Do Voters Vote Sincerely?*

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ABSTRACT

In this paper we address the following question: To what extent is the hypothesis that voters vote sincerely testable or falsifiable? We show that using data only on how individuals vote in a single election, the hypothesis that voters vote sincerely is irrefutable, regardless of the number of candidates competing in the election. On the other hand, using data on how the same individuals vote in multiple elections, the hypothesis that voters vote sincerely is potentially falsifiable, and we provide general conditions under which the hypothesis can be tested. We then consider an application of our theoretical framework and assess whether the behavior of voters is consistent with sincere voting in U.S. national elections in the post-war period. We find that by and large sincere voting can explain virtually all of the individual-level observations on voting behavior in presidential and congressional U.S. elections in the data.

***JEL* D72, C12, C63; *Keywords*: voting, spatial models, falsifiability, testing.**

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1 Introduction

Voting is a cornerstone of democracy and voters' decisions in elections and referenda are fundamental inputs in the political process that shapes the policies adopted by democratic societies. Hence, understanding observed patterns of voting represents an important step in the understanding of democratic institutions. Moreover, from a theoretical standpoint, voters are a fundamental primitive of political economy models. Different assumptions about their behavior have important consequences on the implications of these models and, more generally, on the equilibrium interpretation of the behavior of politicians, parties and governments they may induce.[1]

An important question is whether voters vote "sincerely" based on their ideological views, or whether other factors (like for example strategic considerations, or their assessment of candidates' personal characteristics), determine the way individuals vote. Clearly, this is an empirical question and in order to address it we must first define what we mean by sincere voting.

Consider a situation where a group of voters is facing some contested elections (i.e., there is at least one election and two or more candidates in each election). Suppose that each voter and each candidate has political views that can be represented by a position in some common ideological (metric) space. We say that a voter votes sincerely in an election based on ideological considerations if she casts her vote in favor of the candidate whose ideological position is closest to her own (given the ideological positions of all the candidates in the election).[2]

Given this definition, it follows immediately that if the ideological positions of all voters and candidates as well as the voting decisions of all voters were observable, we could then directly assess whether or not the behavior of each voter in any election is consistent with sincere voting. However, this is generally not the case. While there exist surveys containing

[1] In the citizen-candidate framework, for example, equilibrium policies differ depending on how citizens vote (see, e.g., Besley and Coate (1997) and Osborne and Slivinski (1996)). The recent survey by Merlo (2006) presents a general overview of the implications of alternative theories of voting in political economy.

[2] In this paper, we ignore the issue of abstention. For recent surveys of alternative theories of voter turnout see, e.g., Dhillon and Peralta (2002) and Merlo (2006).

information on how individuals vote in a number of elections (e.g., the American National Election Study, the Canadian National Election Study and the British Election Survey), and data sets containing measures of the ideological positions of politicians based on their observed behavior in a variety of public offices (e.g., Poole and Rosenthal (1997) and Hix, Nouri and Roland (2006)), the ideological positions of voters are not directly observable.[3]

The relevant empirical question thus becomes: To what extent is the hypothesis that voters vote sincerely testable or falsifiable (in a Popperian sense)?[4] This is the question we address in this paper.[5]

The first result of our analysis is that using data only on how individuals vote in a single isolated election, the hypothesis that voters vote sincerely is irrefutable, regardless of the number of candidates competing in the election. Given any configuration of distinct candidates' positions, any observed vote can be rationalized by a voter voting sincerely based on some voter's ideological positions. This result holds for any number of dimensions of the ideological space.

Another result of our analysis is that using data on how the same individuals vote in multiple elections it is possible to construct a meaningful test of whether the behavior of voters is consistent with sincere voting. In other words, the hypothesis that voters vote sincerely in multiple elections is potentially falsifiable, and we provide conditions under which the hypothesis can be tested. We show that in general environments where individual voting decisions and candidates' ideological positions are observable, but voters' ideological positions are not, the hypothesis that individuals vote sincerely in multiple elections with any number of candidates is falsifiable if the number of elections is greater than the number of

[3]Note that in order to directly assess whether the behavior of voters is consistent with sincere voting one would need a consistent set of observations on the ideological positions of all voters and candidates in the same metric space. Hence, measures of citizens' self-reported ideological placements that are contained in some surveys (like, for example, the variable contained in the American National Election Studies, where voters are asked to place themselves on a 7-point liberal-conservative scale), cannot be used for this purpose, since, for instance, different people may interpret the scale differently.

[4]See, e.g., Popper (1935).

[5]Chiappori and Donni (2005) address similar issues in the context of bargaining models, where they investigate the extent to which Nash bargaining theory is testable.

dimensions of the ideological space. Given any configuration of distinct candidates' positions in two or more simultaneous elections, there always exists at least a voting profile (that is, a vector of votes in all elections) that cannot be rationalized by a voter voting sincerely in these elections based on some voter's ideological positions.

Finally, we characterize the maximum number of voting profiles that are consistent with sincere voting as a function of the number of elections, the number of candidates in each election, and the number of dimensions of the ideological space. All our results are formally stated in Section 2, and are proved in the Appendix.

In Section 3, we then consider an application where the hypothesis that voters vote sincerely is falsifiable, and assess whether the observed behavior of voters is consistent with sincere voting in U.S. national elections in the post-war period. In order to address this issue, we use individual-level data on how individuals voted in presidential and congressional elections in seven presidential election years between 1970 and 2000, as well as data on the positions of candidates in these elections. The two data sources we use are the American National Election Studies, and the Poole and Rosenthal NOMINATE Common Space Scores, respectively. We find that by and large sincere voting can explain virtually all of the individual-level observations on voting behavior in presidential and congressional elections in the data. We also explore the robustness of these findings with respect to the choice of the number of elections and the number of dimensions of the ideological space. We conclude in Section 4 with some general remarks about the critical role played by auxiliary assumptions in testing voting theories.

2 Theoretical Analysis

Consider a situation where a group of voters N is facing $m \geq 1$ simultaneous elections. There is a common ideological space $Y = R^k$, $k \geq 1$, and let $d(x, z)$ denote the Euclidean distance between any two points $x, z \in R^k$ (i.e., $d(x, z) = \sqrt{(x_1 - z_1)^2 + \cdots + (x_k - z_k)^2}$). Each voter $i \in N$ is characterized by an ideological position $y^i \in Y$. For any election $e \in \{1, ..., m\}$, let $q_e = |J^e| \in \{2, ..., \overline{q}\}$, denote the number of candidates competing in the election, where J^e is the set of candidates. Each candidate $j \in \{\cup_{e=1}^m J^e\}$ is characterized by a distinct ideological position $y^j \in Y$, which is known to the voters.

For each voter $i \in N$, let $v^i = (v^i_1, ..., v^i_m) \in V^m$ denote i's voting profile, where $v^i_e \in J^e$ denotes voter i's vote in election $e = 1, ..., m$, and V^m is the set of all possible distinct voting profiles in the m elections. Note that $|V^m| = \Pi^m_{e=1} q_e$, and let $v \in V^m$ denote a generic voting profile.[6]

Definition 1: *Voter i votes sincerely in election e if $d\left(y^i, y^{j_e}\right) < d(y^i, y^{\ell_e})$ for all $\ell_e \in J^e$, $\ell_e \neq j_e \in J^e$, implies that $v^i_e = j_e$. Voter i votes sincerely if she votes sincerely in all elections $e = 1, ..., m$.*

Clearly, given the voting profiles of voters, if a researcher could observe the ideological positions of electoral candidates and of voters, it would be possible to directly establish whether or not each voter votes sincerely in each election. Suppose instead that a researcher has access only to limited information, and consider a situation where the researcher observes the voting profiles of voters and the ideological positions of electoral candidates, but does not observe the voters' ideological positions. We are interested in determining the conditions under which the hypothesis that voters vote sincerely could be potentially falsified, and is therefore testable.

Let

$$P = \left\langle k, m, \{q_e\}^m_{e=1}, \{J^e\}^m_{e=1}, \left\{y^j\right\}_{j \in \left\{\cup^m_{e=1} J^e\right\}} \right\rangle \tag{1}$$

denote the primitives of the environment.

Definition 2: *Given P, a voting profile $v \in V^m$ is consistent with sincere voting if there exists some $Y^v \subseteq Y = R^k$ such that if a voter i's ideological position is $y^i \in Y^v$ and i votes sincerely, then $v^i = v$. If it exists, then Y^v is the sincere support of v.*

We can now define the notion of falsifiability.

Definition 3: *Given P, the hypothesis that voters vote sincerely is falsifiable if there exists at least a voting profile $v \in V^m$ that is not consistent with sincere voting.*

In the analysis that follows, we first consider the case of two-candidate elections, and then investigate the general case of elections with any number of candidates.

[6] For example, if there are two elections, 1 and 2, with candidates a_1 and b_1 competing in election 1, and candidates a_2 and b_2 competing in election 2, the set of the four possible voting profiles is $V^2 = \{(a_1, a_2), (a_1, b_2), (b_1, a_1), (b_1, b_2)\}$.

2.1 Two-candidate elections

We begin our analysis by considering the case of two-candidate elections (i.e., $q_e = 2$ for all $e = 1, ..., m$). For each election $e \in \{1, ..., m\}$, let $y^{j_e}, y^{\ell_e} \in Y = R^k, y^{j_e} \neq y^{\ell_e}$, denote the ideological positions of the two candidates $j_e, \ell_e \in J^e$ in the election, and let $H^e = \left\{y \in Y : d\left(y, y^{j_e}\right) = d(y, y^{\ell_e})\right\}$ be the set of points in the ideological space Y that are equidistant from the candidates' positions.

Since $d(\cdot)$ is the Euclidean distance, it follows that for each election e there exists a non-zero vector $\lambda^e = (\lambda_1^e, ..., \lambda_k^e) \in R^k$ and a $\mu_e \in R$ such that

$$H^e = \{y \in Y : \lambda^e y' = \mu^e\}, \; e = 1, ..., m, \tag{2}$$

where y' denotes the transpose of $y = (y_1, ..., y_k)$. Hence, each election $e = 1, ..., m$ implies an hyperplane H^e in R^k which partitions the ideological space Y into two regions (or half spaces),

$$Y^{j_e} = \{y \in Y : \lambda^e y' < \mu_e\}$$

and

$$Y^{\ell_e} = \{y \in Y : \lambda^e y' > \mu^e\},$$

where Y^{j_e} (Y^{ℓ_e}) is the set of ideological positions that are closer to the position of candidate j_e (ℓ_e) than to the position of the other candidate, or equivalently, is the sincere support of voting for candidate j_e (ℓ_e) in election e.[7]

It follows that the collection of the m hyperplanes, $\{H^1, ..., H^m\}$, partitions the ideological space Y into $r_m \leq 2^m$ convex regions, where each region is the sincere support of a distinct voting profile $v \in V^m$. Since in the case of two-candidate elections the number of possible voting profiles is $|V^m| = 2^m$, it follows that the hypothesis that voters vote sincerely in two-candidate elections is falsifiable if and only if $r_m < 2^m$.

We can now state our first set of results.

Proposition 1: *Given P, if $q_e = 2$ for all $e = 1, ..., m$, the hypothesis that voters vote sincerely is falsifiable if $m > k$. If $m \leq k$, the hypothesis is generically not falsifiable.*

[7] Note that $Y^{j_e} \cap Y^{\ell_e} = \varnothing$ and $Y^{j_e} \cup Y^{\ell_e} \cup H^e = Y$.

Corollary 1: *Given P, the hypothesis that voters vote sincerely in a single election with two candidates is not falsifiable for all $k \geq 1$.*

All proofs are contained in the Appendix.[8] In order to illustrate the result that in two-candidate elections the hypothesis that voters vote sincerely is falsifiable only if the number of elections is larger than the number of dimensions of the ideological space, consider an example in the two-dimensional space, $Y = R^2$. In this case, each election implies a line that partitions the plane into two regions, and generically the lines implied by any two elections must intersect.[9]

Figure 1 depicts a situation where there are three elections $e = 1, 2, 3$, the set of candidates in each election is $J^e = \{a_e, b_e\}$, and the candidates' ideological positions y^{a_e} and y^{b_e} are such that the region to the left of each line H^e is closer to the position of a_e than to that of b_e for each election e. Several observations emerge from this figure. If we consider any single election $e \in \{1, 2, 3\}$ in isolation (i.e., $m = 1$), then it is obvious that each voting profile $v \in \{a_e, b_e\}$ is consistent with sincere voting (since the two half planes determined by H^e are the sincere supports of a_e and b_e, respectively). This is also true if we consider any pair of elections $e, f \in \{1, 2, 3\}$, $e \neq f$, (i.e., $m = 2$), since H^e and H^f partition the ideological space in four regions that represent the sincere supports of each of the four possible voting profiles (a_e, a_f), (a_e, b_f), (b_e, a_f), and (b_e, b_f). However, when we consider all three elections together (i.e., $m = 3$), we see that H^1, H^2 and H^3 partition the ideological space in only seven regions, while there are eight possible voting profiles. In this example, there do not exist ideological positions such that the voting profile (a_1, b_2, a_3) is consistent with sincere voting (that is, there does not exist a sincere support for (a_1, b_2, a_3)).

It is should also be clear from the example that increasing the number of elections would increase the number of voting profiles that are inconsistent with sincere voting. In fact, the following proposition characterizes the upper bound on the number of voting profiles that are consistent with sincere voting (i.e., the number of regions r_m), as a function of the number

[8] The proof of Corollary 1 follows directly from the proof of Proposition 1 and is therefore omitted.

[9] While there exist configurations of candidates' positions such that these lines would be parallel (a case that would occur, for example, if the pair of candidates' positions in one election is a linear transformation of the pair of candidates' positions in another election), this case is non generic.

of elections m and the number of dimensions of the ideological space k.[10]

Proposition 2: *Given P, if $q_e = 2$ for all $e = 1, ..., m$, then $r_m \leq \rho(m, k)$, where*

$$\rho(m,k) = \sum_{t=0}^{k} \binom{m}{t}. \tag{3}$$

Note that if $m \leq k$, Proposition 2 implies that

$$\rho(m,k) = \sum_{t=0}^{m} \binom{m}{t} = 2^m$$

and this bound is generically attained (Proposition 1). If, on the other hand, $m > k$, then for example in a two-dimensional ideological space with three, four, and five elections, we have that $\rho(3,2) = 7$, $\rho(4,2) = 11$, and $\rho(5,2) = 16$, respectively. This implies that when there are three elections at most 7 out of the 8 possible voting profiles are consistent with sincere voting; when there are four elections at most 11 out of the 16 possible voting profiles are consistent with sincere voting; and when there are five elections the maximum number of voting profiles that are consistent with sincere voting is 16 out of 32 possible profiles.

2.2 Multi-candidate elections

Consider now the general case where the number of candidates may vary across elections and any election may have more than two candidates (i.e., $q_e \in \{2, ..., \overline{q}\}$, $e = 1, ..., m$). For each election $e \in \{1, ..., m\}$, let $y^{j_e} \in Y = R^k$, denote the distinct ideological position of a generic candidate $j_e \in J^e$ in the election, and $Y^{j_e} = \{y \in Y : d(y, y^{j_e}) < d(y, y^{\ell_e}), \forall \ell_e \in J^e$, $\ell_e \neq j_e\}$ be the set of points in the ideological space Y that are closer to y^{j_e} than to the position of any other candidate in the election.

Since $d(\cdot)$ is the Euclidean distance, it follows that for each pair of candidates in election e, $j_e, \ell_e \in J^e$, the set of points in the ideological space Y that are equidistant from y^{j_e} and y^{ℓ_e} is an hyperplane H^{j_e,ℓ_e}, which partitions the ideological space Y into two regions (or half

[10] The issue we are considering corresponds to the problem of counting the number of regions in arrangements of hyperplanes in k-dimensional Euclidean space. This problem has been extensively studied in computational and combinatorial geometry (see, e.g., Orlik and Terao (1992)), and Proposition 2 follows from a general result that was first proved by Buck (1943).

spaces), $Y^{j_e}_{\ell_e}$ and $Y^{\ell_e}_{j_e} = Y\backslash\{Y^{j_e}_{\ell_e} \cup H^{j_e,\ell_e}\}$, where $Y^{j_e}_{\ell_e}$ ($Y^{\ell_e}_{j_e}$) is the set of ideological positions that are closer to the position of candidate j_e (ℓ_e) than to the position of candidate ℓ_e (j_e). Hence, for each candidate $j_e \in J^e$, Y^{j_e} is an intersection of the half spaces determined by the $q_e - 1$ hyperplanes $\{H^{j_e,\ell_e}\}_{\ell_e \in J^e \backslash j_e}$ (i.e., $Y^{j_e} = \cap_{\ell_e \in J^e \backslash j_e} Y^{j_e}_{\ell_e}$). Note that for all candidates $j_e \in J^e$ and all elections $e \in \{1, ..., m\}$, Y^{j_e} is non empty and convex. Hence, each election $e \in \{1, ..., m\}$ implies a partition T^e of the ideological space Y into q_e convex regions, $\{Y^{j_e}\}_{j_e \in J^e}$, where each region Y^{j_e} is the sincere support of voting for candidate j^e in election e.[11] For each election $e \in \{1, ..., m\}$, the set $T^e = \{Y^{j_e}\}_{j_e \in J^e}$ defines a Voronoi tessellation of R^k and each region Y^{j_e}, $j_e \in J^e$, is a k-dimensional Voronoi polyhedron.[12] Figure 2 illustrates an example of the Voronoi tessellation that corresponds to an election with 5 candidates, $\{a, b, c, d, e\}$, with positions $\{y_a, y_b, y_c, y_d, y_e\}$ in the two-dimensional ideological space $Y = R^2$, and introduces some useful terms.

It follows that the collection of the m tessellations, $\{T^1, ..., T^m\}$, partitions the ideological space Y into $r_m \leq \Pi^m_{e=1} q_e$ convex regions, where each region is the sincere support of a distinct voting profile $v \in V^m$. Since in the general case where the number of candidates may vary across elections the number of possible voting profiles is $|V^m| = \Pi^m_{e=1} q_e$, it follows that the hypothesis that voters vote sincerely is falsifiable if and only if $r_m < \Pi^m_{e=1} q_e$.

We can now state our second set of results.

Proposition 3: *Given P, the hypothesis that voters vote sincerely in a single election with any number of candidates is not falsifiable for all $k \geq 1$.*

Proposition 3 generalizes Corollary 1. In order to illustrate the result consider the following example in the two-dimensional space, $Y = R^2$. Figure 3 depicts a situation where there is a single election $e = 1$, and the set of candidates in the election is $J^1 = \{a_1, b_1, c_1\}$. Given the candidates' ideological positions, y^{a_1}, y^{b_1}, and y^{c_1}, for each $j_1 \in J^1$, Y^{j_1} is the sincere support of voting for candidate j_1 in the election. Hence, it follows immediately that each voting profile $v \in V^1 = \{a_1, b_1, c_1\}$ is consistent with sincere voting. In fact, it should be clear that this result holds for any number of candidates, any distinct candidates' positions,

[11] Note that $Y^{j_e} \cap Y^{\ell_e} = \varnothing$ for all $j_e, \ell_e \in J^e$, $j_e \neq \ell_e$, and $\cup_{j_e \in J^e}\{Y^{j_e} \cup_{\ell_e \in J^e \backslash j_e} H^{j_e}_{\ell_e}\} = Y$.

[12] For a comprehensive treatment of Voronoi tessellations and their properties, see, e.g., Okabe et al. (2000).

and any number of dimensions of the ideological space.

Proposition 4: *Given P, if $q_e \in \{2, ..., \overline{q}\}$, $e = 1, ..., m$, the hypothesis that voters vote sincerely is falsifiable if $m > k$.*

When the number of elections is greater than the number of dimensions of the ideological space, the hypothesis that voters vote sincerely is always falsifiable regardless of the number of candidates in each election. Hence, Proposition 4 extends the result of the first part of Proposition 1. However, for the case where $1 < m \leq k$, while the hypothesis is generically not falsifiable when each election has two candidates, when there are more than two candidates in at least one election, this is no longer the case. In fact, there exist configurations of candidates' positions, $\{y^j\}_{j \in \{\cup_{e=1}^m J^e\}}$, such that the hypothesis that voters vote sincerely is falsifiable, and configurations such that the hypothesis is not falsifiable.

In order to illustrate this result consider the following example in the two-dimensional space, $Y = R^2$. Suppose that in addition to election 1 depicted in Figure 3, there is a second election with two candidates (i.e., $e \in \{1, 2\}$, $q_1 = 3$ and $q_2 = 2$). The set of candidates in election 2 is $J^2 = \{a_2, b_2\}$, and the candidates' ideological positions are such that for each $j_2 \in J^2$, Y^{j_2} is the sincere support of voting for candidate j_2 in election 2. Figures 4 and 5 depict two possible situations that correspond to different configurations of the positions of the two candidates in election 2. As we can see from Figure 4, of the six possible voting profiles in elections 1 and 2, (a_1, a_2), (a_1, b_2), (b_1, a_2), (b_1, b_2), (c_1, a_2), and (c_1, b_2), only five have a sincere support in Y. In this example, there do not exist ideological positions such that the voting profile (a_1, b_2) is consistent with sincere voting (that is, there does not exist a sincere support for (a_1, b_2)). However, this is not the case in Figure 5, where there exists a sincere support for each of the six possible voting profiles in the two elections. Each one of the two cases illustrated in Figures 4 and 5 is robust to small perturbations of the candidates' positions, and is therefore generic. Similar examples can be constructed for any combination of the number of candidates in $m > 1$ elections, $q_1, ..., q_m$, where $q_e > 2$ for at least one $e \in \{1, ..., m\}$.

When the ideological space is either one- or two-dimensional (i.e., $k \leq 2$), we can also characterize the upper bound on the number of voting profiles that are consistent with sincere voting (i.e., the number of regions r_m), as a function of the number of elections m and the number of candidates in each election, $q_1, ..., q_m$.[13]

Proposition 5: *Given P, if $k \leq 2$, then $r_m \leq \tau_k(m, q_1, ..., q_m)$, where*

$$\tau_1(m, q_1, ..., q_m) = 1 + \sum_{e=1}^{m} (q_e - 1) \tag{4}$$

and

$$\tau_2(m, q_1, ..., q_m) = 1 + \sum_{e=1}^{m} \left[(q_e - 1) \left(1 + \sum_{f=e+1}^{m} (q_f - 1) \right) \right]. \tag{5}$$

Note that if $m = 1$, $\tau_1(1, q_1) = \tau_2(1, q_1) = q_1$, and if $m = 2$, $\tau_1(2, q_1, q_2) = q_1 + q_2 - 1 < \tau_2(2, q_1, q_2) = q_1 q_2$. Furthermore, when $k \leq 2 < m$, $\tau_1(m, q_1, ..., q_m) < \tau_2(m, q_1, ..., q_m) < \Pi_{e=1}^{m} q_e$, and the number of voting profiles that are not consistent with sincere voting increases both with the number of elections and with the number of candidates in an election. For example, if the ideological space is two-dimensional, then if $m = 3$ and $q_1 = q_2 = q_3 = 3$, $\tau_2(3, 3, 3, 3) = 19$ (i.e., at most 19 out of the 27 possible voting profiles are consistent with sincere voting); if $m = 4$ and $q_1 = q_2 = q_3 = q_4 = 3$, $\tau_2(4, 3, 3, 3, 3) = 33$ (i.e., at most 33 out of the 81 possible voting profiles are consistent with sincere voting); and if $m = 3$, $q_1 = q_2 = 3$, and $q_3 = 4$, $\tau_2(3, 3, 3, 4) = 24$ (i.e., at most 24 out of 36 possible voting profiles are consistent with sincere voting).

3 Evidence from U.S. National Elections

In the previous section, we have characterized general conditions under which the hypothesis that voters vote sincerely is falsifiable. We now turn our attention to assessing empirically the extent to which, in environments where the hypothesis is falsifiable, the observed behavior of voters is consistent with sincere voting. Our empirical analysis is simply meant as an illustration of the theoretical framework presented above, and focuses on national elections in the United States between 1970 and 2000. The same analysis, however, can also be replicated for other countries, or other types of elections, or other time periods for which there are available data.

[13] The issue we are considering corresponds to the problem of counting the number of regions in arrangements of Voronoi tessellations in k-dimensional Euclidean space. This problem has not yet been studied in computational and combinatorial geometry, and there are no known results in the literature.

Since, as shown in Section 2, this empirical analysis is meaningful only if we have access to data on how individuals vote in multiple elections, we consider the situation faced by U.S. voters in a presidential election year (henceforth, an election year), where presidential and congressional elections occur simultaneously.[14] In any election year, U.S. voters elect the President and, at the same time, each voter faces an election that determines the representative of his or her district in the House of Representatives.[15] Some voters also face a Senate election in their state.[16] Each election is typically contested by two candidates belonging to the Democratic and the Republican party, respectively.[17]

Since the set of candidates competing for a seat in the House of Representatives is different in each congressional district, our unit of analysis is the district. In a generic election year t, a voter i residing in district $h \in \{1, ..., 435\}$ and state $s \in \{1, ..., 50\}$ faces a House election. Let J_t^h denote the set of candidates competing in the House election in congressional district h at time t. Like all other voters in the nation, voter i also faces a presidential election, and let J_t^p denote the set of presidential candidates at time t. If a Senate seat is up for election in state s at time t, then voter i also faces a Senate election, where the set of candidates is J_t^s. Hence, in any given district $h = 1, ..., 435$ in state $s = 1, ..., 50$, a voter i is facing either two or three simultaneous elections, and $v^i = \left(v_p^i, v_h^i\right)$ or $v^i = \left(v_p^i, v_h^i, v_s^i\right)$ denotes i's voting profile, where $v_e^i \in J_t^e$ indicates how voter i votes in election $e = p, h, s$.[18] For example, a

[14] In the United States, citizens are called to participate in national elections to elect the President and the members of Congress. While congressional elections occur every two years, the time between presidential elections is four years. We refer to an election year where both presidential and congressional elections occur simultaneously as a presidential election year.

[15] Citizens who reside in the District of Columbia do not elect a House representative but only a congressional delegate.

[16] Senate elections are staggered, and in any given election year, there are elections to the U.S. Senate in approximately one third of the states. In addition, many voters also face other local elections and referenda. Since data on how individuals vote in these elections is typically not available, we do not consider them here.

[17] In some elections a single candidate runs uncontested. Occasionally, a third, independent candidate also runs. Since data on the positions of independent candidates is not available, in our analysis, we restrict attention to two-candidate presidential and congressional elections.

[18] Recall that here we are ignoring abstention, and only consider the way in which voters vote. For a recent study of the empirical implications of alternative models of voter turnout, see, e.g., Coate and Conlin (2004).

voter facing three elections may vote for the Democratic candidate in each of the elections, or vote for the Democratic presidential candidate and the Republican candidates in the House and Senate elections, and so on.

The data we use for our empirical analysis come from two sources. The first source is the American National Election Studies (NES), which for each election year contains individual voting decisions in presidential and congressional elections of a nationally representative sample of the voting age population. In addition, the NES contains information on the congressional district where each individual resides, the identity of the Democratic and the Republican candidate competing for election in his or her congressional district, and, in the event that a Senate election is also occurring in his or her state, the identity of the candidates competing in the Senate race.[19]

The second source of data is the Poole and Rosenthal NOMINATE Common Space Scores. Using data on roll call voting by each member of Congress and support to roll call votes by each President, Poole and Rosenthal developed a methodology to estimate the positions of all politicians who ever served either as Presidents or members of Congress, in a common two-dimensional ideological space (see, e.g., Poole (1998) and Poole and Rosenthal (1997, 2001)). These estimates, which are comparable across politicians and across time, are contained in their NOMINATE Common Space Scores data set.[20]

We restrict attention to the period 1970-2000, and consider seven election years: 1972, 1976, 1980, 1984, 1988, 1996, and 2000.[21] For each year, Table 1 contains the number of

In Degan and Merlo (2007), we structurally estimate a model of participation and voting in U.S. national elections.

[19] The NES is available on-line at http://www.umich.edu/~nes. For thorough discussions of potential limitations of the survey data in the NES see, e.g., Anderson and Silver (1986) and Wright (1993). Note, however, that the NES represents the best and most widely used source of individual-level data on electoral participation and voting in the U.S.

[20] This data set is also available on-line at http://voteview.com. For a discussion of potential limitations of the methodology proposed by Poole and Rosenthal see, e.g., Heckman and Snyder (1997). For a comparison of alternative estimation procedures see Clinton, Jackman and Rivers (2004). Note, however, that none of the other procedures has been used to generate a comprehensive data set similar to the one by Poole and Rosenthal.

[21] The NES data for the election year 1992 contains a mistake in the variable that identifies the con-

observations in the NES sample of individuals who reported how they voted in the presidential and House elections, as well as in the sub-sample of individuals who were also facing a senatorial election in their state, and reported how they voted in the presidential, House, and Senate elections.[22]

For each of the seven years we consider, we match each voter in the NES sample with the positions of the presidential candidates, as well as the positions of the House candidates running in his or her congressional district, and the positions of the Senate candidates running in his or her state, if applicable. Consistent with the general environment described in Section 2, we assume that the voters know the positions of all candidates in all the elections they face. These positions, however, may or may not be observable to the econometrician.

In order to measure the candidates' positions in each election year, we adopt the following procedure. For all presidential candidates, and all congressional candidates who have an entry in the Poole and Rosenthal data set, we assume that their position is given by their NOMINATE score.[23] For all other congressional candidates, we assume that their positions are drawn from the empirical distributions of the NOMINATE scores for Democratic and Republican members of the House or the Senate in the same election year, and we allow these distributions to differ across regions in the U.S.[24]

For all the cases where we observe the positions of all the candidates competing in the elections faced by the voters residing in a district, following the analysis in Section 2, we

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gressional district of residence of the individuals in the sample (see ftp://ftp.nes.isr.umich.edu/ftp/nes/studypages/1992prepost/int1992.txt). Hence, it cannot be used for the purpose of our analysis.

[22] Obviously, we only consider congressional elections that are contested, and observations for which the voters' district and state of residence are not missing.

[23] Note that Michael Dukakis, the Democratic presidential candidate in 1988, who at the time was the governor of Massachusetts, is the only presidential candidate during the period we consider for whom there is no entry in the Poole and Rosenthal data set. Following Gaines and Segal (1988), we approximate Dukakis' position in the ideological space with that of the Democratic Massachusetts senator in 1988 (Ted Kennedy).

[24] We consider four different regions: Northeast, South, Midwest and West. Alternative ways of constructing the empirical distributions are also possible. Note, however, that it would be unfeasible to characterize a separate empirical distribution for each party in each state (let alone district) in each year, since the number of representatives or senators of either party in each state in any given year is too small.

then directly assess whether or not each observed individual voting profile in those districts is consistent with sincere voting. For each case where we do not observe the position(s) of some candidate(s) competing in the elections faced by the voters residing in a district, we consider instead all possible realizations of these candidates' positions (that is, all the points in the support of the relevant empirical distributions of candidates' positions), and determine whether for any of these positions each observed individual voting profile in that district is consistent with sincere voting.[25] The outcome of our calculations is the fraction of the observed individual voting profiles that are consistent with sincere voting in each election year.

In order to perform these calculations, we need to specify the number of elections m we consider, and the number of dimensions of the ideological space k (where it has to be the case that $m > k$). We begin by ignoring Senate elections, and evaluate the extent to which the observed voting behavior of all individuals in the NES samples who voted in the presidential and House elections is consistent with sincere voting when we restrict attention to a unidimensional liberal-conservative ideological space.[26] We then take into consideration that while some voters only face the presidential and a House election, some voters also face a Senate election, and evaluate the extent to which the observed behavior of voters in presidential and congressional (House or House and Senate) elections is consistent with sincere voting, while still maintaining the assumption of a unidimensional ideological space. Finally, we restrict attention to the sub-samples of individuals in the NES who voted in three elections (presidential, House, and Senate), and perform our calculations for the case where the ideological space is two-dimensional.[27] Table 2 contains our results, where each column corresponds to one of the three scenarios.

[25] Note that since we are assuming that voters know the true position of each candidate, for all voters in the same district (state) we assess whether their observed voting profiles are consistent with sincere voting using the same realizations of candidates' positions in their district (state).

[26] In particular, we only consider the first dimension of the Poole and Rosenthal NOMINATE scores. Note that according to Poole and Rosenthal (1997; p.5), "from the late 1970s onward, roll call voting became largely a matter of positioning on a single, liberal-conservative dimension."

[27] Recall that the hypothesis that voters vote sincerely in presidential and House elections only is not falsifiable if $k = 2$.

As we can see from the first column in Table 2, sincere voting can explain virtually all of the individual-level observations on voting behavior in presidential and House elections in the data. Its worst "failure" amounts to the inability of accounting for 5.1% of the observations in 1980. Overall, by combining all the samples in the seven election years we consider, we have that only 3.3% of the observed individual voting profiles are not consistent with sincere voting. Note that "errors" of this magnitude are within the margin of tolerance when one allows for sampling (or measurement) error.[28]

Columns 2 and 3 in Table 2 help us to assess the robustness of these findings with respect to the choice of the number of elections and the number of dimensions of the ideological space. From the analysis in Section 2, we know that given the number of dimensions of the ideological space, an increase in the number of elections increases the number of voting profiles that cannot be rationalized by a voter voting sincerely in these elections. This increases the extent to which the hypothesis that voters vote sincerely may fail to explain the data. Consistent with this result, we find that increasing the number of elections while maintaining the dimensionality of the ideological space fixed, worsens the empirical performance of the sincere-voting hypothesis (Column 2). Nevertheless, under the maintained assumption that the ideological space is unidimensional, over 92% of the observed individual voting profiles in presidential and congressional (House or House and Senate) elections between 1970 and 2000 are still consistent with sincere voting. Moreover, in a two-dimensional ideological space, the hypothesis that voters vote sincerely in presidential and congressional (House and Senate) elections only fails to account for less than 1% of the observations in each of the seven election years we consider (Column 3). We conclude that a compelling case cannot be made on empirical grounds to dismiss a sincere-voting interpretation of the behavior of voters in U.S. national elections.[29]

[28] One potential source of measurement error in the data, for example, is that individuals in the NES samples may be assigned to the wrong congressional district (a possibility that arises whenever the location where an individual is interviewed does not correspond to his or her permanent residence).

[29] Note that our analysis is not a test of "sincere vs. strategic" voting. The objective of the quantitative exercise is simply to assess the extent to which observed voting behavior is consistent with individuals voting sincerely, given our definition of sincere voting provided in Section 2.

4 Concluding Remarks

Do voters vote sincerely based on ideological considerations? In this paper, we have provided general conditions under which the hypothesis that voters vote sincerely can be falsified. A key result of our analysis is that, when voters' ideological positions are not observed, falsifiability of the sincere-voting hypothesis hinges on the availability of data on how individuals vote in multiple elections. Furthermore, the number of elections has to be greater than the number of dimensions of the ideological space. Given the dimensionality of the ideological space, the larger the number of elections, the larger the number of voting profiles that cannot be rationalized by a voter voting sincerely in these elections. Hence, the larger the number of elections for which there are data on how individuals vote in each election, the higher the possibility of "rejecting" the sincere-voting hypothesis.

These results raise the following important considerations. The choices of the number of elections and the number of dimensions of the ideological space are auxiliary assumptions that are typically dictated by data availability. The conclusions of any empirical analysis that tries to assess the performance of the sincere-voting hypothesis are therefore conditional on these maintained assumptions. In particular, a "statistically significant failure" to explain the data may lead to a rejection of the hypothesis. This failure, however, may simply be due to limitations of the auxiliary assumptions, and could also be interpreted as grounds for rejecting these assumptions instead. It is therefore important to consider alternative specifications of the environment that correspond to different sets of auxiliary assumptions surrounding the main hypothesis that is being tested. The same considerations apply to testing any alternative theory of voting using the same data.

Appendix

Proof of Proposition 1: Let $q_e = 2$ for all $e \in \{1, ..., m\}$. We first show that for all $k \geq 1$ and $m \geq 1$, if $Y = R^k$ and $m \leq k$, then generically $r_m = 2^m$. The reason why the result is true is that if $m \leq k$ then the intersection of the m hyperplanes $H^1, ..., H^m$ defined in (2) is generically non-empty. Hence, each hyperplane H^e, $e \in \{1, ..., m\}$, partitions each of the 2^{m-1} regions in R^k given by the intersections of the half spaces determined by the other $m-1$ hyperplanes in two.

Formally, the hyperplanes $H^1, ..., H^m$ in R^k define a system of m linear equations in k variables

$$\Lambda y' = \mu, \tag{6}$$

where

$$\Lambda = \begin{bmatrix} \lambda_1^1 & \cdots & \lambda_k^1 \\ \vdots & & \vdots \\ \lambda_1^m & \cdots & \lambda_k^m \end{bmatrix},$$

and

$$\mu = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_m \end{bmatrix}.$$

Since generically the vectors $\lambda^e = (\lambda_1^e, ..., \lambda_k^e)$, $e = 1, ..., m$, are linearly independent, the rank of Λ is equal to m. Hence, for $m \leq k$ a solution to the system of linear equations (6) exists and the dimension of the space of solutions is $k - m$. In particular, when $m = k$ the unique solution to (6) is a point in R^k where all the hyperplanes $H^1, ..., H^k$ intersect.

Next, we show that for all $k \geq 1$, if $Y = R^k$ and $m > k$, then $r_m < 2^m$. Given the m hyperplanes $H^1, ..., H^m$ defined in (2), consider an arbitrary collection containing k of these hyperplanes. From the previous part of the proof we know that generically a collection of k hyperplanes partitions R^k into 2^k regions. Since each hyperplane can at most partition each region in two, in order to prove that $r_m < 2^m$ it is enough to show that adding another hyperplane to the collection can never partition R^k into 2^{k+1} regions. In other words, an additional hyperplane can not partition all of the 2^k regions given by the intersections of the half spaces determined by k other hyperplanes.

Without loss of generality, consider the collection of k hyperplanes, $H^1, ..., H^k$. Let $y^* \in R^k$ denote the intersection of $H^1, ..., H^k$, that is

$$y^* = \Lambda_k^{-1} \mu^k$$

is the unique solution to

$$\Lambda_k y' = \mu^k,$$

where

$$\Lambda^k = \begin{bmatrix} \lambda_1^1 & \cdots & \lambda_k^1 \\ \vdots & & \vdots \\ \lambda_1^k & \cdots & \lambda_k^k \end{bmatrix},$$

and

$$\mu^k = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_k \end{bmatrix}.$$

Consider the linear transformation

$$x' = \Lambda_k y' - \mu^k \tag{7}$$

that maps Y into X (where $Y = R^k$ and $X = R^k$). This transformation maps each hyperplane H^j in Y, $j = 1, ..., k$, into the j^{th} coordinate of X, and y^* into the origin of X. Furthermore, it maps each hyperplane H^h in Y, $h = k+1, ..., m$, into a hyperplane Z^h in X, $Z^h = \left\{x \in X : \beta^h x' = \gamma_h\right\}$, where $\beta^h = \lambda^h \Lambda_k^{-1}$ and $\gamma_h = \mu_h - \lambda^h \Lambda_k^{-1} \mu_k$. Without loss of generality, suppose that $\beta^{k+1} > 0$ and $\gamma_{k+1} > 0$. Then, for all $x < 0$, $\beta^{k+1} x' < \gamma_{k+1}$, which implies that the hyperplane Z^{k+1} does not partition the negative orthant of X. This implies that the hyperplane H^{k+1} does not partition the region in Y that corresponds to the negative orthant of X under the linear transformation (7). It follows that for any collection of $k < m$ hyperplanes, there always exists at least a region in Y given by some intersection of the half spaces determined by these hyperplanes that is not partitioned by some other hyperplane. ■

Proof of Proposition 2: Proposition 2 follows from a general result in combinatorial geometry on the maximum number of regions in arrangements of hyperplanes in k-dimensional

Euclidean space. The proof we report here is an adaptation of a proof by Edelsbrunner (1987; pp. 8-10).

Let $H = \{H^1, ..., H^m\}$ denote the collection of the m hyperplanes defined in (2), which defines a partition of R^k into connected objects of dimensions 0 through k, called an arrangement $A(H)$ of H. We use the term vertex to denote a 0-dimensional object in $A(H)$ (that is, a point generated by the intersection of k hyperplanes), and refer to an l-dimensional object in $A(H)$, $1 \leq l \leq k$, as an l-region. We are interested in characterizing the maximum number of k-regions in an arrangement $A(H)$, $\rho(m, k)$.

For $m \leq k$ the first part of the proof of Proposition 1 implies that

$$\rho(m, k) = 2^m = \sum_{t=0}^{m} \binom{m}{t} = \sum_{t=0}^{k} \binom{m}{t}.$$

Hence, we only need to prove the case $m > k$. The proof is by induction on the number of dimensions of the ideological space, k. The assertion is trivial in one dimension, where m points—that is, 0-dimensional hyperplanes—partition R into at most $m+1$ intervals—that is, 1-regions (where the "at most" qualifier follows from the fact that although the positions of all candidates are distinct, the mid-points between any pairs of candidates, one pair in each election, may coincide). Thus, assume that the assertion holds for all dimensions less than k.

Any k hyperplanes intersect in at most one point in R^k (and generically in exactly one point). Hence, $A(H)$ contains $d \leq \binom{m}{k}$ vertices. Consider a new hyperplane

$$h(s) = \left\{ y \in R^k : y_1 = s \right\}$$

that sweeps through $A(H)$ as the parameter s varies from $-\infty$ to $+\infty$. Without loss of generality assume that no hyperplane in H is vertical and that no two vertices in $A(H)$ share the same y_1-coordinate. Let $s_1 < s_2 < \cdots < s_d$ be the y_1-coordinates of the d vertices in $A(H)$. We say that vertex i, $i = 1, ..., d$, lies behind $h(s)$ if $s_i < s$, and that a k-region lies behind $h(s)$ if the y_1-coordinates of all the points in the region are less than s.

Let $A_s(H)$ denote the intersection of $A(H)$ with $h(s)$. Hence, $A_s(H)$ is an arrangement of m hyperplanes in R^{k-1}, which by induction hypothesis contains at most $\rho(m, k-1)$

$(k-1)$-regions, where

$$\rho(m, k-1) = \sum_{t=0}^{k-1} \binom{m}{t}.$$

Furthermore, each $(k-1)$-region in $A_s(H)$ is contained in a unique k-region of $A(H)$.

To complete the proof we count the number of k-regions in $A(H)$ that either lie behind or intersect the hyperplane $h(s)$ as it sweeps through $A(H)$ (that is, as s varies from $-\infty$ to $+\infty$). Clearly, when $s = -\infty$, no k-region lies behind $h(s)$, when $s_1 < s < s_2$ one k-region lies behind $h(s)$, and as $h(s)$ passes each other vertex in $A(H)$, one more k-region lies behind $h(s)$. During the entire sweep, $h(s)$ passes d vertices, which implies that at most $\binom{m}{k}$ k-regions lie behind $h(s)$, and for $s > s_d$ the remaining k-regions in $A(H)$ intersect $h(s)$. It follows that

$$\rho(m, k) = \binom{m}{k} + \sum_{t=0}^{k-1} \binom{m}{t} = \sum_{t=0}^{k} \binom{m}{t}. \blacksquare$$

Proof of Proposition 3: The proof follows directly from the observation that for a generic election e, the set Y^{j_e} for each candidate $j_e \in J^e$ is a Voronoi polyhedron, which is always non empty. Hence, an election partitions Y into q_e convex regions, where each region is the sincere support of the vote for a different candidate in the election. $\blacksquare$

Proof of Proposition 4: Since $q_e \geq 2$ for all $e = 1, ..., m$, consider an arbitrary pair of candidates in each election. Given this subset of $2m$ candidates, Proposition 1 implies that if $m > k$, there must exist at least one combination of m candidates, one for each election, such that the voting profile corresponding to that combination of candidates is not consistent with sincere voting. This establishes the result. $\blacksquare$

Proof of Proposition 5: For the case where $k = 1$, the derivation of $\tau_1(m, q_1, ..., q_m)$ is straightforward and follows directly from the observation that each election $e = 1, ..., m$, with $q_e \in \{2, ..., \overline{q}\}$ candidates implies $(q_e - 1)$ points that partition the line into q_e regions. Hence, starting from the case of no elections, where the number of regions in R is 1, adding each election $e = 1, ..., m$ one at the time increases the number of regions by at most $(q_e - 1)$.

Now consider the case where $k = 2$. Then each election $e \in \{1, ..., m\}$ defines a Voronoi diagram in the plane with q_e regions. Note that, given any collection of Voronoi diagrams that partitions the plane into Q regions, if we superimpose an additional diagram with q_j

regions, the total number of regions becomes $Q + (q_j - 1) + n$, where n is the number of intersection points of the edges of the additional Voronoi diagram with the edges of the other diagrams.

Let the union of the edges of the Voronoi diagram defined by election e be denoted by U_e, $e = 1, ..., m$. Then for each pair of elections, $e, f \in \{1, ..., m\}$, $e \neq f$, the cardinality n of the intersection of U_e and U_f is at most $(q_e - 1)(q_f - 1)$. To see that this is the case, note that the number of regions in the superimposition of the two Voronoi diagrams is at most $q_e q_f$. But, as noted above, it is also equal to $q_e + (q_f - 1) + n$. It follows that $n \leq (q_e - 1)(q_f - 1)$.

Starting with the Voronoi diagram defined by election $e = 1$, superimposing the remaining $m - 1$ Voronoi diagrams defined by elections $2, ..., m$ one at the time, we obtain a number of regions r_m that is at most

$$
\begin{aligned}
& q_1 + (q_2 - 1) + (q_2 - 1)(q_1 - 1) + (q_3 - 1) + (q_3 - 1)(q_1 - 1 + q_2 - 1) \\
& + \cdots + (q_m - 1) + (q_m - 1)(q_1 - 1 + \cdots + q_{m-1} - 1)
\end{aligned}
$$

or, equivalently,

$$
1 + \sum_{e=1}^{m} \left[(q_e - 1) \left(1 + \sum_{f=e+1}^{m} (q_f - 1) \right) \right]. \blacksquare
$$

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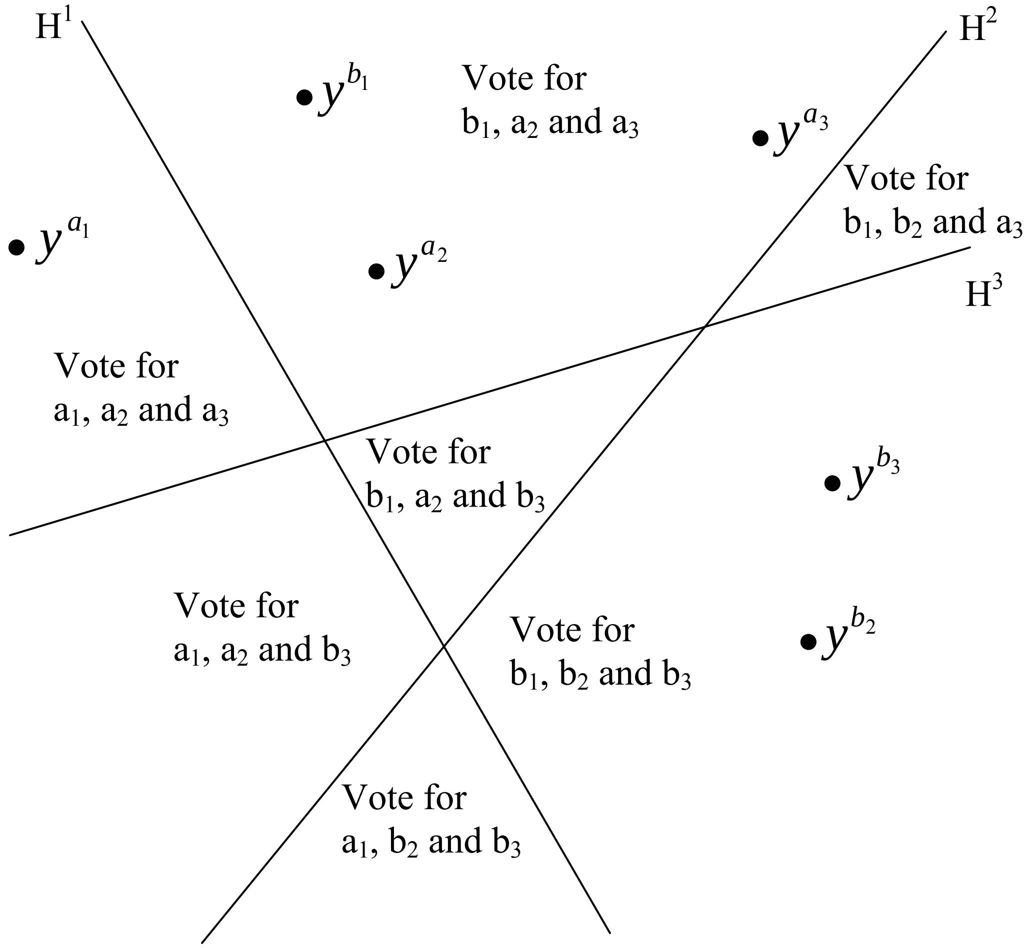

FIGURE 1: Three 2-candidate elections in a two-dimensional ideological space

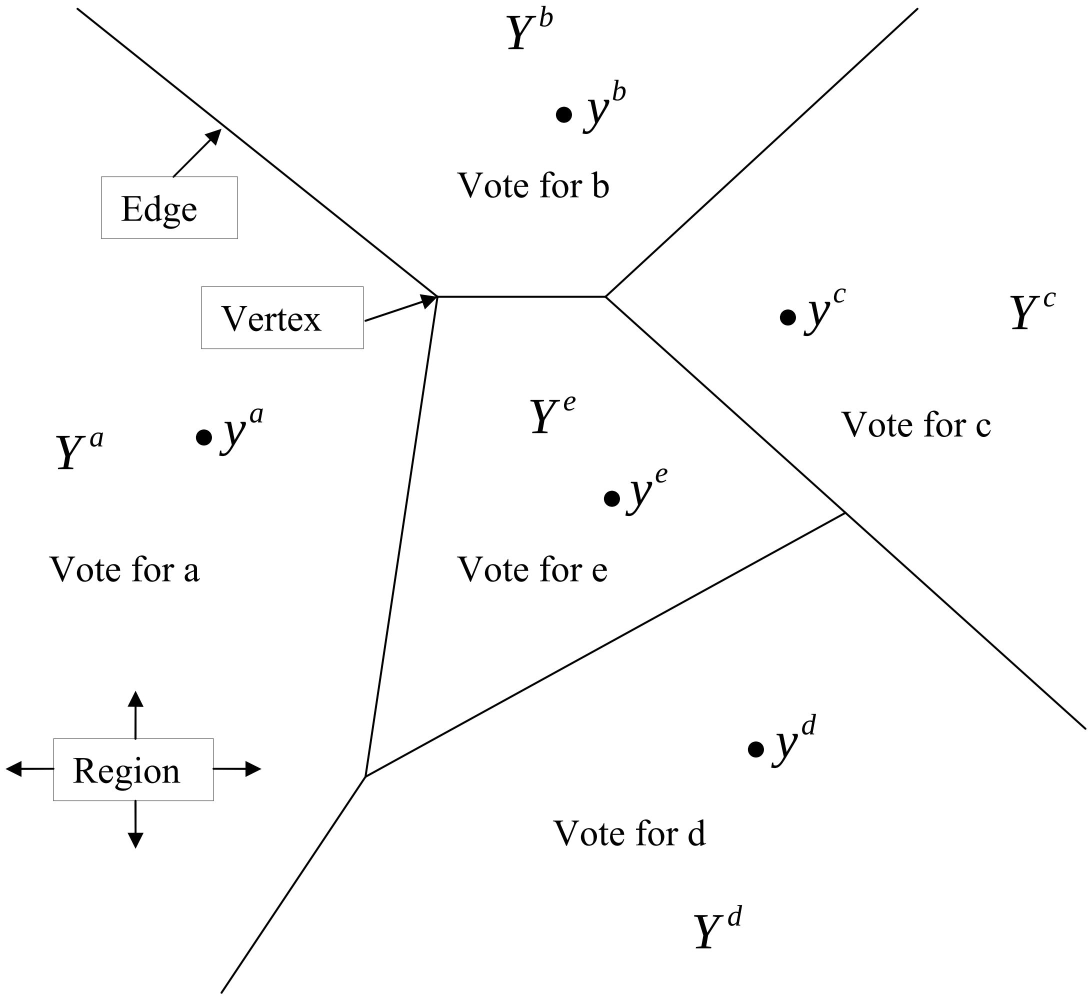

FIGURE 2: The Voronoi tessellation corresponding to a 5-candidate election in a two-dimensional ideological space.

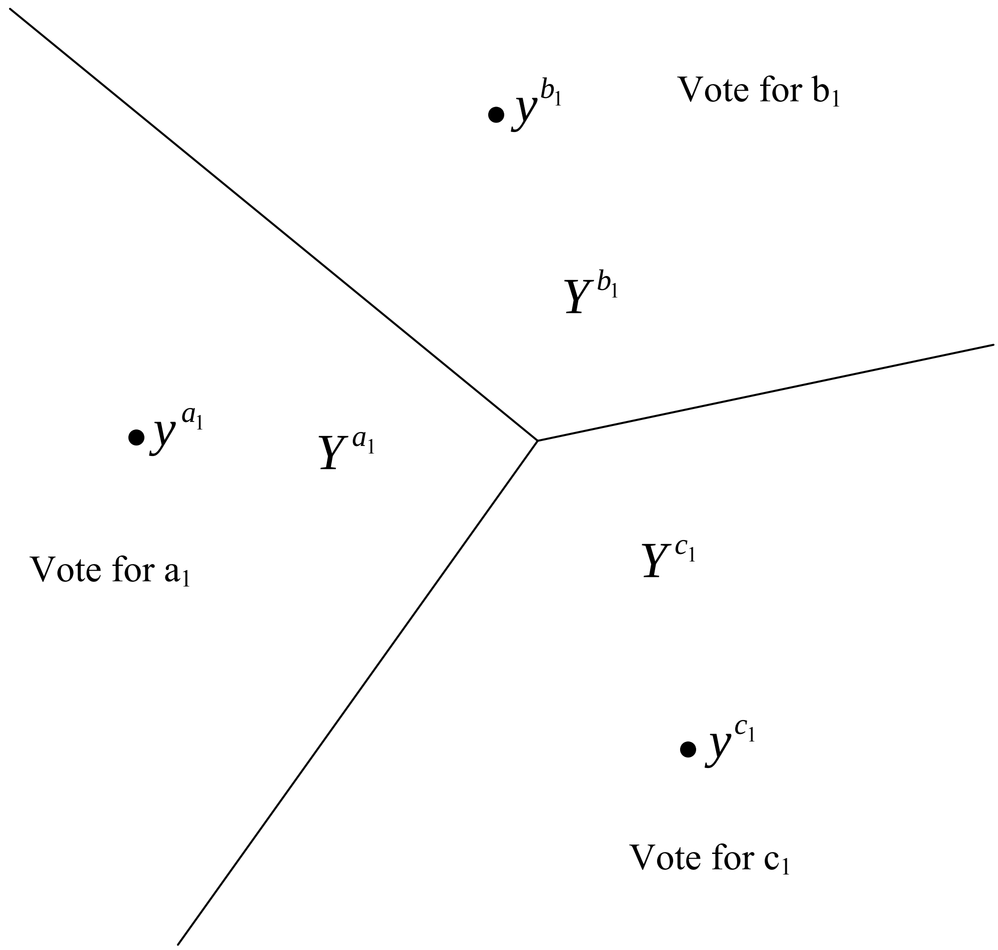

FIGURE 3: A 3-candidate election in a two-dimensional ideological space

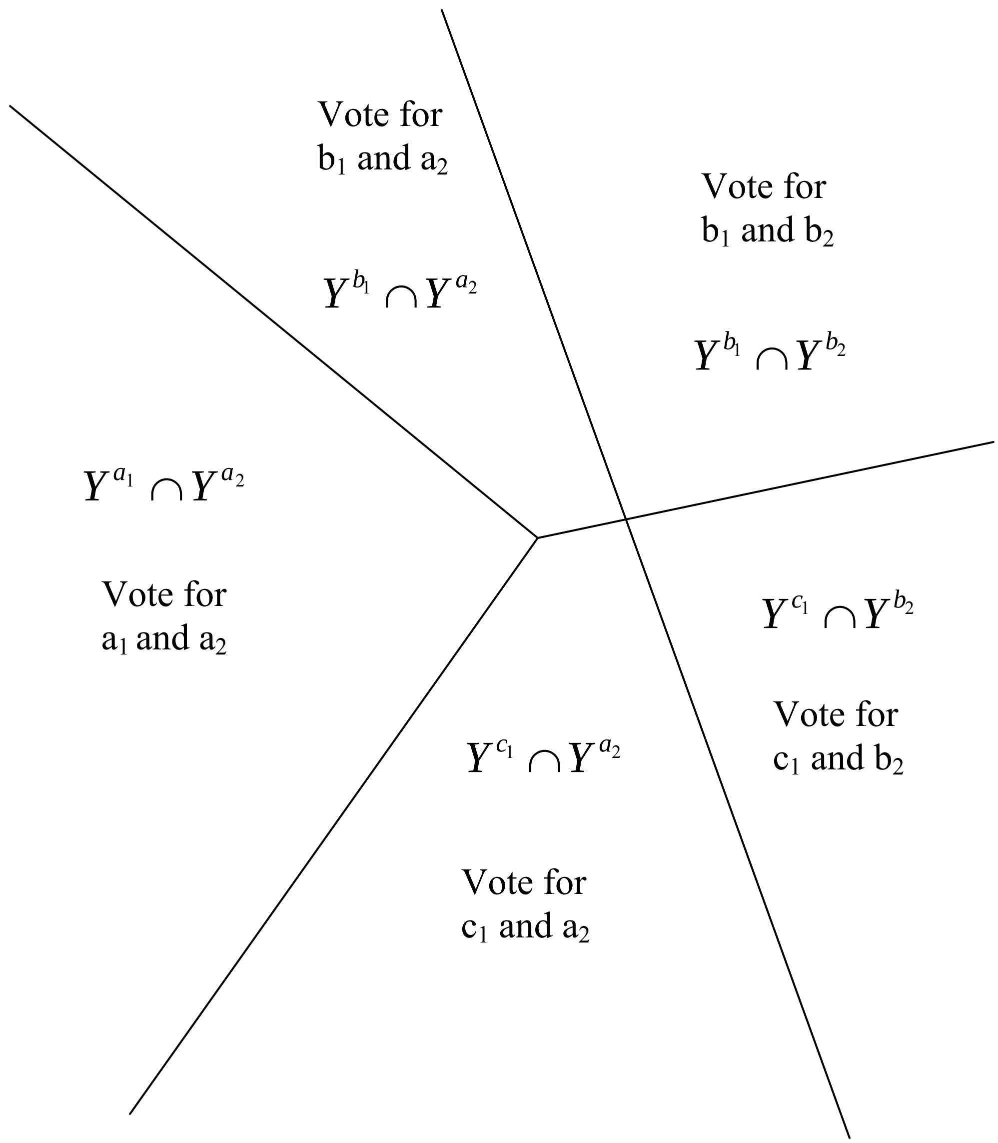

FIGURE 4: An example of a 3-candidate election and a 2-candidate election in a two-dimensional ideological space where the hypothesis that voters vote sincerely is falsifiable

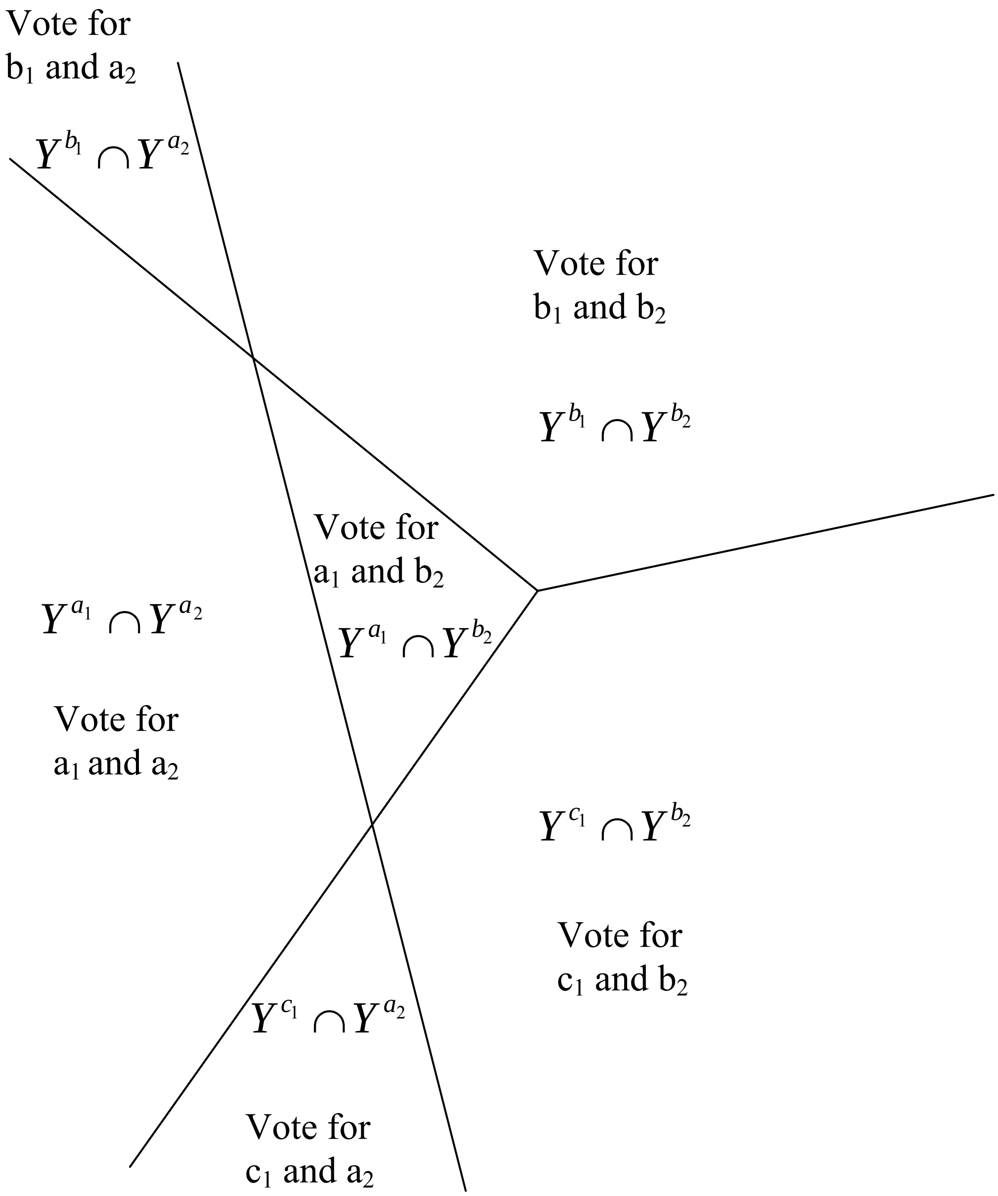

FIGURE 5: An example of a 3-candidate election and a 2-candidate election in a two-dimensional ideological space where the hypothesis that voters vote sincerely is not falsifiable

TABLE 1: Number of observations

Year	Number of voters in presidential and House elections	Number of voters in presidential, House and Senate elections
1972	1121	515
1976	968	561
1980	641	440
1984	1046	538
1988	797	590
1996	885	490
2000	781	565
Overall	6239	3699

TABLE 2: Percentage of observations consistent with sincere voting

Year	Voters in presidential and House elections (unidimensional space)	Voters in presidential and House, or presidential, House and Senate elections (unidimensional space)	Voters in presidential, House and Senate elections (two-dimensional space)
1972	96.5%	91.4%	99.2%
1976	96.2%	91.1%	99.6%
1980	94.9%	91.5%	99.5%
1984	96.5%	92.4%	99.8%
1988	98.4%	91.8%	99.7%
1996	96.2%	92.1%	100.0%
2000	98.1%	95.6%	99.8%
Overall	96.7%	92.2%	99.7%