Some Extensions to the Theory of Optimal Income Taxation.

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1 Introduction

We mean by "the" theory of optimal income taxation the literature that has developed out of two fundamental contributions: the paper by James Mirrlees[1] on the optimal non linear income tax; and the paper by Eytan Sheshinsky[2] on the optimal linear income tax.[3] The model of the household on which this literature is based is that of the single worker/consumer dividing his time between market labour supply and leisure. On the other hand, a central policy issue is, in our view, that of how to tax two-earner couples. Typically, households consist of two adult members, with or without children, and the single-person household model provides only limited insight into the real problems of tax policy.[4] Accordingly, in this paper we first of all extend the Mirrlees analysis of

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[1] See Mirrlees (1971).

[2] See Sheshinski (1972).

[3] For quite comprehensive expositions of this literature see Atkinson and Stiglitz (1988), Myles (1995) and Salanié (2002).

[4] Moreover, as has been recognised in public finance for a long time, when considering the variation in utility possibilities across households, which is a central consideration when dealing with the redistributional aspects of income taxation, it is necessary to take account of household production. That is, the simple dichotomization of time into market work and "leisure" is insufficient to allow a satisfactory analysis of, in particular, the equity implications of taxation. The problem is that this dichotomization implies that money income is always an appropriate indicator of household utility possibilities, which in any realistic context is not the case. In this paper however we do not pursue this point further.

optimal non linear income taxation to the case of two-person households. Since Mirrlees's model is a one-dimensional screening model, this extension is essentially an application of the theory of two-dimensional screening models to the problem of optimal taxation.

Mirrlees's model is also that of a one-shot adverse selection game. In reality, taxation is clearly a repeated game. If the "planner" is able to commit not to use the information about a taxpayer's type, gained in the first period, in setting taxes in subsequent periods, this is not a problem, but in fact it is hard to see that this commitment possibility exists. Taxation seems to us to be a repeated adverse selection game with no commitment. This has important implications for the solution of the optimal tax problem, and also for the usefulness of the entire approach for the design of income tax policy. The second extension in this paper is therefore to apply the theory of repeated adverse selection games[5] to the optimal tax problem.

The extension of the model of linear taxation to two-person households has already been carried out by Michael Boskin and Eytan Sheshinsky.[6] Indeed their results are now characterised as the "conventional wisdom" on how to tax couples. This status is not however justified, at least by the analysis Boskin and Sheshinski gave, and so the last part of this paper presents a simple formal model designed to explore this point.

In any analysis of the optimal taxation of couples, the issue arises of the relationship between the distributional preferences of the "social planner" on the one hand, and of the household on the other.[7] The social welfare function is formulated in terms of individual utilities, and optimal taxation will reflect the planner's preferences toward the distribution of individual welfares. At the same time the household by its resource allocation decisions determines the distribution of utilities of the individuals within it. If there is non-identity or *dissonance* between the distributional preferences of the planner and the household, this will affect the form of optimal taxation. To avoid that complication in the present paper we assume that no such dissonance exists: the household distributes welfare exactly as the planner would wish it to.

2 Optimal Taxation: The Mirrlees Model

Consumers/workers in the economy are partioned into subsets of those with, respectively, low and high productivity, where productivity is exogenously given and measured by the market wage rate. On standard assumptions, at a competitive market equilibrium high productivity workers achieve higher utility than low productivity workers. Thus a planner with a social welfare function exhibiting at least as high a degree of inequality aversion as that of a utilitarian (for which this degree is zero) would want to redistribute income from high to low

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[5] The pioneers here were Laffont and Tirole, with their model of a repeated regulation game. A general framework has more recently been supplied by Bester and Strausz (2001).

[6] For an empirically-oriented discussion see also Feldstein and Feenburg (1996).

[7] This issue is discussed at greater length in Apps and Rees (1988).

productivity workers.

Why might the planner *not* be able to use lump sum taxes to carry out this redistribution? One answer often given relates to the difficulty of finding a tax base which is truly non-distortionary. Thus, taxing wage income distorts labour supply decisions; taxing wealth would distort saving and consumption decisions; taxing goods distorts the pattern of expenditures.[8] On the other hand, if we really know or can observe a consumer's innate productivity type, what is there to rule out the simple instruction: Pay to the tax collector T units of the consumption good? We do not need to relate the tax to anything except the productivity type of the individual.

Mirrlees points out that this is precisely the problem. If the planner cannot observe the productivity type of the individual, then lump sum taxation of this kind becomes infeasible. The tax has to be applied to something observable, most likely wage income, and so the trade-off between equity and efficiency comes into play - increased equity cannot be achieved without the sacrifice of efficiency. We then have to find the second best optimal tax policy.

We now summarise briefly the Mirrlees model and its main results, as a point of departure and also as a way of introducing notation. The two types of consumer in this economy have identical utility functions

$$u_i = u(x_i) - v(l_i) \quad i = H, L \tag{1}$$

where $x_i \geq 0$ is consumption and $l_i \in [0,1]$ is labour supply of a consumer of type i. 1 is total time available to a consumer. We assume $u', v', v'' > 0$, $u'' < 0$, so the utility functions are strictly concave, and at any optimum l_i lies strictly inside this interval. The additively separable form of this utility function is somewhat special but is useful later in extending the model to two-person households.

Consumers differ in respect of w_i, their productivity in production of the consumption good, with $w_H > w_L > 0$, so we will refer to types L and H as the low and high productivity types respectively. The proportion of consumers of type i in the population is $\phi_i > 0$, with $\sum_i \phi_i = 1$. With no or only lump sum taxation the consumer equilibrium is characterized by the condition

$$\left[\frac{dx_i}{dl_i}\right]_{u_i^*} = \frac{v'(l_i^*)}{u'(x_i^*)} = w_i \quad i = H, L \tag{2}$$

which is of course the condition for Pareto efficiency in this economy. "Lump sum" redistribution takes the form of a tax $T_i \gtreqless 0$, that is simply an amount of the consumption good that must be paid by, if $T_i > 0$, or is transferred to, if $T_i < 0$, a consumer of type i. The amount of this tax depends simply on the consumer's type, and is not based on her choice of consumption and labour supply, though these will of course in general be influenced by the tax, as a result

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of the income effect. The taxes must satisfy the government budget constraint

$$\sum_i \phi_i T_i \geq G \tag{3}$$

where $G \geq 0$ is a per capita revenue requirement.[9]

What lump sum taxes would be chosen by a utilitarian planner? From the consumers' budget constraints in the presence of lump sum taxation

$$x_i = w_i l_i - T_i \quad i = H, L \tag{4}$$

we have

$$T_i = w_i l_i - x_i \quad i = H, L \tag{5}$$

Now let us assume that the planner chooses the x_i and l_i directly. Thus we formulate the planner's problem as

$$\max_{x_i, l_i} W(u_L, u_H) = \sum_{i=H,L} \phi_i [u(x_i) - v(l_i)] \tag{6}$$

$$s.t. \sum_{i=H,L} \phi_i [w_i l_i - x_i] = 0 \quad (\lambda) \tag{7}$$

We derive immediately from the first order conditions the Pareto efficiency condition

$$\frac{v'(l_i^*)}{u'(x_i^*)} = w_i \quad i = H, L \tag{8}$$

From the first order conditions we have

$$u'(x_L^*) = \lambda^* = u'(x_H^*) \tag{9}$$

implying $x_L^* = x_H^*$. Thus, although the utilitarian planner is not averse to inequality (of utilities), she will equalize consumptions of the two types of consumer (a direct consequence of the assumption of identical utilities). We also have

$$\frac{v'(l_L^*)}{v'(l_H^*)} = \frac{w_L}{w_H} < 1 \tag{10}$$

implying

$$v'(l_H^*) > v'(l_L^*) \tag{11}$$

Since the $v(.)$ function is strictly convex, this implies $l_H^* > l_L^*$. Thus the planner requires a larger labour supply from the more productive worker, but gives her the same consumption as the less productive. It follows immediately from $x_L^* = x_H^*$ and $l_H^* > l_L^*$ that the low productivity type enjoys a higher utility than the high productivity type at the planner's optimal allocation

$$u(x_L^*) - v(l_L^*) > u(x_H^*) - v(l_H^*) \tag{12}$$

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[9] The case $G = 0$ will be referred to as pure redistribution.

and this must be brought about by

$$w_H l_H^* > x_H^* = x_L^* > w_L l_L^* \tag{13}$$

We now consider Mirrlees's analysis. First, we reformulate the model in a way that is more convenient for the analysis of the problem under asymmetric information. Define $y_i = w_i l_i$ as type i's *gross wage income*, and, since $l_i = y_i/w_i$, we can rewrite the utility function as

$$u_i = u(x_i) - v(\frac{y_i}{w_i}) \equiv u(x_i) - \psi_i(y_i) \quad i = H, L \tag{14}$$

with

$$\psi_i'(y_i) = \frac{v'\left(\frac{y_i}{w_i}\right)}{w_i} > 0 \tag{15}$$

$$\psi_i''(y_i) = \frac{v''\left(\frac{y_i}{w_i}\right)}{w_i^2} > 0 \tag{16}$$

In the absence of taxation, the individual's choice problem becomes:

$$\max_{x_i, y_i} u_i \quad s.t. \ x_i \le y_i \tag{17}$$

with the first order condition

$$\left[\frac{dx_i}{dy_i}\right]_{u_i^*} = \frac{\psi_i'(y_i^*)}{u'(x_i^*)} = \frac{v'(l_i^*)}{w_i u'(x_i^*)} = 1 \tag{18}$$

Clearly this model of the consumer is equivalent to the previous one.

Consider how the marginal rate of substitution at a given point (y_i^0, x_i^0) varies with w_i

$$\frac{\partial}{\partial w_i}\left[\frac{\psi_i'(y_i^0)}{u'(x_i^0)}\right] = \frac{\partial}{\partial w_i}\left[\frac{v'\left(\frac{y_i^0}{w_i}\right)}{w_i u'(x_i^0)}\right] \tag{19}$$

$$= \frac{\left(-\frac{y_i^0}{w_i^2}\right) w_i u' v'' - v' u'}{(w_i u')^2} < 0 \tag{20}$$

This is the single-crossing condition. Note that it implies that in the absence of taxation a type H consumer has both higher gross income and consumption than that of a type L.

As we saw earlier, a key feature of the solution to the lump sum tax problem is that at the planner's optimum high productivity types are worse off than low productivity types. It is essential therefore that the planner can observe an individual's type. Suppose that this is not the case. A consumer's type is private information, unavailable to the planner. This information asymmetry then creates an adverse selection problem. All individuals would claim to be

low productivity types, and if the planner took this at face value and applied the tax accordingly, the budget constraint would be violated.

The solution to this adverse selection problem is well-known. We introduce an incentive compatibility (IC) constraint, which requires that the equilibrium allocation be such that the high productivity type has no incentive to lie. Thus we formulate the planner's problem[10] as:

$$\max_{x_i y_i} W = \sum_{i=H,L} \phi_i [u(x_i) - \psi_i(y_i)] \tag{21}$$

$$s.t. \sum_{i=H,L} \phi_i (y_i - x_i) - G \geqslant 0 \quad (\lambda) \tag{22}$$

$$u(x_H) - \psi_H(y_H) \geqslant u(x_L) - \psi_H(y_L) \quad (\mu) \tag{23}$$

The main results of the model for optimal values $\hat{y}_i$, $\hat{x}_i$ are:
(i) *"No distortion at the top"*:

$$\left[\frac{dx_H}{dy_H} \right]_{\hat{u}_H} = \frac{\psi_H'(\hat{y}_H)}{u'(\hat{x}_H)} = 1 \tag{24}$$

This is precisely the condition on type 2's allocation that results from the lump sum tax problem, hence the name of this result. It caused quite a stir, because it implies that the marginal rate of tax on the gross income of the high productivity type is zero. This does not of course mean that she pays no tax. In fact, she pays a lump sum tax. But it does conflict with conventional notions of the progressivity of the tax system, since most empirical tax schedules have marginal tax rates increasing with taxable income.

(ii) *"Distortion at the bottom":* We can express the first order conditions in the following way. Define

$$\delta \equiv \psi_L'(\hat{y}_L) - \psi_H'(\hat{y}_L) \tag{25}$$

We show that $\delta > 0$. Thus we have

$$\delta \equiv \frac{v'\left(\frac{\hat{y}_L}{w_L}\right)}{w_L} - \frac{v'\left(\frac{\hat{y}_L}{w_H}\right)}{w_H} \tag{26}$$

Then since $w_L < w_H$, $\frac{\hat{y}_L}{w_L} > \frac{\hat{y}_L}{w_H}$, and the convexity of $v(.)$ implies $v'\left(\frac{\hat{y}_L}{w_L}\right) > v'\left(\frac{\hat{y}_L}{w_H}\right)$, while dividing these by respectively w_L and w_H strengthens the inequality.

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[10]Note that in this formulation we implicitly assume there will be an interior solution, with all variables strictly positive. However, it is quite possible that for sufficiently small ϕ_L, the optimum would involve $y_L = 0$. To allow for this case we should really impose the constraint $y_L \geq 0$. This possibility is not without economic interest: it is optimal to pay the low productivity type to be unemployed. However we will ignore this possibility throughout this paper.

Then we can write

$$\left[\frac{dx_L}{dy_L}\right]_{\hat{u}_L} = \frac{\psi_L'(\hat{y}_L)}{u'(\hat{x}_L)} = 1 - \frac{\hat{\mu}\delta}{\phi_L\hat{\lambda}} < 1 \qquad (27)$$

Given the strict convexity of the indifference curves, this implies that consumption and gross income (therefore labour supply) of the low productivity type are reduced relative to the levels that correspond to the first best condition.

This second best optimal allocation can be implemented by a tax system as follows. As already suggested, the high productivity consumers each pay a lump sum tax $\hat{T}_H = \hat{y}_H - \hat{x}_H$, which gives them the budget constraint $x_H = y_H - \hat{T}_H$ and "guides" them to their allocation $(\hat{y}_H, \hat{x}_H)$.

For low productivity consumers, to induce them to choose the second best optimal allocation $(\hat{y}_L, \hat{x}_L)$, they have to be offered a budget constraint $x_L = (1 - \hat{t})y_L + \hat{a}$. In that case they choose $(\hat{y}_L, \hat{x}_L)$ and satisfy the condition

$$\left[\frac{dx_L}{dy_L}\right]_{\hat{u}_1} = \frac{\psi_L'(\hat{y}_L)}{u'(\hat{x}_L)} = 1 - \hat{t} \qquad (28)$$

where the optimal marginal tax rate is

$$\hat{t} = \frac{\hat{\mu}\delta}{\phi_L\hat{\lambda}} \qquad (29)$$

In order to ensure that they have the right amount of consumption they receive the lump sum payment

$$\hat{a} = (\hat{x}_L - \hat{y}_L) + \hat{t}\hat{y}_L \qquad (30)$$

which more than repays them their tax bill $\hat{t}\hat{y}_L$. Note also that when this tax function is offered, it must be specified to apply only to $y \leq \hat{y}_L$, since without such a quantity limitation the high productivity type would choose it.

This relatively simple two-type version of Mirrlees' model is sufficient to bring out most, but not quite all, of the main results of his analysis. It does not allow us to analyse the way in which the marginal tax rate, the degree of distortion of the allocation for types below the highest productivity type, varies with gross income, *i.e.* it does not allow us to study the structure of the optimal tax function. For this we need a model with a continuum of types.[11] However, since the main purpose of the next section is to generalize Mirrlees's model to two-person households, and this can only be done tractably in the two-type case, this simple model suffices for purposes of comparison.

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[11]On the other hand it should be said that, in the absence of specific assumptions on the social welfare function, consumer utility functions and the distribution of innate productivities (which realistically should not simply be identified with the empirical wage distribution), not much can be said about this function except that it has a slope between zero and 1, and may exhibit non differentiabilities, or so-called "bunching". On the other hand the two-type case does perhaps give undue emphasis to the "no distortion at the top" result, while also not allowing the complementary "no distortion at the bottom" result that is obtained in the continuum-of-types case.

3 The Mirrlees Model with Two-Person Households and Imperfect Assortative Matching

We now make the simplest possible extension to Mirrlees' model to take account of two-person households.[12] Thus suppose households consist of two individuals who may be of either productivity type, so that there are four possible household types. Call the first individual f and the second m. A household's type is then described by the pair (w_i, w_j), $i, j = H, L$, with w_i the wage of f and w_j that of m. A household of type (w_i, w_j) has the utility function

$$u_{ij} = u(x_{ij}) - v(\frac{y_i}{w_i}) - v(\frac{y_j}{w_j}) \equiv u(x_{ij}) - \psi_i(y_i) - \psi_j(y_j) \quad i, j = H, L \quad (31)$$

All individuals in all households have identical preferences, and the properties of the functions v and ψ are just as in the previous discussion of the Mirrlees model. Given the household's budget constraint in the absence of taxation

$$x_{ij} \leq y_i + y_j \quad (32)$$

it is easy to see that the household's equilibrium with no or only lump sum taxation is characterized by

$$\frac{\psi_i^{'}(y_i^*)}{u^{'}(x_{ij}^*)} = \frac{\psi_j^{'}(y_j^*)}{u^{'}(x_{ij}^*)} = 1 \quad (33)$$

As we know from the literature on two-dimensional screening models,[13] the analysis of this kind of model depends heavily on the nature of the joint distribution of types. For example, if there were perfect positive correlation of types across households - high wage f's and m's form households only with each other, and similarly for low wage f's and m's, the case we call *perfect assortative matching* - then we could in this model ignore the two-person nature of households and just apply the results of the Mirrlees model to the sets of partners taken separately. The empirical evidence however suggests that although there is quite a strong positive correlation of wage types across households, it is by no means perfect. This motivates the following.

Let ϕ denote the proportion of f's who are high wage, and ϕ_H the proportion of their partners who are also high wage, while ϕ_L denotes the proportion of partners of low wage f's who are high wage. Then we assume:

Imperfect assortative matching: $1 > \phi_H > \phi_L > 0$.

Let us assume (and this is just for convenience) that ϕ is also the proportion of m's who are high wage. The effect of this assumption is to simplify notation by giving the same proportions of "mixed" couples (w_H, w_L) and (w_L, w_H), as can be seen from the following description of the distribution of types

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[12] The paper by F Schroyen (2003) analyses a much richer and more complex model than that considered here, even incorporating household production. The comparative advantage of the present model is its simplicity and transparency.

[13] See in particular Armstrong and Rochet (1999).

$(w_H, w_H) : \phi\phi_H \equiv \phi_{HH}$
$(w_H, w_L) : \phi(1 - \phi_H) \equiv \phi_M$
$(w_L, w_H) : (1 - \phi)\phi_L \equiv \phi_M$
$(w_L, w_L) : (1 - \phi)(1 - \phi_L) \equiv \phi_{LL}$
with $\phi = \phi\phi_H + (1 - \phi)\phi_L$.

Since mixed household types (w_H, w_L) and (w_L, w_H) are essentially identical and present in the same proportions, we use the subscript M to denote them. Note that we have to allow the possibility that the allocation received by an individual of type $j = H, L$ will depend on the type of household to which he or she belongs, and so we attach a subscript to the y-variable to indicate this. Thus y_{Mj} is the gross income of a j-productivity individual in a mixed household, y_{jj} that of each individual in a matched household.

We again assume that the planner is utilitarian, and so the social welfare function W is

$$\phi_{HH}[u(x_{HH}) - 2\psi_H(y_{HH})] + 2\phi_M[u(x_M) - \psi_H(y_{MH}) - \psi_L(y_{ML})] + \phi_{LL}[u(x_{LL}) - 2\psi_L(y_{LL})] \tag{34}$$

while the government budget constraint is

$$\phi_{HH}(2y_H - x_{HH}) + 2\phi_M[y_{MH} + y_{ML} - x_M] + \phi_{LL}[2y_{LL} - x_{LL}] \geq G \tag{35}$$

Under symmetric information, where the planner can observe everyone's type, it is straightforward to show that the optimal lump sum taxes, found by maximizing W subject to (35), imply a straightforward extension of the results for single-person households. Taxation is essentially individual, everyone receives the same consumption and high productivity types supply more labour, regardless of the type of their partner.[14]

Under asymmetric information, we have to introduce incentive compatibility constraints. A potential difficulty here is the multiplicity of logically possible constraints. However, a substantial simplification is available because of the assumption of imperfect assortative matching. Armstrong and Rochet show that, in their model, this would imply that only "downward" constraints may be binding. We conjecture that this will also hold in the case of the present model[15], and so we solve the problem in the presence only of the three downward constraints:

$$u(x_{HH}) - 2\psi_H(y_{HH}) \geq u(x_M) - \psi_H(y_{MH}) - \psi_H(y_{ML}) \tag{36}$$

$$u(x_M) - \psi_H(y_{MH}) - \psi_L(y_{ML}) \geq u(x_{LL}) - \psi_H(y_{LL}) - \psi_L(y_{LL}) \tag{37}$$

$$u(x_{HH}) - 2\psi_H(y_{HH}) \geq u(x_{LL}) - 2\psi_H(y_{LL}) \tag{38}$$

It is straightforward to show that all three constraints cannot be binding. Setting the three inequalities as strict equalities can be shown to imply the condition

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[14] Note again the point made in the Introduction: we are ignoring the issue of possible dissonance between the planner's desired and the household's actual allocation between household members.

[15] Though this has still to be verified.

$\phi_{LL} = 2\phi_M$.There is no reason to expect this to hold ingeneral and so we rule this case out. Moreover, it can be shown that assuming the first two constraints are binding and the third non binding, or the last two constraints binding and the first non binding, leads to a contradiction, and that only the case is possible in which the first and third constraints are binding and the second non binding. Thus we formulate the second best optimal taxation problem as that of maximizing W subject to the budget constraint and the IC constraints (36),(38).

From the first order conditions for this problem, with λ and μ_M, μ_L the multipliers attached to the budget and incentive constraints respectively, we derive the following results:[16]

Result 3.1: The optimal allocation for the members of HH households is characterized by the condition

$$u'(x^*_{HH}) = \psi'_H(y^*_{HH}) = \frac{\lambda^* \phi_{HH}}{\phi_{HH} + \mu^*_M + \mu^*_L} \tag{39}$$

implying "no distortion at the top".

Result 3.2: H-types in M households have an allocation characterized by

$$u'(x^*_M) = \psi'_H(y^*_{MH}) = \frac{\lambda^* 2\phi_M}{2\phi_M - \mu^*_M} \tag{40}$$

so that for high productivity individuals in mixed households there is no distortion. The reason for this is that at any point (x^0, y^0), the marginal rates of substitution $\psi'_H(y^0)/u'(x^0)$ are the same for both household types. Thus there is no gain in terms of extra redistribution from distorting the equilibrium of the high productivity type in the mixed household. On the other hand, this non distortion relates to the equilibrium condition, not the values at the optimum, which are different from those in the lump sum tax case, as we show below.

Result 3.3: L-types in M households have an allocation characterized by

$$\frac{\psi'_L(y^*_{ML})}{u'(x^*_M)} = 1 - t^*_{ML} \tag{41}$$

where

$$t^*_{ML} = \frac{\mu^*_M \delta_{ML}}{2\phi_M - \mu^*_M} \tag{42}$$

and

$$\delta_{ML} = \frac{\psi'_L(y^*_{ML})}{u'(x^*_M)} - \frac{\psi'_H(y^*_{ML})}{u'(x^*_M)} > 0 \tag{43}$$

Thus there is a downward distortion in their labour supply as compared to the first best, brought about by a positive marginal tax rate.

Result 3.4: The optimal allocation for members of LL households is characterized by

$$u'(x^*_{LL}) = \frac{\lambda^* \phi_{LL}}{\phi_{LL} - \mu^*_L} \tag{44}$$

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[16]Proofs of all these results are given in the Appendix.

and

$$\frac{\psi'_L(y^*_{LL})}{u'(x^*_{LL})} = 1 - t^*_{LL} \tag{45}$$

where

$$t^*_{LL} = \frac{\mu^*_L \delta_{LL}}{2\phi_{LL} - \mu^*_L} \tag{46}$$

and

$$\delta_{LL} = \frac{\psi'_L(y^*_{LL})}{u'(x^*_{LL})} - \frac{\psi'_H(y^*_{LL})}{u'(x^*_{LL})} > 0 \tag{47}$$

Thus again there is a downward distortion of labour supply as compared to the first best.

Result 3.5: At the optimum the utility values are ordered as $u^*_{HH} > u^*_M > u^*_{LL}$. Since this is the reverse of that which would follow from optimal lump sum taxation, we again have binding limits on redistribution imposed by the IC constraints. Moreover, it implies that H-types are better off by forming households with H-types, while L-types do better by forming households with H-types. To the extent that taxation has implications for household formation this could be an interesting result. However, in this paper the formation of households is taken as exogenous.

Result 3.6: At the optimum we have gross incomes ordered as $y^*_{MH} > y^*_{HH} > y^*_{LL} > y^*_{ML}$. Since these are induced by tax rates (lump sum in the first two cases, marginal in the last two), we have that how you are taxed depends on your household type. In terms of the debate about whether the individual or the household should be the "unit of taxation", we see that for second best non linear (as opposed to linear) taxation, the answer is the household.[17]

4   Repeated Taxation

To keep what is a fairly complicated analysis as simple as possible, we revert to the assumption that the household consists only of a single individual. There are two periods, the first with a length normalised at 1, the second with a finite length $\tau \in (0, \infty)$. It is a standard proposition that if the planner can commit fully to a "long term tax contract" she would repeat the single period Mirrlees optimal second best allocation. However, in the absence of any commitment possibilities, the planner would naturally use full revelation of types in the first period to implement the first best optimal lump sum tax system in the second, and since this is bad for the high productivity types, they would not truthfully reveal their types in the first period. The approach to the solution is illustrated in Figure 1.[18]

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[17] This result is already clear from Schroyen (2003).

[18] The analysis here draws heavily on Laffont and Tirole (1993) and Bester and Strausz (2001). For simplicity we take the polar case of pure redistribution, but where the government's budget constraint must balance over both periods taken together, implying that it has borrowing/lending possibilities.

Figure 1 about here.

We introduce a degree of pooling in the first period expressed by the variable $\gamma \in [0,1]$, which is the proportion of high productivity types that will be offered the same first period allocation (x_P, y_P) as all low productivity types, while the proportion $1 - \gamma$ of high productivity types reveal their type in the first period and receive the allocation (x_{H1}, y_{H1}).

In the second period, the planner has the proportion $1 - \gamma$ of high productivity types that she knows to be so, and offers them the allocation (x_{H2}, y_{H2}), and offers a Mirrlees-type separating allocation (x_{PH2}, y_{PH2}) to the proportion γ of high productivity types and (x_{L2}, y_{L2}) to all low productivity types, who received the pooled allocation in the first period. To formalise and solve the planner's problem of choosing these allocations optimally subject to its budget constraint and to incentive compatability conditions, we begin with the second period problem. In this period, the planner maximises

$$\phi(1-\gamma)[u(x_{H2})-\psi_H(y_{H2})]+\phi\gamma[u(x_{PH2})-\psi_H(y_{PH2})]+(1-\phi)[u(x_{L2})-\psi_L(y_{L2})] \tag{48}$$

$$s.t. \quad \phi(1-\gamma)[y_{H2}-x_{H2}] + \phi\gamma[y_{PH2}-x_{PH2}] + (1-\phi)[y_{L2}-x_{L2}] \geq G_2 \tag{49}$$

$$u(x_{PH2}) - \psi_H(y_{PH2}) \geq u(x_{L2}) - \psi_H(y_{L2}) \tag{50}$$

Here G_2 in the budget constraint is a parameter that will be optimally determined in period 1. Likewise γ is here a parameter that will be optimally determined in the first period. Only the incentive constraint for previously pooled consumers is required. From the conditions for solution of this problem we have the following results:[19]

Result 4.1: There is no distortion at the top, so both the known H-types and those who pooled in the first period pay lump sum taxes, while L-types have a positive marginal tax rate, found essentially along the lines of the standard Mirrlees solution set out in the previous section.

Result 4.2: We have

$$y_{H2}^* \quad > \quad y_{PH2}^* \tag{51}$$

$$x_{PH2}^* \quad > \quad x_{H2}^* \tag{52}$$

and so

$$0 < y_{PH2}^* - x_{PH2}^* < y_{H2}^* - x_{H2}^* \tag{53}$$

and

$$u_{PH2}^* = u(x_{PH2}^*) - \psi_H(y_{PH2}^*) > u_{H2}^* = u(x_{H2}^*) - \psi_H(y_{H2}^*) \tag{54}$$

Thus H-types who pooled in the first period pay a lower lump sum tax and have higher utility in the second period than the H-types who revealed their types in the first period. This is because of the incentive compatibility constraint. Thus the latter group of H-types contributes more toward the redistribution in favour of L-types in the second period than do the H-types who pooled in the first period.

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[19] Proofs are given in the Appendix.

Result 4.3: From the IC constraint we have

$$u(x^*_{PH2}) - \psi_H(y^*_{PH2}) = u(x^*_{L2}) - \psi_H(y^*_{L2}) > u(x^*_{L2}) - \psi_L(y^*_{L2}) = u^*_{L2} \quad (55)$$

since $\psi_H(y^*_{L2}) < \psi_L(y^*_{L2})$. This is the "limit to redistribution" arising out of the IC constraint.

Result 4.4: Denote the maximised level of social welfare in the second period by $W^*_2 = W(G_2, \gamma)$, and note that by the Envelope Theorem

$$\frac{\partial W^*_2}{\partial G_2} = -\lambda^*_2 \quad (56)$$

where λ^*_2 is the shadow price of the government budget constaint in the second period, and

$$\frac{\partial W^*_2}{\partial \gamma} = \phi\{u^*_{PH2} + \lambda^*_2(y^*_{PH2} - x^*_{PH2}) - [u^*_{H2} + \lambda^*_2(y^*_{H2} - x^*_{H2})]\} \quad (57)$$

Turning now to the first period, the planner chooses the allocations $(x_{H1}, y_{H1}), (x_P, y_P)$, and the values of γ and G_2, by maximising

$$\phi(1-\gamma)[u(x_{H1}) - \psi_H(y_{H1})] + \phi\gamma[u(x_P) - \psi_H(y_P)] + (1-\phi)[u(x_P) - \psi_L(y_P)] + \tau W^*_2 \quad (58)$$

$$s.t. \quad \phi(1 - \gamma)[y_{H1} - x_{H1}] + [\phi\gamma + (1 - \phi)](y_P - x_P) + \tau G_2 \geq 0 \quad (59)$$

$$u(x_{H1}) - \psi_H(y_{H1}) + \tau u^*_{H2} = u(x_P) - \psi_H(y_P) + \tau u^*_{PH2} \quad (60)$$

$$1 \geq \gamma \geq 0 \quad (61)$$

The first constraint is the budget constraint (assuming pure redistribution) while the second could be called the *mixed strategy constraint (MSC):* the value of γ must be chosen in such a way that high productivity types are in the first period indifferent between revealing their types and being pooled with the low productivity types. Since those H-types who reveal themselves in the first period do worse in the second period than those who pool, this constraint implies that they must be compensated for this in the first period. This compensation must be greater, the larger is τ. Moreover, since the compensation must be at the expense of all pooled consumers, there is in effect a redistribution away from low productivity consumers in the first period, which is costly to the planner.

The strict equality in the MSC constraint assumes implicitly that $\gamma \in (0, 1)$ at the optimum. If it were optimal to have $\gamma = 0$, then $u(x_{H1}) - \psi_H(y_{H1}) + \tau u^*_{H2} \geq u(x_P) - \psi_H(y_P) + \tau u^*_{PH2}$, since then nobody wants to be pooled, while in the converse case of $\gamma = 1$ being optimal we have $u(x_{H1}) - \psi_H(y_{H1}) + \tau u^*_{H2} \leq u(x_P) - \psi_H(y_P) + \tau u^*_{PH2}$ and nobody wants to be separated. Note further that, since $\psi_H(y_P) < \psi_L(y_P)$, and $u^*_{PH2} > u^*_{H2}$, this constraint ensures that H-types who reveal themselves in the first period would never want to mimic L-types.[20]

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[20] It seems plausible in the present context to rule out the "take-the-money-and-run strategy identified by Laffont and Tirole in the regulation setting, where L-types would claim to be H-types in the first period and then quit in the second period.

The solution to this problem yields the following results:

Result 4.5: H-types who reveal themselves as such are undistorted, while all consumers who pool have an allocation which implies distortions for both their types. Since $\psi'_H(y_P) < \psi'_L(y_P)$, we have at the pooled optimum

$$\frac{\psi'_L(y_P^*)}{u'(x_P^*)} > 1 > \frac{\psi'_H(y_P^*)}{u'(x_P^*)} \tag{62}$$

It is interesting to consider how pooling could be implemented by a tax system, since in reality the planner cannot choose quantities directly. The answer is that the pooled consumers are offered the same linear tax system, with a marginal tax rate equal to one *minus* the marginal rate of substitution $\psi'_L(y_P^*)/\,u'(x_P^*)$ of the low productivity types at the optimal pooling point, and a uniform lump sum designed to achieve the correct level of consumption. From (.) we see that in fact the marginal tax rate has to be negative, i.e. pooled consumers are paid a marginal subsidy, and the lump sum is then a tax. Offered this, and given the single crossing condition, high productivity types would want to earn more gross income than y_P^*, so this has to be accompanied by a 100% tax rate on gross income greater than y_P^*. (See Figure 2).

Figure 2 about here

Result 4.6: The optimal choice of γ (assumed positive but less than one) together with the MSC implies that at the optimum

$$(y_P^* - x_P^*) - (y_{H1}^* - x_{H1}^*) = \tau[(y_{H2}^* - x_{H2}^*) - (y_{PH2}^* - x_{PH2}^*)] \tag{63}$$

We know from Result 4.2 that the right hand side of this equation is positive. It says simply that the higher tax contribution of the non-pooled H-types in the second period must be balanced by a lower tax contribution, or possibly even a subsidy, in the first period.

5   The Boskin-Sheshinski Model

This model, based on the optimal linear income tax analysis of Sheshinski (1972), could be viewed as making the smallest possible extension to the model of the individual worker/consumer just necessary to analyse taxation of two-person households. Its main result is to make precise the intuition that selective taxation is optimal because the elasticity of female labour supply is higher than that of male labour supply. In fact the paper by Boskin and Sheshinski presents the general results incorporating both efficiency and equity considerations, and these general results do *not* imply the "conventional wisdom", that the tax rate on women should be lower than that on men. They present a numerical example that has this result, and their general discussion gives the impression that this result is necessarily the case, but they do not carry out the kind of empirical study that would be necessary to establish this. Part of the problem is that their model is somewhat complex, and so the issue is not as clear as it could be. Here, we present a simple model which makes it much easier to see what is at stake.

A household has the utility function $u(x, l_f, l_m)$, where x is a market consumption good, and $l_i \geq 0$, $i = f, m$, is the labour supply of household member[21] i. The household faces the budget constraint

$$x = a + \sum_{i=f,m} (1 - t_i)y_i$$

where a is the lump sum transfer in a linear tax system and t_i is the marginal tax rate on $i's$ gross income $y_i \equiv w_i l_i$, with w_i the exogenously given market wage. Thus a household is characterised by a pair of wage rates (w_f, w_m), otherwise households are identical. Since this is a linear tax problem we do not have to assume that a household's wage pair is observable. There is a given population joint density function $f(w_f, w_m)$, everywhere positive on $\Omega = [w_f^0, w_f^1] \otimes [w_m^0, w_m^1] \subset \mathbf{R}_+^2$, which tells us how households are distributed according to the innate productivities in market work of their members, as measured by their market wage rates.

To focus attention on what we regard as the most important aspects of the results, we assume that the household utility function[22] takes the quasilinear form

$$u = x - u_f(l_f) - u_m(l_m) \quad u_i^{'} > 0, u_i^{''} > 0$$

which, however, we find more convenient to write in terms of gross incomes

$$u = x - v_f(y_f) - v_m(y_m) \quad v_i^{'} = u_i^{'}/w_i, v_i^{''} = u_i^{''}/w_i^2$$

Solving the household's utility maximisation problem yields demands $x(a, t_f, t_m)$, $y_i(t_i)$ and the indirect utility function $v(a, t_f, t_m)$ such that

$$\frac{\partial v}{\partial a} = 1; \quad \frac{\partial v}{\partial t_i} = -y_i \quad \frac{\partial v}{\partial w_i} = (1 - t_i)l_i$$

Note that

$$y_i^{'}(t_i) = w_i \frac{dl_i}{dt_i}$$

is a compensated derivative, because of the absence of income effects. For the same reason, it is straightforward to confirm that labour supplies and gross incomes are strictly increasing in the wage rate and decreasing in the tax rate. Thus household utility is strictly increasing in household income. Note that the choice of utility function sets the effects of one partner's wage on the labour supply of the other to zero. This makes it much easier to derive the main insights of the analysis without doing too much injustice to the facts.[23]

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[21] Although it could just as well be thought of as referring to a single individual with two sorts of labour supply or leisure.

[22] Clearly, as we pointed out in the Introduction, the model can say nothing about the within-household welfare distribution.

[23] Empirical evidence seems to suggest no significant effects of a wife's wage on husband's labour supply and only very weak negative effects of husband's wage on wife's labour supply.

To find the optimal tax system we introduce the social welfare function $W(.)$, which is strictly increasing, strictly concave and differentiable in the utility of every household, and the planner's problem is then

$$\max_{a,t_f,t_m} \iint_{\Omega} W[v(a,t_f,t_m)]f(w_f,w_m)dw_f dw_m$$

subject to the tax revenue constraint

$$\iint_{\Omega} [t_f y_f + t_m y_m]f(w_f,w_m)dw_f dw_m - a - G \geq 0$$

where $G \geq 0$ is a per household revenue requirement.

Result 5.1: The optimal lump sum a satisfies

$$\iint_{\Omega} \frac{W'}{\lambda} f(w_f,w_m)dw_f dw_m = 1$$

where $\lambda > 0$ is the marginal social cost of tax revenue and W'/λ the marginal social utility of income to a household with characteristic (w_f, w_m). Thus the optimal a equates the average marginal social utility of income to the marginal cost of the lump sum. We denote a household's marginal social utility of income W'/λ by s, and its mean by $\bar{s}$. Thus the condition sets $\bar{s} = 1$. Because of the assumptions on $W(.)$, households with relatively low wage pairs will have values of s above the average, those with relatively high wage pairs, below.

Result 5.2: The marginal tax rates satisfy

$$t_i^* = \frac{Cov[s,y_i]}{\bar{y}_i'} \quad i = f,m$$

where

$$Cov[s,y_i] = \iint_{\Omega} (\frac{W'}{\lambda} - 1)x_i f(w_f,w_m)dw_f dw_m$$

is the covariance of the marginal social utility of household income and the gross household income of individual i, and

$$\bar{y}_i' = \iint_{\Omega} y_i'(t_i^*)f(w_f,w_m)dw_f dw_m$$

is the average compensated derivative of gross income with respect to the tax rate, and is negative.

Now the argument that $t_f^* < t_m^*$ is based on the empirical evidence suggesting that $-\bar{y}_f' > -\bar{y}_i'$, but this clearly considers only part of the optimal tax formula, and is in general neither necessary nor sufficient for the result. In other words, though taxing women at a given rate creates a higher average deadweight loss

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than taxing men at the same rate, the policy maker's willingness to trade off efficiency for equity might imply that the tax rate on women could optimally be higher than that on men, if the covariance between the marginal social utility of household income and women's gross income is in absolute value sufficiently higher than that of men, so that the corresponding redistributive effects make that worthwhile. But as far as we are aware there is no empirical evidence that establishes what these redistributive effects are, either in Boskin-Sheshinski or in the earlier, less formal treatments of the subject. It is certainly true that equality of the marginal tax rates appears as a highly special case, and so joint taxation is very unlikely to be optimal, but the results of this model so far do not make a conclusive case for taxing women at a lower rate than men, as the conventional wisdom assumes. The optimal tax analysis suggests a departure from income splitting, but it does not tell us much about the appropriate direction of this departure. In fact, the analysis is unnecessary to give us the basic result, since joint taxation amounts to imposing on the optimal tax problem the constraint that the marginal tax rates be equal, and such a constraint cannot increase the value of the objective function at the optimum.

To make this a little more precise, write

$$Cov[s, y_i] = \rho_i \sigma_i \sigma_s \quad i = f, m$$

with ρ_i the correlation coefficient between s and y_i, σ_i the standard deviation of y_i, and σ_s the standard deviation of s. Then we have

Result 5.3:

$$t_f^* < t_m^* \Leftrightarrow \frac{\rho_f \sigma_f}{\rho_m \sigma_m} < \frac{\bar{y}_f'}{\bar{y}_m'}$$

It is an open question empirically, whether this condition is satisfied. We would conjecture that the variance of female market income is greater than that of the male income, but we know nothing about the relevant correlation coefficients.

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