\right]$$

where δ is the a common discount factor and S_1 and S_2 are given by the expressions defined above. The first-order conditions imply the following:

$$B_1'(e_1^1) = B_2'(e_2^1); \qquad B_1'(e_1^2) = B_2'(e_2^2)$$

$$\frac{D_1'(S^2)+D_2'(S^2)}{B_1'(e_1^2)} = \frac{D_1'(S^2)+D_2'(S^2)}{B_2'(e_2^2)} = 1$$

and

$$\frac{D_1'(S^1)+D_2'(S^1)+\delta(1-\gamma)\left[D_1'(S^2)+D_2'(S^2)\right]}{B_1'(e_1^1)}$$

$$= \frac{D_1'(S^1)+D_2'(S^1)+\delta(1-\gamma)\left[D_1'(S^2)+D_2'(S^2)\right]}{B_2'(e_2^1)} = 1$$

The marginal benefits of emissions are equalized across countries in each of the two periods. In period 2, the sum of marginal damages is equal to each country's marginal benefit from emissions, while in period 1, each country's marginal benefit equals the sum of the two countries' period 1 marginal damages and the discounted, decay-adjusted period 2 marginal damages. Hence, full efficiency requires that the total level of emissions be efficient, and that emissions be efficiently allocated across countries and across periods.

The Two-Period Equilibrium

In period 1, countries 1 and 2 offer matching rates m_1^1 and m_2^1 and make direct abatement commitments a_1^1 and a_2^1, respectively. Their actual emissions are:

$$e_1^1 \equiv \bar{e}_1 - a_1^1 - m_1^1 a_2^1 + q_1^1, \quad e_2^1 \equiv \bar{e}_2 - a_2^1 - m_2^1 a_1^1 + q_2^1$$

Similarly, emissions in period 2 are:

$$e_1^2 \equiv \bar{e}_1 - a_1^2 - m_1^2 a_2^2 + q_1^2, \quad e_2^2 \equiv \bar{e}_2 - a_2^2 - m_2^2 a_1^2 + q_2^2$$

These actual emissions result in stocks of pollution in each period given by:

$$S^1 \equiv (1-\gamma)S^0 + e_1^1 + e_2^1 = (1-\gamma)S^0 + (\bar{e}_1 - a_1^1 - m_1^1 a_2^1) + (\bar{e}_2 - a_2^1 - m_2^1 a_1^1)$$

$$S^2 \equiv (1-\gamma)S^1 + e_1^2 + e_2^2 = (1-\gamma)[(1-\gamma)S^0 + e_1^1 + e_2^1] + e_1^2 + e_2^2$$

$$= (1-\gamma)[(1-\gamma)S^0 + +(\bar{e}_1 - a_1^1 - m_1^1 a_2^1) + (\bar{e}_2 - a_2^1 - m_2^1 a_1^1)] + (\bar{e}_1 - a_1^2 - m_1^2 a_2^2) + (\bar{e}_2 - a_2^2 - m_2^2 a_1^2)$$

We characterize the two-period equilibrium by backward induction starting with period 2.

Period 2

Since emission quotas cannot be transferred across periods, the decisions in the first period (a_1^1, a_2^1, m_1^1, m_2^1) will only affect the period 2 equilibrium through their effects on the pollution stock at the end of the first period, S^1. It is also straightforward to see that, for a given level of S_1, the three-stage abatement process that countries face in period 2 is essentially the same as in the basic one-period case, and the equilibrium will have the same characteristics. In particular, the equilibrium in period 2 will be fully efficient, given the pollution stock S_1. Denote the efficient total abatements in the second period by A_1^{2*} and A_2^{2*}, where $A_1^{2*} \equiv a_1^2 + m_1^2 a_2^2$ and $A_2^{2*} \equiv a_2^2 + m_2^2 a_1^2$. The demand for quotas by countries 1 and 2 satisfy, respectively, $B_1'(\bar{e}_1 - A_1^{2*} + q_1^2) = p^2$ and $B_2'(\bar{e}_2 - A_2^{2*} + q_2^2) = p^2$, with $\partial q_i^2/\partial A_i^2 = 1$ and $\partial q_i^2/\partial A_j^2 = 0$ for $i, j = 1, 2$, and can be written as $q_1^2(p^2, A_1^{2*})$ and $q_2^2(p^2, A_2^{2*})$. In equilibrium, $q_1^2(\cdot) + q_2^2(\cdot) = 0$ and $p^2(A_1^{2*}, A_2^{2*}) = B_1'(\bar{e}_1 - A_1^{2*} + q_1^2) = B_2'(\bar{e}_2 - A_2^{2*} + q_2^2)$.

Given that the outcome in period 2 is fully efficient, the marginal effect of the period 1 pollution stock on total abatements in period 2 can be derived from the condition that characterizes the social optimum:

$$f(\cdot) \equiv \frac{D_1'(S^2)}{B_1'(e_1^2)} + \frac{D_2'(S^2)}{B_2'(e_2^2)}$$

$$\equiv \frac{D_1'\left((1-\gamma)S^1 + \bar{e}_1 - A_1^{2*} + \bar{e}_2 - A_2^{2*}\right)}{B_1'\left(\bar{e}_1 - A_1^{2*} + q_1^2(\cdot)\right)} + \frac{D_2'\left((1-\gamma)S^1 + \bar{e}_1 - A_1^{2*} + \bar{e}_2 - A_2^{2*}\right)}{B_2'\left(\bar{e}_2 - A_2^{2*} + q_2^2(\cdot)\right)} = 1$$

Differentiating the above and using $\partial q_i^2/\partial A_i^2 = 1$ and $\partial q_i^2/\partial A_j^2 = 0$ for $i, j = 1, 2$, we have:

$$f_{A_1^2} = f_{A_2^2} = -\frac{D_1''(S^2)}{B_1'(e_1^2)} - \frac{D_2''(S^2)}{B_2'(e_2^2)}$$

$$f_{S^1} = \frac{(1-\gamma)D_1''(S^2)}{B_1'(e_1^2)} + \frac{(1-\gamma)D_2''(S^2)}{B_2'(e_2^2)} = -(1-\gamma)f_{A_1^2} = -(1-\gamma)f_{A_2^2}$$

from which we obtain:

$$\frac{\partial A_i^{2*}}{\partial S^1} = -\frac{f_{S^1}}{f_{A_i^{2*}}} = 1 - \gamma$$

Consequently, the change in the net benefit of country 1 in period 2 resulting from a change in the stock of pollution at the end of period 1 is given by

$$\begin{aligned}\frac{d(B_1(e_1^2)-D_1(S^2)-p^2q_1^2)}{dS^1} =& B_1'(e_1^2)\cdot\left[-\frac{\partial A_1^{2*}}{\partial S^1}+\frac{\partial q_1^2}{\partial A_1^{2*}}\frac{\partial A_1^{2*}}{\partial S^1}\right] \\ & -D_1'(S^2)\cdot\left[(1-\gamma)-\frac{\partial A_1^{2*}}{\partial S^1}-\frac{\partial A_2^{2*}}{\partial S^1}\right]-p^2\left[\frac{\partial q_1^2}{\partial A_1^{2*}}\frac{\partial A_1^{2*}}{\partial S^1}\right] \\ =& -(1-\gamma)\left[B_1'(e_1^2)-D_1'(S^2)\right]<0\end{aligned}$$

A similar expression holds for country 2. An increase in the stock of pollution in period 1, of which a proportion $(1-\gamma)$ will remain in period 2, will induce an increase in the total abatement of country 1 in period 2, reducing the period 2 net benefit of country 1 by an amount equal to the difference between its benefit from emission and its own damages from pollution $\left(B_1'(e_1^2)-D_1'(S^2)\right)$. Let $\Pi_1^2(S^1)$ and $\Pi_2^2(S^1)$ denote the second period net benefits of countries 1 and 2, respectively.

Period 1

Since quota trading in the third stage does not affect the stock of pollution at the end of period 1, the quota trading process has no impact on the second period. Therefore, the quota trading equilibrium has the same properties as in the static one-period case, and there is no need to characterize it again.

In Stage 2, country 1 chooses its direct abatement a_1^1, taking matching rates (m_1^1, m_2^1) and country 2's direct abatement a_2^1 as given and anticipating the effect of a_1^1 on the second period equilibrium, in order to maximize the discounted sum of its net benefits over both periods. Thus, it solves the following:

$$\max_{\{a_1^1\}} B_1(e_1^1)-D_1(S^1)-p^1q_1^1+\delta\Pi_1^2(S^1)$$

for which the first-order condition is

$$F(\cdot)\equiv -B_1'(e_1^1)+(1+m_2^1)D_1'(S^1)+\delta(1-\gamma)\left[-B_1'(e_1^2)+D_1'(S^2)\right]\left[-(1+m_2^1)\right]=0$$

This condition can be written as

$$\frac{D_1'(S^1)}{B_1'(e_1^1)}-\frac{\delta(1-\gamma)\left[D_1'(S^2)-B_1'(e_1^2)\right]}{B_1'(e_1^1)}=\frac{1}{1+m_2^1}$$

The second term in the expression above is the discounted reduction in country 1's second period net benefits resulting from higher first period pollution as a ratio of the marginal benefit of first period emissions. Country 1 chooses its level of direct abatement such that the sum of this discounted cost and of the ratio of first period marginal damages to marginal benefits of emissions equals the effective cost to country 1 of reducing world emissions by one unit, given that its own abatements are matched at the rate m_2^1 by country 2. The solution to this condition gives the reaction function of country 1, which can be shown to have the following properties:

$$\begin{aligned}
\frac{\partial a_1^1}{\partial a_2^1} &= -\frac{F_{a_2^1}}{F_{a_1^1}} = -\frac{1+m_1^1}{1+m_2^1} \\
\frac{\partial a_1^1}{\partial m_1^1} &= -\frac{F_{m_1^1}}{F_{a_1^1}} = -\frac{a_2^1}{1+m_2^1} \quad (= 0 \text{ at } a_2^1 = 0) \\
\frac{\partial a_1^1}{\partial m_2^1} &= -\frac{F_{m_2^1}}{F_{a_1^1}} = -\frac{a_1^1}{1+m_2^1} - \frac{D_1'(S^1) + \delta(1-\gamma)[-B_1'(e_1^2) + D_1'(S^2)]}{F_{a_1^1}} \\
&= -\frac{D_1'(S^1) + \delta(1-\gamma)[-B_1'(e_1^2) + D_1'(S^2)]}{F_{a_1^1}} > 0 \text{ at } a_1^1 = 0
\end{aligned}$$

The analog holds for country 2. Hence, the two countries' reaction curves have similar properties as in the one-period case. In particular, reaction curves are linear and parallel in the (a_1, a_2)–space for any matching rates (m_1^1, m_2^1). A demonstration analog to the one we used in the one-period model could be constructed to show that the equilibrium matching rates in Stage 1 are such that $m_1^1 m_2^1 = 1$, and that the subgame perfect equilibrium has same properties as in the one-period case. Hence, the equilibrium replicates the social optimum derived earlier, so both intra-temporal efficiency and inter-temporal efficiency are achieved. Total emissions are efficient, and they are efficiently allocated across countries and across periods.

The results of this section are summarized below.

Proposition 4. *In a two-period setting where both countries can commit to match each others abatements and engage in emissions quota trading in both periods, the subgame perfect equilibrium is such that:*

i. The properties listed in Proposition 1 apply in each period;

ii. Inter-temporal efficiency is achieved: emissions are efficiently allocated across periods.

5 Adding Contributions to an International Public Good

In this section, we explore how the introduction of an international public good provided through the voluntary contributions of countries will affect the pollution abatement process. For ease of exposition, we return to the basic one-period two-country case. Let the level of provision of the international public good be denoted by G and the contributions of each country by g_1 and g_2. Contributions are assumed to be perfect substitutes, so $G = g_1 + g_2$.

Utility in country i is a function of the international public good G and of private consumption x_i according to $u^i(G, x_i)$, which is increasing and quasi-concave in both arguments. Both G and x_i are assumed to be normal, and the latter is given by

$$x_i = w_i - g_i + B_i\Big(\overline{e}_i - a_i - m_i a_j + q_i\Big) - D_i\Big(\overline{e}_1 - (1+m_2)a_1 + \overline{e}_2 - (1+m_1)a_2\Big) - pq_i$$

where w_i is the initial endowment of country i. This formulation assumes that the benefits of emissions, net of damages, as well as the revenues from emissions quota trading are perfect substitutes for consumption.

The timing of decisions is important. We assume that countries choose their level of pollution abatement first, and then contribute to the international public good. With this order of decisions we find that, even without matching rate commitments and quota trading, both the level of emissions and the allocation of emissions across countries are efficient. Although we will not go through the analysis of the case where contributions to the public good are determined first, it is straightforward to show that, in this case, the equilibrium of the abatement process will is only efficient if countries are making matching rate commitments and are engaging in emission quota trading, as in the basic case without contributions to a public good.

As mentioned, with contributions to the public good determined after abatement decisions, commitments to matching abatements and emission quota trading turn out to be irrelevant. The sequence of decisions is simply as follows. In Stage 1, the two countries simultaneously choose emission abatements a_i. Both countries then set their contributions to the international public good g_i in Stage 2. We consider Stage 2 first.

Stage 2: Choosing Contributions to the International Public Good g_i

At the beginning of this stage, the available resources of the two countries are $w_1 + B_1(\bar{e}_1 - a_1) - D_1(\bar{e}_1 - a_1 + \bar{e}_2 - a_2)$ and $w_2 + B_2(\bar{e}_2 - a_2) - D_2(\bar{e}_1 - a_1 + \bar{e}_2 - a_2)$, given the levels of abatements (a_1, a_2) chosen in the previous stage. Country i chooses its contribution to maximize $u^i(g_1 + g_2, w_i - g_i + B_i(\cdot) - D_i(\cdot))$, taking the contribution of the other country as given. Assuming an interior solution to public good contributions, g_i is such that $u^i_G/u^i_x = 1$. The provision of the public good is inefficiently low given that efficient contributions would satisfy $u^1_G/u^1_x + u^2_G/u^2_x = 1$. More importantly, the well-known Neutrality Theorem (Shibata, 1971; Warr, 1983; Bergstrom, Blume, and Varian, 1986) implies that the private consumptions of the two countries and the level of public good provision will depend only on the sum of resources, and not on the distribution of resources across the two countries. The sum of resources here is $w_1 + w_2 + I$, where

$$I \equiv B_1(\bar{e}_1 - a_1) - D_1(\bar{e}_1 - a_1 + \bar{e}_2 - a_2) + B_2(\bar{e}_2 - a_2) - D_2(\bar{e}_1 - a_1 + \bar{e}_2 - a_2)$$

Thus, the two countries' utilities after the second stage can be written as $u^1[G(I), x_1(I)]$ and $u^2[G(I), x_2(I)]$, since $w_1 + w_2$ is constant. Given that G, x_1, and x_2 are normal goods, and that utilities are increasing in both arguments, maximizing I will also maximize the utility of each country. As a result, the objectives of the two countries in Stage 1 will be perfectly aligned.

Stage 1: Choosing Emission Abatements a_i

In this stage, the countries choose their abatement efforts, anticipating the outcome of Stage 2. The problem of country i consists in choosing a_i, given a_j, to maximize $u^i[G(I), x_1(I)]$, and the first-order condition implies that

$$-B_1'(\bar{e}_1 - a_1) + D_1'(\bar{e}_1 - a_1 + \bar{e}_2 - a_2) + D_2'(\bar{e}_1 - a_1 + \bar{e}_2 - a_2) = 0$$

It is immediately clear that the first-order conditions for the two countries taken together coincide with the conditions characterizing the social optimum derived in Section 2, i.e. $D_1'/B_1' + D_2'/B_2' = 1$ and $B_1' = B_2'$. Remarkably, the equilibrium is such that the level of emissions and the allocation of emissions across countries are efficient despite the fact that countries do not commit to match each other's abatements and there is no emission quota trading.

Moreover, even if countries are able to commit to matching the abatement efforts of each other, they cannot derive any gain from making such commitments. To see this, suppose that contributions to the international public good are chosen after the three-stage abatement process considered in Section 3. As above, utilities after contributions to the public good can still be written as $u^1[G(I), x_1(I)]$ and $u^2[G(I), x_2(I)]$, but where I is now

$$I \equiv B_1\Big(\bar{e}_1 - a_1 - m_1 a_2 + q_1(\cdot)\Big) - D_1\Big(\bar{e}_1 - (1+m_2)a_1 + \bar{e}_2 - (1+m_1)a_2\Big) - pq_1(\cdot)$$
$$+B_2\Big(\bar{e}_2 - a_2 - m_2 a_1 + q_2(\cdot)\Big) - D_2\Big(\bar{e}_1 - (1+m_2)a_1 + \bar{e}_2 - (1+m_1)a_2\Big) - pq_2(\cdot)$$

As usual, quota trading in Stage 3 leads to $B_1'(\cdot) = B_2'(\cdot) = p$. Using this, the first-order conditions for the choices of a_1 and a_2 in Stage 2 can be written as

$$F^1(\cdot) = -B_1' - m_2 B_2' + (1+m_2)(D_1' + D_2') \leqslant 0 \tag{14}$$

$$F^2(\cdot) \equiv -B_2' - m_1 B_1' + (1+m_1)(D_1' + D_2') \leqslant 0 \tag{15}$$

Suppose that matching rates are initially zero for both countries, $m_1 = m_2 = 0$. Since quota trading ensures that $B_1' = B_2'$, conditions (14) and (15) are the same. The reaction curves of the two countries are coinciding and equilibrium abatements satisfy

$$F^1(\cdot) = F^2(\cdot) = -B_1' + D_1' + D_2' = -B_2' + D_1' + D_2' = 0$$

The level and the allocation of abatements are efficient, but because we now allow for the possibility of quota trading, the allocation of direct abatement commitments between the two countries in Stage 2 is indeterminate. For a given level of total abatements, any change in the allocation of abatement commitments across the two countries will be undone through quota trading and will have no effect on I. In turn, this implies that countries will not have any incentives to induce a higher level of abatement commitment in Stage 2 from the other country by offering a stictly positive matching rate. To see this formally, differentiate country 1's objective function in Stage 1 with respect to m_1:

$$\frac{du^1}{dm_1} = -\frac{\partial a_1}{\partial m_1}[B_1' + m_2 B_2' - (1+m_2)(D_1' + D_2')]$$
$$-\frac{\partial a_2}{\partial m_1}[B_2' + m_1 B_1' - (1+m_1)(D_1' + D_2')] - a_2(B_2' - D_1' - D_2')$$

It is immediately clear that if $F^1 = F^2 = 0$, then $du^1/dm_1 = 0$. A similar result holds for country 2. Since $F^1 = F^2 = 0$ holds when $m_1 = m_2 = 0$, both countries choosing zero matching rates is an equilibrium in Stage 1.

The main results of this section are stated below.

Proposition 5. *If countries are making voluntary contributions to pollution abatement and then contribute voluntarily to an international public good, the equilibrium has the following properties:*

i. If contributions to the public good are strictly positive for both countries, the level of emissions and the allocation of emissions across countries are efficient without any matching rate commitments and quota trading;

ii. Countries cannot gain by offering strictly positive matching rates;

ii. Contributions to the public good are inefficient.

6 Concluding Remarks

Our purpose in this paper has been to characterize a process of pollution emissions reduction in which countries can commit to match each others' abatement efforts and subsequently engage in emissions quota trading. The mechanism that we considered is non-cooperative in the sense that each country, acting in its own self-interest, voluntarily offers to match the emission abatements of the other countries at some announced rates, anticipating the subsequent abatement equilibrium and the outcome of emissions quota trading. The analysis has shown that this mechanism leads to a fully efficient outcome. The level of emissions is efficient, as well as the allocation of emissions across countries. This result holds independently of the number of countries involved, and in an environment where countries have different abatement technologies as well as different benefits from emissions. In a dynamic setting where the quality of the environment depends on cumulative emissions over two periods, the mechanism is found to achieve both intra-temporal and inter-temporal efficiency.

The mechanism also has appealing distributional implications. The equilibrium set of matching rates implies that the effective cost at which any given country can reduce world emissions is equal to that country's marginal valuation of pollution reduction relative to its marginal benefits of emissions. Thus, the initial allocation of emission quotas across countries (before trading) emerges endogenously without any form of central coordination and reflects each country's net marginal benefits from reducing pollution. This result also implies that all countries find it in their own interest to participate. Therefore, the mechanism does not take the participation of countries as given. Countries with relatively low net marginal valuations for pollution reduction will face relatively low effective costs of abatement, given the set of equilibrium matching rates.

We extended the model by considering the case where countries are voluntarily contributing to an international public good in addition to undertaking pollution abatement. We found that if public good contributions are determined after abatement efforts, the level of emissions is efficient even in the absence of any matching abatement commitments. In fact, the incentive for countries to match the abatements of each other vanishes entirely. Moreover, the allocation of emissions across countries is efficient even in the absence of emissions quota trading.

Throughout, our analysis has assumed that all countries were able to commit to match the other countries' abatements. It would be interesting to extend the analysis to characterize the pollution abatement process when only a subset of countries are able to commit. In this case, different forms of commitment could emerge as well as different distributions of the gains from achieving more efficient allocations.

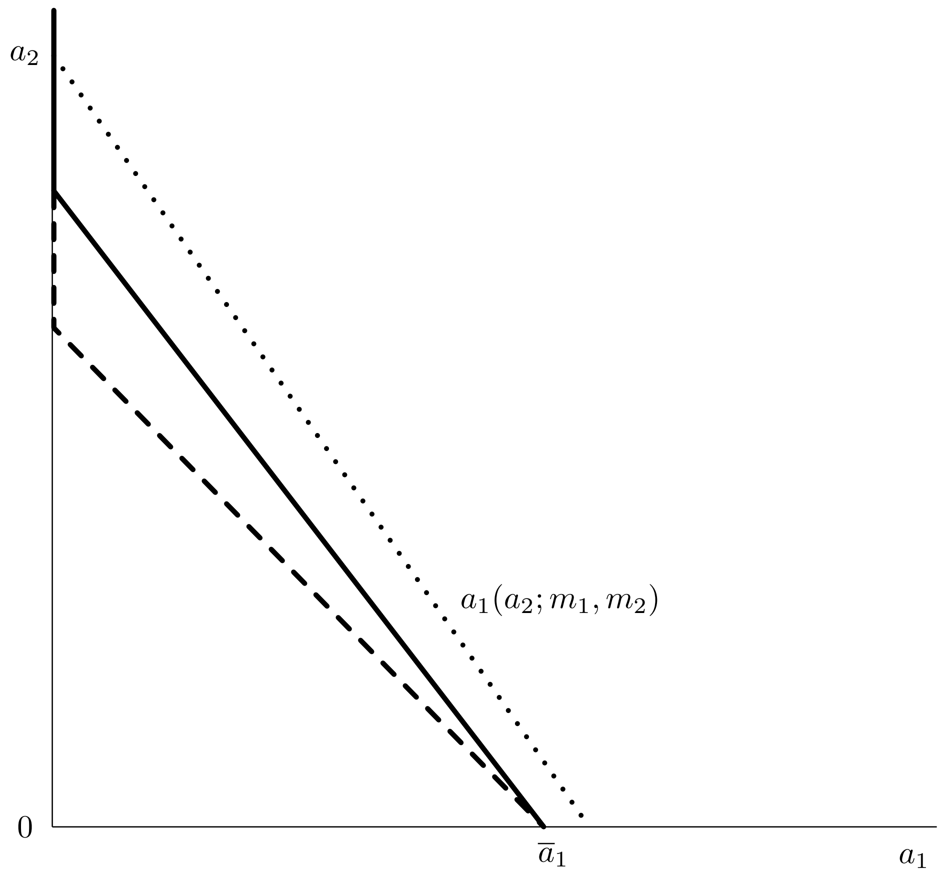

Figure 1. Effects of an increase in m_1 and m_2 on country 1's reaction curve

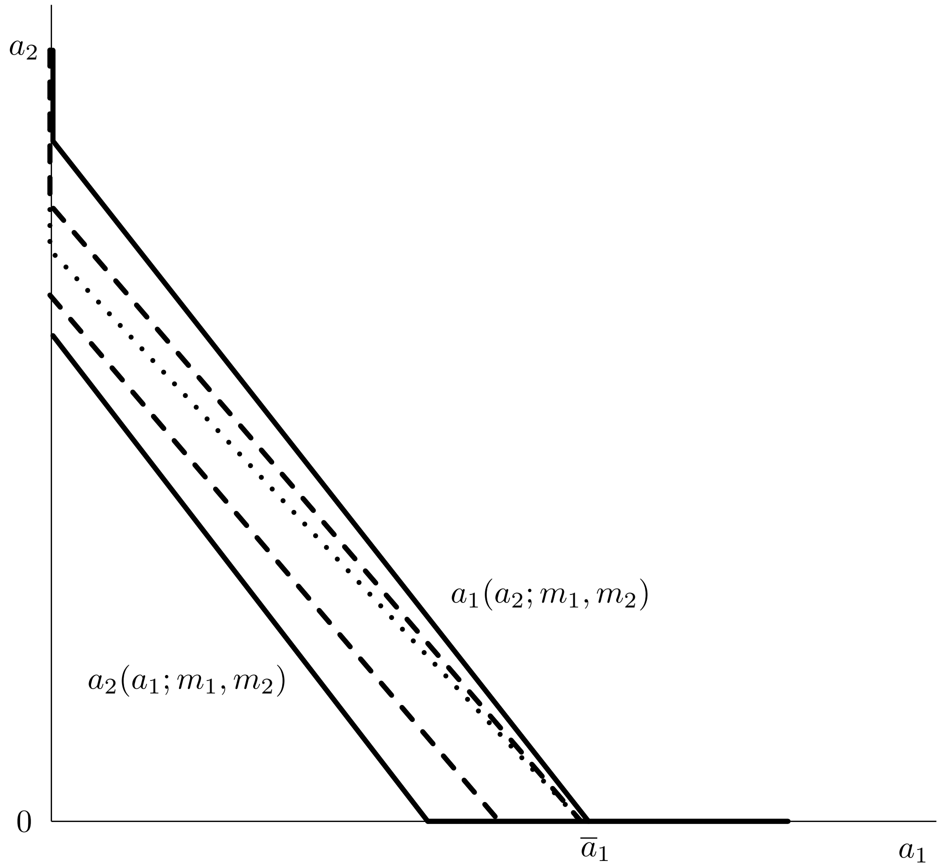

Figure 2. Effect of an increase in m_1, given m_2, in the Stage 2 Nash Equilibrium

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