Taxing sin goods and subsidizing health care

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Abstract

We consider a two-period model. In the first period, individuals consume two goods: one is sinful and the other is not. The sin good brings pleasure but has a detrimental effect on second period health and individuals tend to underestimate this effect. In the second period, individuals can devote part of their saving to improve their health status and thus compensate for the damage caused by their sinful consumption. We consider two alternative specifications concerning this second period health care decision: either individuals acknowledge that they have made a mistake in the first period out of myopia or ignorance, or they persist in ignoring the detrimental effect of their sinful consumption. In either case, there is call for government intervention that operates through linear taxes or subsidies.

1 Introduction

In our everyday life we consume a number of goods that all bring us utility. For most of them, that is all. For some, today's consumption can have some effects on tomorrow's health. For example, smoking leads to shorter lives or excess sugar to diabetes.[1]

To the extent that we impose costs on ourselves there is no need of government action except if out of ignorance or myopia we don't take into account the delayed damage done to our health. If this is the case, then there is a "paternalistic" mandate for public action, assuming that the government has a correct perception of the health damage generated by our sinful consumption.

When the damage is done, there is a possibility of alleviating it through health care. We consider two possibilities. In the first one, individuals persist in their ignorance of the link between health on the one hand and sinful consumption or health care on the other hand. In the second possibility, individuals realize their error in the second period and make the appropriate decision concerning health care.

In both cases, government action is needed. In the first one, the action is purely paternalistic. Through Pigouvian tax or subsidy the government should induce individuals into choosing the appropriate amount of sinful consumption and health care. In an identical individuals setting, the first-best can be decentralized with Pigouvian instruments. In the second case, there is some time inconsistency as in the second period individuals acknowledge to have made an error and try to compensate for it. The social planner faces a problem with changing preferences.

Here again, with identical individuals, the first-best can be decentralized with a sin tax and a subsidy on saving. The idea is that in the second period of his life the individual makes the right health care decision, but he needs to have enough resources for both second period consumption and health spending.

Not surprisingly, when individuals differ not only in incomes but also in myopia (or ignorance), things become more difficult. If, for example, low income individuals

[1] We are not concerned here by two important issues: addiction and externalities associated with sinful activities. In our model, there is no addiction and the damage done by the sinful consumption is internal (through lower health condition).

don't see the damage their sinful consumption does to their health and hence end up consuming more sin goods than the high income individuals, the issue of sin taxation becomes tricky.

The rest of the paper is organized as follows. In section 2, the model, the first-best solutions and the decentralization conditions are presented. Then in section 3 we turn to the second-best problem when individuals persist in their ignorance. In section 4, we study the alternative second-best problem, that is when individuals realize having made a mistake. A final section concludes.

2 First-best and decentralization

2.1 Model

We consider a society consisting of a number of types of individuals i. Each type is characterized by a wealth endowment w_i and a subjective and objective health parameter α_i and β_i. Each individual's life spans over two periods. In the first one, he consumes a numeraire good c_i and a sin good x_i. He also saves s_i for future expenses. In the second period, he consumes and amount d_i of the numeraire and he invests e_i in health improvement. In this second period, he enjoys a quality of health $\beta_i h(x_i, e_i)$, on which x_i has a negative effect and e_i a positive effect. For reason of ignorance or myopia, the individual has a perception of this function that underestimates the impact of both arguments. In other words, he perceives a health function equal to $\alpha_i h(x_i, e_i)$ with $\alpha_i < \beta_i$.

His two-period utility function can be written as:

$$U_i = u(c_i) + \varphi(x_i) + u(d_i) + \alpha_i h(x_i, e_i) \quad (1)$$

with budget constraints:

$$\begin{aligned} w_i &= (1+\tau) s_i + (1+\theta) x_i + c_i - a \\ d_i &= s_i - (1+\sigma) e_i \end{aligned}$$

where τ, θ, σ are tax rates and a is a demogrant. For simplicity reasons, we assume a zero time discount rate and a zero rate of interest.

2.2 First-best

We assume that the government is a paternalistic utilitarian. In other words, it adopts an objective made of the sum of utilities (1) in which β_i replaces α_i. As a benchmark, we derive the first-best (FB) conditions by solving the following Lagrangian:

$$\begin{aligned}\mathcal{L}_1 &= \sum n_i \left[u\left(c_i\right)+\varphi\left(x_i\right)+u\left(d_i\right)+\beta_i h\left(x_i,e_i\right)\right. \\ &\quad \left. -\mu\left(c_i+x_i+d_i+e_i-w_i\right)\right]\end{aligned}$$

where μ is the Lagrangian multiplier associated with the resource constraints and n_i the relative number of type i's individuals. The FOC's yield:

$$u'\left(c_i\right)=u'\left(d_i\right)=\varphi'\left(x_i\right)+\beta_i h_x\left(x_i,e_i\right)=\beta_i h_e\left(x_i,e_i\right)=\mu$$

with $h_x<0$ and $h_e>0$. Denote the first best solution by c_i^*, x_i^*, d_i^* and e_i^*. Note that c_i^* and d_i^* are equal and the same for all. If $\alpha_i=\alpha$ and $\beta_i=\beta$, x_i^* and e_i^* would also be identical for all. We can also define $s_i^*=d_i^*+e_i^*$, the (implicit) individual savings at the first-best solution.

It is interesting to contrast these conditions with the *laissez-faire* (LF) ones; that are obtained by maximizing:

$$\begin{aligned}U_i &= u\left(w_i-(1+\tau)\,s_i-(1+\theta)\,x_i+a_i\right) \\ &\quad +\varphi\left(x_i\right)+\alpha_i h\left(x_i,e_i\right)+u\left(s_i-(1+\sigma)\,e_i\right).\end{aligned}$$

In the LF, $\tau=\sigma=\theta=a_i=0$ and we have:

$$u'\left(c_i\right)=u'\left(d_i\right)=\varphi'\left(x_i\right)+\alpha_i h_x\left(x_i,e_i\right)=\alpha_i h_e\left(x_i,e_i\right).$$

2.3 Decentralization with persisting ignorance

To decentralize the above optimum, we need individualized redistributive lump sum taxes a_i and individualized corrective taxes or subsidies on the sin good and health expenditure.

$$\theta_i=\frac{\left(\alpha_i-\beta_i\right)h_x\left(x_i^*,e_i^*\right)}{\beta_i h_e\left(x_i^*,e_i^*\right)}>0 \tag{2}$$

and

$$\sigma_i = \frac{(\alpha_i - \beta_i)}{\beta_i} < 0. \tag{3}$$

Those taxes and subsidies are individualized. Naturally, with $\alpha_i = \alpha$ and $\beta_i = \beta$, they would be identical for all. There is no need to influence saving.

2.4 Decentralization with dual self

Up to now we have assumed that individuals stick to their beliefs in the second period. Let us now make the reasonable assumption that in the second period they realize that they have made a mistake out of ignorance or myopia and will accordingly modify their decision concerning health care. In behavioral economics, one then speaks of dual self. When the "reasonable" self prevails in the second period, the choice of e_i is determined by the equality

$$(1+\theta)u'(s_i - e_i) = \beta_i h_e(x_i, e_i).$$

Is it possible to decentralize the first-best optimum in these conditions with our linear instruments that are chosen in the first period? In fact, this is possible using τ_i and θ_i plus a_i. With these instruments, one obtains the optimal values x_i^* and s_i^*, given that s_i^* implies e_i^*. Define e_i^{P*} such that

$$\alpha_i h_e(x_i^*, e_i^{P*}) = u'(s_i^* - e_i^{P*}).$$

In words, e_i^{P*} is the planned level of e_i when the tax instruments are set to decentralize the first-best. Implementing tax rates must satisfy

$$\begin{aligned} &-(1+\tau_i)u'(c_i^*) + u'(s_i^* - e_i^{P*}) = 0 \\ &-(1+\theta_i)u'(c_i^*) + \varphi'(x_i^*) + \alpha_i h_x(x_i^*, e_i^{P*}) = 0 \end{aligned}$$

and are thus given by

$$\tau_i = \frac{u'(s_i^* - e_i^{P*}) - u'(c_i^*)}{u'(c_i^*)}, \tag{4}$$

$$\theta_i = \frac{\varphi'(x_i^*) + \alpha_i h_x(x_i^*, e_i^{P*}) - u'(c_i^*)}{u'(c_i^*)} = \frac{\alpha_i h_x(x_i^*, e_i^{P*}) - \beta_i h_x(x_i^*, e_i^*)}{u'(c_i^*)}. \tag{5}$$

Note that assuming $\alpha_i = \alpha$ and $\beta_i = \beta$, $\tau_i = \tau$ and $\theta_i = \theta$. In words, a_i makes everyone identical and the Pigouvian tax and subsidy rates are identical.

To illustrate this point, assume a single individual with $\alpha = 0 < \beta = 1$. We then have $e_i^{P*} = 0$ so that the implementing tax rates or subsidy are

$$\begin{aligned}\tau &= \frac{u'(s^*)}{u'(c^*)} - 1 < 0\\ \theta &= \frac{-h_x(x^*, e^*)}{u'(c^*)} > 0.\end{aligned}$$

It is interesting to compare the sin taxes obtained under the two specifications. To make the comparison easier, we assume that $\alpha_i = \alpha > 0$ and $\beta_i = \beta$. We thus have (with S for single self and D for dual self):

$$\theta^S = \frac{(\alpha - \beta) h_x(x^*, e^*)}{u'(c^*)}$$

and

$$\theta^D = \frac{(\alpha - \beta) h_x(x^*, e^*) + \alpha \left[h_x\left(x^*, e^{P*}\right) - h_x(x^*, e^*)\right]}{u'(c^*)}.$$

To interpret these formula, one has to make an assumption on h_{xe}. We can indeed show that

$$\theta^S \gtrless \theta^D \quad \text{iff} \quad h_{xe} \gtrless 0.$$

The assumption $h_{xe} > 0$ seems to be the most natural. It implies that the marginal effect of health care on health increases as x increases. We will make this assumption throughout the paper.[2] With this assumption the sin tax is smaller when the individual acknowledges his mistake in the second period of his life.

We will make another assumption in the remaining of this paper: $\beta_i = \beta > \alpha_i$. In other words, the objective effect of both e and x on health is the same for all.

3 Second-best in the case of persistent errors

We now turn to the second-best setting with linear tax instruments and uniform demogrant. We first consider the case when the individuals never acknowledge that the

[2] An example verifying this assumption is $h(x, e) = f\left(e - x^2/2\right)$ with $f' > 0$ and $f'' < 0$.

true health parameter is β. In that case, restricting the instruments to linear taxes and uniform demogrant we write the new Lagrangian as:

$$\begin{aligned}\mathcal{L}_2 &= \sum n_i \left[u\left(w_i - s_i(1+\tau) - x_i\left(1+\theta\right) + a\right) + \varphi\left(x_i\right) + u\left(s_i - \left(1+\sigma\right)e_i\right)\right. \\ &\left. +\beta_i h\left(x_i, e_i\right) + \mu\left(a - \tau s_i - \theta x_i - \sigma e_i\right)\right]\end{aligned}$$

where s_i, x_i and e_i are functions of a, θ, τ and σ and are obtained from the following optimal conditions for individual choices:

$$-u'\left(c\right)\left(1+\tau\right) + u'\left(d\right) = 0 \tag{6}$$

$$-u'\left(c\right)\left(1+\theta\right) + \varphi'\left(x\right) + \alpha h_x\left(x, e\right) = 0 \tag{7}$$

$$-u'\left(d\right)\left(1+\sigma\right) + \alpha h_e\left(x, e\right) = 0. \tag{8}$$

Assuming interior solutions, the FOC's of the social problem are given by:

$$\begin{aligned}\frac{\partial \mathcal{L}_2}{\partial a} &= Eu'\left(c\right) + E\left[h_x\left(x,e\right)\frac{\partial x}{\partial a} + h_e\left(x,e\right)\frac{\partial e}{\partial a}\right]\left(\beta - \alpha\right) \\ &+\mu E\left[1 - \tau\frac{\partial s}{\partial a} - \theta\frac{\partial x}{\partial a} - \sigma\frac{\partial e}{\partial a}\right] = 0 \\ \frac{\partial \mathcal{L}_2}{\partial \tau} &= -Eu'\left(c\right)s + E\left[h_x\left(x,e\right)\frac{\partial x}{\partial \tau} + h_e\left(x,e\right)\frac{\partial e}{\partial \tau}\right]\left(\beta - \alpha\right) \\ &-\mu E\left[s + \tau\frac{\partial s}{\partial \tau} + \theta\frac{\partial x}{\partial \tau} + \sigma\frac{\partial e}{\partial \tau}\right] = 0 \\ \frac{\partial \mathcal{L}_2}{\partial \theta} &= -Eu'\left(c\right)x + E\left[h_x\left(x,e\right)\frac{\partial x}{\partial \theta} + h_e\left(x,e\right)\frac{\partial e}{\partial \theta}\right]\left(\beta - \alpha\right) \\ &-\mu E\left[x + \tau\frac{\partial s}{\partial \theta} + \theta\frac{\partial x}{\partial \theta} + \sigma\frac{\partial e}{\partial \theta}\right] = 0 \\ \frac{\partial \mathcal{L}_2}{\partial \sigma} &= -Eu'\left(d\right)e + E\left[h_x\left(x,e\right)\frac{\partial x}{\partial \sigma} + h_e\left(x,e\right)\frac{\partial e}{\partial \sigma}\right]\left(\beta - \alpha\right) \\ &-\mu E\left[e + \tau\frac{\partial s}{\partial \sigma} + \theta\frac{\partial x}{\partial \sigma} + \sigma\frac{\partial e}{\partial \sigma}\right] = 0.\end{aligned}$$

In these expressions, we have used the operator E for $\sum n_i$.

In compensated terms,[3] these expressions can be written as:

$$\frac{\partial\tilde{\mathcal{L}}_2}{\partial\tau} = -Eu'(c)\,s + Eu'(c)\,Es + E\left[h_x(x,e)\frac{\partial\tilde{x}}{\partial\tau} + h_e(x,e)\frac{\partial\tilde{e}}{\partial\tau}\right](\beta-\alpha)$$
$$-\mu E\left[\tau\frac{\partial\tilde{s}}{\partial\tau} + \theta\frac{\partial\tilde{x}}{\partial\tau} + \sigma\frac{\partial\tilde{e}}{\partial x}\right] = 0$$

$$\frac{\partial\tilde{\mathcal{L}}_2}{\partial\theta} = -Eu'(c)\,x + Eu'(c)\,Ex + E\left[h_x(x,e)\frac{\partial\tilde{x}}{\partial\theta} + h_e(x,e)\frac{\partial\tilde{e}}{\partial\theta}\right](\beta-\alpha)$$
$$-\mu E\left[\tau\frac{\partial\tilde{s}}{\partial\theta} + \theta\frac{\partial\tilde{x}}{\partial\theta} + \sigma\frac{\partial\tilde{e}}{\partial\theta}\right] = 0$$

$$\frac{\partial\tilde{\mathcal{L}}_2}{\partial\sigma} = -Eu'(c)\,e + Eu'(c)\,Ee + E\left[h_x(x,e)\frac{\partial\tilde{x}}{\partial\sigma} + h_e(x,e)\frac{\partial\tilde{e}}{\partial\sigma}\right](\beta-\alpha)$$
$$-\mu E\left[\tau\frac{\partial\tilde{s}}{\partial\sigma} + \theta\frac{\partial\tilde{x}}{\partial\sigma} + \sigma\frac{\partial\tilde{e}}{\partial\sigma}\right] = 0.$$

Note that with either identical individuals or individualized lump sum transfers a_i, the first-best optimum is obtained with just σ and θ. In what follows, we assume that $\tau = 0$, namely there is no tax on saving. With this assumption, we have:

$$\mu\theta = \frac{-cov(u'(c),c)\,E\frac{\partial\tilde{e}}{\partial\sigma} + cov(u'(c),e)\,\frac{\partial\tilde{e}}{\partial\theta} + E(\beta-\alpha)\,H_\theta E\frac{\partial\tilde{e}}{\partial\sigma} - E(\beta-\alpha)\,H_\sigma E\frac{\partial\tilde{e}}{\partial\theta}}{E\frac{\partial\tilde{x}}{\partial\theta}E\frac{\partial\tilde{e}}{\partial\sigma} - E\frac{\partial\tilde{e}}{\partial\theta}E\frac{\partial\tilde{x}}{\partial\sigma}}$$

$$\mu\sigma = \frac{-cov(u'(c),c)\,E\frac{\partial\tilde{x}}{\partial\theta} + cov(u'(c),x)\,\frac{\partial\tilde{x}}{\partial\sigma} + E(\beta-\alpha)\,H_\sigma\frac{\partial\tilde{x}}{\partial\theta} - E(\beta-\alpha)\,H_\theta E\frac{\partial\tilde{x}}{\partial\sigma}}{E\frac{\partial\tilde{x}}{\partial\theta}E\frac{\partial\tilde{e}}{\partial\sigma} - E\frac{\partial\tilde{e}}{\partial\theta}E\frac{\partial\tilde{x}}{\partial\sigma}}$$

where $H_\theta = h_x\frac{\partial\tilde{x}}{\partial\theta} + h_e\frac{\partial\tilde{e}}{\partial\theta}$ and $H_\sigma = h_x\frac{\partial\tilde{x}}{\partial\sigma} + h_e\frac{\partial\tilde{e}}{\partial\sigma}$.

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[3]
$$\begin{aligned}
\frac{\partial\tilde{\mathcal{L}}_2}{\partial\tau} &= \frac{\partial\mathcal{L}_2}{\partial\tau} + \frac{\partial\mathcal{L}_2}{\partial a}Es \\
\frac{\partial\tilde{\mathcal{L}}_2}{\partial\theta} &= \frac{\partial\mathcal{L}_2}{\partial\theta} + \frac{\partial\mathcal{L}_2}{\partial a}Ex \\
\frac{\partial\tilde{\mathcal{L}}_2}{\partial\sigma} &= \frac{\partial\mathcal{L}_2}{\partial\sigma} + \frac{\partial\mathcal{L}_2}{\partial a}Ee.
\end{aligned}$$

In the case we assume that the cross derivatives are negligible, namely $\dfrac{\partial \tilde{x}}{\partial \sigma} \to 0$ and $\dfrac{\partial \tilde{e}}{\partial \theta} \to 0$, we can write:

$$\theta = \frac{-\text{cov}\left(u'\left(c\right), x\right) + Eh_x\left(x, e\right)\left(\beta - \alpha\right)\dfrac{\partial \tilde{x}}{\partial \theta}}{\mu E \dfrac{\partial \tilde{x}}{\partial \theta}} \tag{9}$$

$$\sigma = \frac{-\text{cov}\left(u'\left(c\right), e\right) + Eh_e\left(x, e\right)\left(\beta - \alpha\right)\dfrac{\partial \tilde{e}}{\partial \sigma}}{\mu E \dfrac{\partial \tilde{e}}{\partial \sigma}}. \tag{10}$$

Note first that with identical individuals, (9) and (10) reduce to (2) and (3). With different individuals, the covariance term is expected to be negative; both derivatives are also negative as well as μ. The first term of the numerator of (9) and (10) reflects redistributive consideration. It depends on the concavity of $u(\cdot)$, the initial inequality of earnings and the correlation between α_i and w_i. With a positive correlation the absolute value of the covariance is likely to be higher than with a zero correlation. The second term of the numerator of (9) and (10) is the Pigouvian term found in (2) and (3) summed over all individuals with weights equal to the effect of the tax on individual demands of either x or e.

Turning to the complete formula, we now have the cross effects that can either increase or reduce the gap between θ and σ.

4 Second-best with dual self

Now we assume that the individuals realize after one period that they made a mistake and that the only corrective decision they can make is the choice of health expenditure. We thus distinguish between the planned investment $e^P$and the *ex post* choice. The latter corresponds to the FOC:

$$-\left(1+\sigma\right)u'\left(s - e\left(1+\sigma\right)\right) + \beta h_e\left(x, e\right) = 0 \tag{11}$$

where s and x are predetermined by (6) and (7). The indirect utility function used by the social planner in its welfare maximization has to take into account these two values of e which yields two values of d.

Which demand function are to be used? In the first period, the functions $x(\tau,\theta,\sigma,a)$, $s(\tau,\theta,\sigma,a)$ and $e^P(\tau,\theta,\sigma,a)$ are obtained as the solution to:

$$-u'(c)(1+\tau)+u'\left(d^P\right)=0 \tag{12}$$

$$-u'(c)(1+\theta)+\varphi'(x)+\alpha h_x\left(x,e^P\right)=0 \tag{13}$$

$$-u'\left(d^P\right)(1+\sigma)+\alpha h_e\left(x,e^P\right)=0. \tag{14}$$

where $d^P=s-e^P(1+\sigma)>d=s-e(1+\sigma)$.

In the second period we have the effective demand for e defined by (11)

$$e=f\left(s,x,e^P\right)=e(\tau,\theta,\sigma,a). \tag{15}$$

The Lagrangian is given by:

$$\begin{aligned}\mathcal{L}_3 \;=\; & \sum n_i\left[u\left(w_i-s_i(1+\tau)-x_i(1+\theta)+a\right)+\varphi(x_i)+u\left(s_i-(1+\sigma)e_i\right)\right.\\ & \left.+\beta_i h(x_i,e_i)+\mu\left(a-\tau s_i-\theta x_i-\sigma e_i\right)\right]\end{aligned}$$

which is similar to $\mathcal{L}_2$ except that individual choices are now determined by (12), (13) and (15).

$$\begin{aligned}\frac{\partial\mathcal{L}_3}{\partial a} &= Eu'(c)+E\left\{u'[s-e(1+\sigma)]-u'[s-e^P(1+\sigma)]\right\}\frac{\partial s}{\partial a}\\ &+E\left[\beta h_x(x,e)-\alpha h_x\left(x,e^P\right)\right]\frac{\partial x}{\partial a}\\ &+\mu E\left[1-\tau\frac{\partial s}{\partial a}-\theta\frac{\partial x}{\partial a}-\sigma\frac{\partial e}{\partial a}\right]=0.\end{aligned}$$

$$\begin{aligned}\frac{\partial\mathcal{L}_3}{\partial \tau} &= -Eu'(c)s+E\left\{u'[s-e(1+\sigma)]-u'[s-e^P(1+\sigma)]\right\}\frac{\partial s}{\partial \tau}\\ &+E\left[\beta h_x(x,e)-\alpha h_x\left(x,e^P\right)\right]\frac{\partial x}{\partial \tau}\\ &-\mu E\left[s+\tau\frac{\partial s}{\partial \tau}+\theta\frac{\partial x}{\partial \tau}+\sigma\frac{\partial e}{\partial \tau}\right]=0.\end{aligned}$$

$$\begin{aligned}\frac{\partial\mathcal{L}_3}{\partial \theta} &= -Eu'(c)x+E\left\{u'[s-e(1+\sigma)]-u'[s-e^P(1+\sigma)]\right\}\frac{\partial s}{\partial \theta}\\ &+E\left[\beta h_x(x,e)-\alpha h_x\left(x,e^P\right)\right]\frac{\partial x}{\partial \theta}\\ &-\mu E\left[x+\tau\frac{\partial s}{\partial \theta}+\theta\frac{\partial x}{\partial \theta}+\sigma\frac{\partial e}{\partial \theta}\right]=0.\end{aligned}$$

$$\frac{\partial \mathcal{L}_3}{\partial \sigma} = -Eu'\left(s - e\left(1+\sigma\right)\right)e + E\left\{u'[s - e\left(1+\sigma\right)] - u'[s - e^P\left(1+\sigma\right)]\right\}\frac{\partial s}{\partial \sigma}$$
$$+E\left[\beta h_x\left(x,e\right) - \alpha h_x\left(x,e^P\right)\right]\frac{\partial x}{\partial \sigma}$$
$$-\mu E\left[e + \tau\frac{\partial s}{\partial \sigma} + \theta\frac{\partial x}{\partial \sigma} + \sigma\frac{\partial e}{\partial \sigma}\right] = 0.$$

As above, we assume $\tau = 0$ and use $\frac{\partial \mathcal{L}_3}{\partial a}$ to obtain the compensated expressions of $\frac{\partial \mathcal{L}_3}{\partial \theta}$ and $\frac{\partial \mathcal{L}_3}{\partial \sigma}$.

$$\frac{\partial \tilde{\mathcal{L}}_3}{\partial \theta} = -cov\left(u'\left(c\right),x\right) + E\left(u'\left(d\right) - u'\left(d^P\right)\right)\frac{\partial \tilde{s}}{\partial \theta}$$
$$+E\left(\beta h_x\left(x,e\right) - \alpha h_x\left(x,e^P\right)\right)\frac{\partial \tilde{x}}{\partial \theta} - \mu\left(\theta E\frac{\partial \tilde{x}}{\partial \theta} + \sigma E\frac{\partial \tilde{e}}{\partial \theta}\right) = 0,$$

$$\frac{\partial \tilde{\mathcal{L}}_3}{\partial \sigma} = -cov\left(u'\left(c\right),e\right) + E\left(u'\left(d\right) - u'\left(d^P\right)\right)\frac{\partial \tilde{s}}{\partial \sigma}$$
$$+E\left(\beta h_x\left(x,e\right) - \alpha h_x\left(x,e^P\right)\right)\frac{\partial \tilde{x}}{\partial \sigma} - \mu\left(\theta E\frac{\partial \tilde{x}}{\partial \sigma} + \sigma E\frac{\partial \tilde{e}}{\partial \sigma}\right) = 0.$$

It is clear from the above that even with identical individual, one cannot achieve the first-best with θ and σ as instruments. Solving for θ and σ, we obtain:

$$\Delta\mu\theta = -\operatorname{cov}\left(u'\left(c\right),x\right)E\frac{\partial \tilde{e}}{\partial \sigma} + \operatorname{cov}\left(u'\left(c\right),e\right)E\frac{\partial \tilde{e}}{\partial \theta} + E\tilde{H}\frac{\partial \tilde{x}}{\partial \theta}E\frac{\partial \tilde{e}}{\partial \sigma} - E\tilde{H}\frac{\partial \tilde{x}}{\partial \sigma}E\frac{\partial \tilde{e}}{\partial \theta}$$
$$+E\left(u'\left(d\right) - u'\left(d^P\right)\right)\frac{\partial \tilde{s}}{\partial \theta}E\frac{\partial \tilde{e}}{\partial \sigma} - E\left(u'\left(d\right) - u'\left(d^P\right)\right)\frac{\partial \tilde{s}}{\partial \sigma}E\frac{\partial \tilde{e}}{\partial \theta}$$

$$\Delta\mu\sigma = -\operatorname{cov}\left(u'\left(c\right),e\right)E\frac{\partial \tilde{x}}{\partial \theta} + \operatorname{cov}\left(u'\left(c\right),x\right)E\frac{\partial \tilde{x}}{\partial \sigma} + E\tilde{H}\frac{\partial \tilde{x}}{\partial \sigma}E\frac{\partial \tilde{x}}{\partial \theta} - E\tilde{H}\frac{\partial \tilde{x}}{\partial \theta}E\frac{\partial \tilde{x}}{\partial \sigma}$$
$$+E\left(u'\left(d\right) - u'\left(d^P\right)\right)\frac{\partial \tilde{s}}{\partial \sigma}E\frac{\partial \tilde{x}}{\partial \theta} - E\left(u'\left(d\right) - u'\left(d^P\right)\right)\frac{\partial \tilde{s}}{\partial \theta}E\frac{\partial \tilde{x}}{\partial \sigma}$$

where $\tilde{H} = \beta h_x\left(x,e\right) - \alpha h_x\left(x,e^P\right)$ and $\Delta = E\frac{\partial \tilde{x}}{\partial \theta}E\frac{\partial \tilde{e}}{\partial \sigma} - E\frac{\partial \tilde{e}}{\partial \theta}E\frac{\partial \tilde{x}}{\partial \sigma}$.

These can be rewritten as follows if we assume that the cross price effects are negligible:

$$\mu\theta = \frac{-\operatorname{cov}\left(u'\left(c\right),x\right) + E\left(u'\left(d\right) - u'\left(d^P\right)\right)\frac{\partial \tilde{s}}{\partial \theta} + E\left[\beta h_x\left(x,e\right) - \alpha h_x\left(x,e^P\right)\right]\frac{\partial \tilde{x}}{\partial \theta}}{E\frac{\partial \tilde{x}}{\partial \theta}} \qquad (16)$$

$$\mu\sigma = \frac{-\operatorname{cov}\left(u'\left(c\right),e\right) + E\left(u'\left(d\right) - u'\left(d^P\right)\right)\dfrac{\partial\tilde{s}}{\partial\sigma}}{E\dfrac{\partial\tilde{e}}{\partial\sigma}}. \tag{17}$$

Again the covariance terms are negative reflecting the equity concern of public policy. The second term of the denominator has the sign of either $\dfrac{\partial\tilde{s}}{\partial\sigma}$ or $\dfrac{\partial\tilde{s}}{\partial\theta}$. If either tax has a positive effect on saving, then it should have a relatively higher value. The third term of the numerator of (16) is the Pigouvian term one finds in (5). With $\alpha = 0$, it is definitively positive.

In the case of identical individuals, (16) and (17) can be rewritten as:

$$\mu\theta = \frac{\left[u'\left(d\right) - u'\left(d^P\right)\right]\dfrac{\partial\tilde{s}}{\partial\theta}}{\dfrac{\partial\tilde{x}}{\partial\theta}} + \left(\beta - \alpha\right)h_x\left(x,e\right) + \alpha\left[h_x\left(x,e\right) - h_x(x,e^P)\right],$$

$$\mu\sigma = \left[u'\left(d\right) - u'\left(d^P\right)\right]\frac{\partial\tilde{s}}{\partial\sigma}\frac{\partial\tilde{e}}{\partial\sigma}.$$

We know from the first-best analysis that to decentralize the optimum one needs to control both saving and sinful consumption. One does not need any health subsidy. Here we don't have any direct control of saving. It can be indirectly controlled through the use of both θ and σ. If any of these instruments stimulate saving, this makes using it more desirable.

References

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