A Bargaining Model of Tax Competition

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20 October 2005

Abstract

This paper develops a model in which competing governments offer financial incentives to individual firms to induce the firms to locate within their jurisdictions. Equilibrium is described under three specifications of the supplementary taxes. There is no misallocation of capital under two of these specifications, and there might or might not be capital misallocation under the third. This result contrasts strongly with that of the standard tax competition model, which does not allow governments to treat firms individually. That model almost always finds that competition among governments leads to the misallocation of capital.

1 Introduction

Toyota announced in June 2005 that it would build a new autombile assembly plant in Woodstock, Ontario, Canada. The plant will cost about US$625 million and will ultimately employ 1300 people. The governments of Ontario and Canada have promised Toyota US$100 million in financial assistance. The government of Ontario had, within the previous year, provided financial assistance to both Ford and General Motors, to ensure that their plans to redevelop existing facilities went ahead.

The tax competition literature assumes that the economy is divided into autonomous regions, and that capital can move freely between the regions.

Company	Location	Investment	Government Aid	Annual Output
Honda	Lincoln, AL	825	248	270
Hyundai	Montgomery, AL	1000	252	300
Nissan	Canton, MS	1400	360	400
Toyota	Princeton, IN	1900	117	400
Toyota	San Antonio, TX	800	133	150
Toyota	Woodstock, ON	625	100	100

Source: *The Globe and Mail*, Toronto, 1 July 2005. Investment and aid are in millions of US dollars; output is in thousands of vehicles.

Table 1: Assembly Plants Built by Asian Auto Makers in North America since 1998

Its objective is to determine the rates at which the regions will tax profits, and the impact of the profits tax on resource allocation. It finds that the tax rates generally vary across regions, and that the variation in tax rates leads to an inefficient allocation of capital: the low tax regions use too much capital and the high tax regions use too little. A limitation of this literature is that it assumes that a government can only attract capital by reducing the rate at which it taxes profits. In reality governments attach so much importance to new capital investment that they will often make substantial financial concessions to get it. The example above, in which the concessions amount to 15% of the new investment, is not atypical. Table 1 shows the concessions recently given to Asian auto makers building assembly plants in North America. These concessions have been as high as 30% of the new investment.

A government hoping to induce a major corporation to locate facilities within its jurisdiction will often offer a package of financial incentives. Smaller corporations tend to be less generously treated than larger corporations, and very small firms—mom and pop grocery stores—can expect no special treatment. Nevertheless, if competition among governments is to be properly understood, the focus must be on the overall financial package, and the way in which that package varies from firm to firm. A model of this sort is described here. The key element is that capital is embodied in heterogeneous firms. Each firm is mobile, and each firm's productivity varies from region to region. The firms differ in the way that their productivity varies across regions. Each firm receives an offer from the government of every region, and the firm locates in the region in which its after-tax profits would be highest. The government uses its tax revenue to provide a public good to the region's citizens.

Casual observation shows that governments are prepared to negotiate with some firms but not with others, so this model is an abstraction. The standard tax competition model, in which no-one gets a special deal, is also an abstraction. Our view is that these two models constitute polar cases in the study of tax competition. Reality lies somewhere between them, but almost all research has been concentrated on one of the two poles. Our hope is that models like this one will ultimately lead to a more balanced view of tax competition.

Our basic model assumes that each government can levy a tax on the profits of firms located within its jurisdiction, and that the government can also levy a lump-sum tax on the incomes of the citizenry. Under these assumptions, the standard tax competition model predicts optimal public goods provision but misallocation of capital. This finding was first presented by Hamada (1966). Hamada's model has been expanded in many different ways, but the core result has remained largely unchanged. The bargaining model, by contrast, predicts both optimal public goods provision and optimal allocation of capital. The resource misallocation that has been the focus of the tax competition literature is simply not there.

Since the existence of such a broadly based lump-sum tax might be viewed with some scepticism, we consider two alternative assumptions. The first is that the lump-sum tax strikes only wages. Capital is again correctly allocated, but optimal provision of the public good is no longer guaranteed. Public goods are optimally provided if the constraint on lump-sum taxation is not binding, and they are underprovided if it is binding. The second is Wilson's (1999) assumption that the profits tax is the only tax.[1] Wilson shows that, under this assumption, the standard tax competition model predicts both underprovision of the public good and misallocation of capital. The bargaining model is more agnostic. There is underprovision and capital misallocation if the typical firm's output would not fall greatly if it moved

[1] Our assumption is actually slightly different from Wilson's, in that we assume that a government that raises too much revenue through the profits tax can return the excess revenue to the citizens through a negative lump-sum tax. Wilson requires an exact match between revenue and public goods expenditure.

from its best location to its second best location; and there is optimal provision and optimal capital allocation if this move causes the typical firm's output to fall dramatically.

An important feature of these results is that they are derived from a general equilibrium model, and hence can be directly compared to those of the standard fixed-rate model of tax competition. A number of earlier papers have offered explanations of the tax breaks given to mobile firms, but these papers have described the negotiations between a single firm and one or two governments. Doyle and van Wijnbergen (1994) examine the intertemporal structure of a firm's tax payments. They note that a mobile firm has greater bargaining power than a firm that has already incurred the sunk costs associated with locating in a particular region. They argue that mobile firms will use their extra bargaining power to extract concessions. Bond and Samuelson (1986) present an alternative explanation of the same phenomenon: a region can offer a tax holiday to a mobile firm to signal that firms that locate there experience high productivity. The firm will willingly pay higher tax rates in later periods because it is very productive, and these high tax rates allow the government to recover the cost of the initial tax holiday. A low-productivity region could not offer the same incentive: firms that located there would relocate when they found that they had low productivity, so that the region would be unable to recover the cost of the initial tax holiday. King, McAfee and Welling (1993) allow the firm to negotiate simultaneously with two governments, and add a stochastic element to the regional productivities. Black and Hoyt (1989), by contrast, present a static model in which the subsidies to mobile firms undo the distortionary effects of average cost pricing of publicly provided services.

Section 2 of this paper sets out an economy in which firms earn locational rents. Section 3 describes the Pareto optimal allocations. Section 4 describes the bargaining model, and derives the major result of the paper: there is no misallocation of capital when governments bargain with firms. Section 5 provides a broader comparison of the bargaining model and the standard model. Section 6 examines the role of the lump-sum tax, and section 7 contains brief conclusions. All proofs are contained in the appendix.

2 Preliminaries

The economy consists of I regions and a continuum of firms. The regions are identified by the elements of the set $\mathcal{I} \equiv \{1, ..., I\}$. A firm is characterized by its ownership structure and by its productivity in the various regions. A firm's ownership structure is represented by the vector $\gamma \equiv (\gamma_1, \cdots, \gamma_I)$, where γ_i is the fraction of the firm owned by residents of region i. The set of all possible ownership structures is

$$\Gamma = \left\{ \gamma \in [0,1]^I : \textstyle\sum_{i=1}^I \gamma_i = 1 \right\}$$

A firm's productivity in region i is governed by the parameter $\theta_i \in \mathbb{R}_+$, and the firm's productivity in each of the regions is described by the vector $\theta \equiv (\theta_1, \cdots, \theta_I)$. The set of all possible productivity vectors is $\Theta \subset \mathbb{R}_+^I$. The distribution of firms is represented by an atomless and σ-finite measure space $(\Theta \times \Gamma, \mathcal{B}, F)$. Here, $\Theta \times \Gamma$ is the sample space of firms, $\mathcal{B}$ is a σ-algebra over the sample space, and $F(X)$ denotes the measure of firms in any set X contained in $\mathcal{B}$. It is assumed that Θ is a compact set and that

$$F(\Theta \times \Gamma) = F(\mathsf{Int}(\Theta \times \Gamma)) > 0$$

where $\mathsf{Int}(\Theta \times \Gamma)$ denotes the interior set of $\Theta \times \Gamma$.[2]

Each firm locates and produces in one of the regions, or in none of them. The firm's output when it locates in region i is denoted y_i. It is determined by θ_i and by n_i, the quantity of labour employed by the firm.

$$y_i = \left(\frac{1}{1-\alpha} \right) \theta_i^\alpha n_i^{1-\alpha} \qquad 0 < \alpha < 1$$

The total quantity of labour available in region i is fixed and equal to N_i. Let $L_i \subset \Theta \times \Gamma$ be the set of firms that locate in region i, and let the distribution of firms across the economy be $L \equiv \{L_1, \cdots, L_I\}$. Let the mapping $n : \Theta \times \Gamma \to \mathbb{R}_+$ describe the quantity of labour employed by firms of each type.

The residents of each region consume two goods, a private good and a public good. One unit of output can be transformed into one unit of either

[2] These restrictions ensure that in equilibrium, the firms that are indifferent between two (or more) locations constitute a set of measure zero. Imposing these restrictions allows us to make stronger statements about the differences among equilibria than would otherwise be possible.

good. Let $c_i \in \mathbb{R}_+$ be the aggregate quantity of the private good in region i, and let $g_i \in \mathbb{R}_+$ be the aggregate quantity of the public good. The social preferences of region i are represented by a social welfare function $s_i : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ that is strictly concave, strictly increasing, and twice continuously differentiable. Let the vectors $c \equiv (c_1, \cdots, c_I)$ and $g \equiv (g_1, \cdots, g_I)$ describe the aggregate quantities of the private and public goods in the economy as a whole.

An *allocation* is a list (L, c, g, n). An allocation is *feasible* if

F1. The sets in L are disjoint, and $\cup_{i=1}^{I} L_i \subseteq \Theta \times \Gamma$

F2. The mapping n satisfies the condition

$$\int_{L_i} n dF = N_i$$

for each i.

F3. The vectors c and g satisfy the condition

$$\sum_{i=1}^{I} c_i + \sum_{i=1}^{I} g_i = \sum_{i=1}^{I} \frac{1}{1-\alpha} \left(\int_{L_i} \theta_i^{\alpha} n^{1-\alpha} dF \right)$$

Note that goods produced in one region can be used to increase the aggregate consumption of another region.

Allocations can differ in ways that do not lead to differences in aggregates or in social welfare. The following concepts will be used to identify allocations with inconsequential differences.

Definition 1 *Any two sets B and B' in $\mathcal{B}$ are measurably identical if*

$$F((B \cup B') - (B \cap B')) = 0$$

Definition 2 *Any two maps $\phi : \Theta \times \Gamma \to \mathbb{R}_+$ and $\varphi : \Theta \times \Gamma \to \mathbb{R}_+$ are measurably identical if*

$$F\{(\theta, \gamma) \in \Theta \times \Gamma : \phi(\theta, \gamma) \neq \varphi(\theta, \gamma)\} = 0$$

3 Pareto Optimal Allocations

A feasible allocation (L, c, g, n) is *Pareto optimal* if there does not exist an alternative feasible allocation (L', c', g', n') such that $s_i(c'_i, g'_i) \geq s_i(c_i, g_i)$ for all i and $s_i(c'_i, g'_i) > s_i(c_i, g_i)$ for some i.

A two-step procedure will be used to characterize the Pareto optimal allocations. Condition F3 and the monotonicity of the social welfare functions imply that any Pareto optimal allocation maximizes the total output of the economy. Total output is entirely determined by L and n, so the first step is to find the conditions under which (L, n) maximizes total output. The second step is to find the restrictions that Pareto optimality places on the allocation of output, that is, on (c, g).

With L given, each region's output is maximized by allocating the available labour so that the marginal product of labour is equalized across firms. That is, under any Pareto optimal allocation, the mapping $n : \Theta \times \Gamma \to \mathbb{R}_+$ satisfies, for each i

$$\theta_i^\alpha n(\theta, \gamma)^{-\alpha} = \widetilde{\theta}_i^\alpha n(\widetilde{\theta}, \widetilde{\gamma})^{-\alpha} \tag{1}$$

for all (θ, γ) and all $(\widetilde{\theta}, \widetilde{\gamma})$ contained in L_i.

Now consider the collection L. Let $k \equiv (k_1, \cdots, k_I)$ be a vector of non-negative real numbers. For each k, let $L(k) \equiv \{L_1(k), \cdots, L_I(k)\}$ be a particular collection of disjoint sets that satisfies these conditions:

C1. For each $i \in \mathcal{I}$, $(\theta, \gamma) \in L_i(k)$ implies that $k_i\theta_i = \max[k_1\theta_1, \cdots, k_I\theta_I]$.

C2. $\cup_{i=1}^I L_i(k) = \Theta \times \Gamma$.

Conditions C1 and C2 do not uniquely define $L(k)$, because C1 does not determine the placement of firms for which the product $k_i\theta_i$ reaches its maximum in more than one region. For each k, a unique collection $L(k)$ is obtained by adding an arbitrary "tie-breaking" rule to C1 and C2.

Define the functions

$$Z_i(k) \equiv \int_{L_i(k)} \theta_i dF \qquad \text{for all } i \in \mathcal{I}$$

The function $Z_i(k)$ aggregates the productivity factors of the firms that locate in region i under $L_i(k)$. It is readily shown that, if labour is allocated according to (1), the output of a firm locating in that region is

$$y_i(\theta_i, k) = \left(\frac{1}{1-\alpha}\right) \theta_i \left(N_i / Z_i(k)\right)^{1-\alpha}$$

and that the region's aggregate output is

$$Y_i(k) = \left(\frac{1}{1-\alpha}\right) Z_i(k)^\alpha N_i^{1-\alpha}$$

Total output is simply the sum of the regions' aggregate outputs. The conditions under which it is maximized are described by the following lemma.

Lemma 1 *Let k^* be the solution to the equation system*

$$k_i = \left(\frac{N_i}{Z_i(k)}\right)^{1-\alpha} \qquad \textit{for all } i \in \mathcal{I} \tag{2}$$

Then:

1. *The vector k^* exists and is unique. Each k_i^* and each $Z_i(k^*)$ is strictly positive.*

2. *If labour is allocated in accordance with (1), total output is maximized if and only if L is measurably identical to $L(k^*)$.*

The intuition behind this lemma is quite simple. If the firms are allocated across regions in accordance with $L(k^*)$, the output of a single firm that chose to locate in region i would be

$$y_i = \left(\frac{1}{1-\alpha}\right) \theta_i k_i^*$$

The firm's output is maximized by locating in the region in which $\theta_i k_i$ is greatest,[3] which is where the firm does locate under $L(k^*)$. That is, if every firm's location is determined by $L(k^*)$, no firm could raise its output by changing its location. Total output would be the same under any L that is measurably identical to $L(k^*)$. Also, some firms locate in every region under any L that is measurably identical to $L(k^*)$.

Let Y^* be maximal total output, and let $R \equiv (R_1, ..., R_I)$ represent the way in which total output is distributed across regions. The definition of

[3] The firm's output in region i depends upon its productivity in that region and the quantity of labour that would be allocated to it in that region. The latter factor is summarized by k_i, which rises with the supply of labour and falls with the number and productivity of the other firms in the region.

a feasible allocation assumed that goods produced in one region could be allocated to any region, so the only restriction on R is that

$$\sum_{i=1}^{I} R_i = Y^* \tag{3}$$

A unit of the produced good can be converted into one unit of either good, so

$$c_i + g_i = R_i \qquad \text{for all } i \in \mathcal{I} \tag{4}$$

The optimal policy in region i is to allocate R_i so that the society's marginal rate of substitution between these goods is just equal to the marginal rate of transformation between them, where the latter rate is fixed at one.

Let $\widehat{n}(k) : \Theta \times \Gamma \rightarrow \mathbb{R}_+$ be the mapping that equalizes the marginal product of labour across all of the firms in each region, when the firms are distributed in accordance with $L(k)$. That is,

$$\widehat{n}(k)(\theta, \gamma) = \theta_i N_i / Z_i(k) \qquad \text{for all } (\theta, \gamma) \in L_i(k) \text{ and all } i \in \mathcal{I}$$

Then:

Theorem 1 *A feasible allocation (L, n, c, g) is Pareto optimal if and only if*

P1. L is measurably identical to $L(k^*)$.

P2. n is measurably identical to $\widehat{n}(k^*)$.

P3. $\dfrac{\partial s_i(c_i, g_i)}{\partial c_i} = \dfrac{\partial s_i(c_i, g_i)}{\partial g_i}$ *for all $i \in \mathcal{I}$.*

The location of the firms and the allocation of labour across firms are measurably identical in every Pareto optimal allocation, but the division of resources between the regions varies substantially across the Pareto optimal allocations. Indeed, every division of resources that satisfies (3) is part of some Pareto optimal allocation.

4 Bargaining over Tax Rates

Governments set the rates at which they will tax profits, but they are prepared to offer financial concessions to attract new investment. The combination of fixed rate taxation with firm-specific concessions implies that the effective rate of taxation varies from firm to firm. The model in this section assumes that both firms and governments are only concerned with the effective rate of taxation, and seeks to explain the way in which this tax rate varies with the characteristics of the firm. It abstracts from the interplay of tax rates and concessions by imagining that each government is able to set a tax rate for each type of firm.

The equilibrium unfolds in two stages:

1. Each government, taking the wage rates as given, chooses the rates (one for every type of firm) at which it will tax profits. The tax rates chosen by the government of region i are represented by the map $t_i : \Theta \times \Gamma \to \mathbb{R}$, and the taxes imposed by all governments are given by $t \equiv (t_1, ..., t_I)$. Each firm, knowing these tax rates and taking the wage rates as given, chooses the region in which it will locate and the quantity of labour that it will employ. The wage rates anticipated by the governments and the firms are the wage rates that ultimately clear the labour markets. The wage in region i is w_i, and the wage rates of all regions are given by the vector $w \equiv (w_1, ..., w_I)$.

2. Each government chooses the rate at which it will tax the incomes that accrue to domestic residents. These incomes consist of wages and the residents' share of the after-tax profits of all firms. Since people are assumed to be immobile, this tax is a lump-sum tax. The proceeds of the tax, along with the proceeds of the profits tax, are used to provide units of the public good.

The assumption that governments, like firms, treat the wage rates as exogenous is appropriate because each government makes concessions to individual firms in an attempt to influence their location decisions. Both the firms and the government believe that the location decision of a single firm will not alter the prevailing wage rate.

The decision-making in stage 2 is very simple. Each region's total resources have already been determined. These resources consist of the wages earned by the region's workers, the government's revenue from the profits tax, and the residents' shares of the after-tax profits of all firms. Under competitive labour markets, the share of output allocated to wages is always $1-\alpha$, so the total resources of region i are

$$R_i = \int_{L_i} y_i[1-\alpha(1-\gamma_i)(1-t_i)]dF + \sum_{j\neq i}\left(\int_{L_j} \alpha\gamma_i(1-t_j)y_j dF\right) \tag{5}$$

The revenue from the profits tax has already accrued to the government, and the government can acquire additional revenue through the lump-sum tax.[4] Since all of the revenue is used to provide units of the public good, choosing the lump-sum tax is equivalent to choosing the quantity of public good provided. The government's choice problem is

$$\max_{(c_i,g_i)} s_i(c_i,g_i) \text{ s.t. } c_i + g_i = R_i \tag{6}$$

The maximum value function associated with this problem, $S_i(R_i)$, is strictly increasing, so the government's objective when it chooses its tax schedule in stage 1 should be the maximization of its resources.

Write (5) as

$$R_i = \int_{\Theta\times\Gamma} r_i dF$$

where r_i is a firm's contribution to region i's resources:

$$r_i = \begin{cases} [1-\alpha(1-\gamma_i)(1-t_i)]y_i & \text{if the firm locates in } i \\ \alpha\gamma_i(1-t_j)y_j & \text{if the firm locates in } j \ (j\neq i) \end{cases} \tag{7}$$

The government's ultimate objective is to maximize its total resources, but it achieves this end if it maximizes the resources contributed by each firm.

The equilibrium in stage 1 consists of taxes t, wage rates w, locations L and a labour allocation n such that:

1. No firm that chooses to locate in region i would have greater after-tax profits if it located elsewhere.

[4] The lump-sum tax can be negative, so that the government can return some of the revenue from the profits tax to the citizens if that tax raised an excessive amount of revenue.

2. For each type $(\theta,\gamma) \in \Theta \times \Gamma$, no government i $(i \in \mathcal{I})$ that can increase the resources r_i contributed by firms of that type by deviating from the equilibrium tax rate $t_i(\theta,\gamma)$ while every other government j adheres to its equilibrium tax rate $t_j(\theta,\gamma)$.

3. Given w and L, the labour allocation n assigns the profit-maximizing quantity of labour to firms of each type.

4. The wage vector w clear each region's labour market.

The equilibrium is derived in two steps. The first step characterizes the tax schedules, location decisions and labour allocation associated with an arbitrary vector w. The second step shows that there exists a unique vector w that clears the regional labour markets.

The firms, knowing the wage rates, can anticipate the quantity of labour that they would hire if they located in a given region. Specifically, the quantity of labour demanded by a firm that locates in i is

$$\widetilde{n}_i(\theta_i, w_i) = \arg\max_n \left[\left(\frac{1}{1-\alpha}\right) \theta_i^\alpha n^{1-\alpha} - w_i n \right] = \theta_i \left(\frac{1}{w_i}\right)^{1/\alpha}$$

The firm is also able to anticipate its output and after-tax profits in that region. These are, respectively,

$$\widetilde{y}_i(\theta_i, w_i) = \left(\frac{1}{1-\alpha}\right) \theta_i \left(\frac{1}{w_i}\right)^{(1-\alpha)/\alpha} \tag{8}$$

$$\widetilde{\pi}_i(\theta_i, w_i, t_i) = \alpha(1-t_i)\widetilde{y}_i(\theta_i, w_i)$$

The location chosen by any given firm is completely determined by its anticipated output in the various regions:

Lemma 2 *Assume that governments can tax the incomes of domestic residents in a lump-sum fashion. Assume that a firm of type (θ,γ) believes that it can hire as much labour as it wants at the wages contained in w. Suppose that this firm would produce its highest output if it located if region i and that it would produce its next highest output if it located in region h. Let $t^*(\theta,\gamma)$ be the vector of equilibrium tax rates. Then, in any equilibrium,*

1. *The firm locates in a region in which it attains maximal output.*

2. The firm's after-tax profits satisfy the conditions

$$\widetilde{y}_h \leq \widetilde{\pi}_i(\theta_i, w_i, t_i^*) \leq \widetilde{y}_i$$

$$\widetilde{\pi}_j(\theta_j, w_j, t_j^*) \leq \widetilde{\pi}_i(\theta_i, w_i, t_i^*) \qquad \textit{for all } j \neq i$$

$$\widetilde{\pi}_j(\theta_j, w_j, t_j^*) = \widetilde{\pi}_i(\theta_i, w_i, t_i^*) \qquad \textit{for some } j \neq i$$

3. The tax rate offered by region i satisfies the condition

$$1 - \frac{1}{\alpha} \leq t_i^*(\theta, \gamma) \leq 1 - \frac{\widetilde{y}_h}{\alpha \widetilde{y}_i}$$

The firm attains its highest after-tax profits in region i and chooses to locate there, but some other region offers the firm the same after-tax profits. (Otherwise, region i would increase its tax rate.) The matching offer might or might not be made by region h; but in either case, it must be sufficiently high that region h has no incentive to improve its offer enough to induce the firm to locate there. Region h has no such incentive if $\widetilde{\pi}_i$ is at least as great as $\widetilde{y}_h$, and this consideration provides the lower bound on $\widetilde{\pi}_i$. Similarly, region i has no incentive to decrease its offer, driving the firm away, if $\widetilde{\pi}_i$ is no greater than $\widetilde{y}_i$.

There are multiple equilibria associated with any wage vector w, and the tax rate actually paid by a firm of a given type varies across equilibria. The *lowest* possible tax rate is always so low that $\widetilde{\pi}_i$ is equal to $\widetilde{y}_i$, implying that the firm is being subsidized by an amount equal to labour's share of output. The *highest* possible tax rate depends upon the firm's next best output. If $\widetilde{y}_h$ is nearly as large as $\widetilde{y}_i$, the firm's lowest possible after-tax profits are nearly equal to $\widetilde{y}_i$, again implying a very high level of subsidization. On the other hand, if $\widetilde{y}_h$ is nearly zero—if the firm has no credible alternative locations—the firm's lowest possible after-tax profits are nearly zero.

The market-clearing wage vector exists and is unique. Once this vector is known, the remaining properties of equilibrium follow directly from the previous lemma.

Theorem 2 *If the governments can tax the incomes of domestic residents in a lump-sum fashion, an equilibrium exists. The market-clearing wages are the same in every equilibrium, and L and n are measurably identical to $L(k^*)$ and $\widehat{n}(k^*)$. Every equilibrium allocation is Pareto optimal.*

The bargaining model shows that tax competition does not necessarily lead to a misallocation of resources. The assumption about the scope of the lump-sum tax is important but not decisive. Section 6 relaxes this assumption.

5 Discussion

Models of tax competition typically assume that each government taxes every firm's profits at the same rate. Each government sets its tax rate to maximize the welfare of its own region, taking the other governments' tax rates as given. An equilibrium is characterized by a tax profile such that no government can raise the welfare of its own region by unilaterally deviating from the tax profile.

This section briefly sets out a two-region fixed-rate model that incorporates the assumptions set out in section 2. It is almost identical to the model described by Burbidge, Cuff and Leach (2005, henceforth BCL), and a number of their results are presented here without proof. The implications of the fixed-rate and bargaining models are then compared.

Let the scalar t_i be the tax rate paid by every firm that locates in region i, and let t_1 and t_2 be given. Let the allocation of firms across regions is given by $L(k)$ for some vector $k \equiv (k_1, k_2)$. Then every firm locates in the region in which its after-tax profits are greatest when k satisfies the conditions

$$k_i = (1 - t_i)\left(\frac{N_i}{Z_i(k)}\right)^{1-\alpha} \qquad i = 1, 2 \tag{9}$$

Let $\widehat{k}(t_1, t_2)$ be the solution to this equation system. For any given pair of tax rates, $L_i(\widehat{k})$ is the set of firms that locate in region i and $Y_i(\widehat{k})$ is the aggregate output of that region.

In a two-region economy the allocation of firms is determined by $\kappa \equiv k_2/k_1$, with an increase in κ shifting firms from region 1 to region 2. Furthermore, κ is related to the tax rates in a very simple fashion. Combining the two equations in (9) and using the linear homogeneity of $Z_i(k)$ gives

$$\kappa \left(\frac{Z_2(1, \kappa)}{Z_1(1, \kappa)}\right)^{1-\alpha} = \tau \left(\frac{N_2}{N_1}\right)^{1-\alpha} \tag{10}$$

where

$$\tau \equiv \frac{1 - t_2}{1 - t_1}$$

Then:

1. Comparing (9) with (2) shows that $\widehat{k}(0,0)$ is equal to k^*. By (10), κ takes the same value whenever the two tax rates are equal, implying

$$\kappa = \frac{\widehat{k}_2(t,t)}{\widehat{k}_1(t,t)} = \frac{\widehat{k}_2(0,0)}{\widehat{k}_1(0,0)} = \frac{k_2^*}{k_1^*}$$

   Thus, (L,n) maximizes total output whenever t_1 and t_2 are equal.

2. Since Z_2 is non-decreasing in k_2 and Z_1 is non-increasing in k_2, there is a unique κ associated with each τ. An increase in τ causes κ to rise.

Thus, an increase in one region's tax rate reduces that region's output and raises the other region's output. Total output rises (falls) if the increase in the tax rate pushes τ towards (away from) one.

Assume, as in the bargaining model, that each government uses a lump-sum tax to optimally divide the region's resources between the public and private goods. Government i's objective when it chooses t_i is then the maximization of its resources R_i, as defined by (5). An equilibrium is a pair (t_1^*, t_2^*) that satisfies the conditions

$$\frac{\partial R_i}{\partial t_i} = 0 \qquad i = 1,2$$

5.1 Pre-Commitment

The standard model of tax competition imagines that, when each government chooses its tax rate, it anticipates both the division of the firms between regions and the impact of this division upon wage rates. That is, it commits itself to a particular policy with full knowledge of the ultimate consequences of that decision. A bargaining model with the same properties could be constructed. Government i would choose a non-linear tax schedule $t_i : \Theta \times \Gamma \to \mathbb{R}$. The equilibrium would consist of a list of tax schedules such that no government, correctly anticipating the movement of firms and the adjustment of the market-clearing wage rates, could increase its own resources by unilaterally changing its own schedule.

It is not evident, however, that this kind of model would accurately reflect the behaviour of governments. Suppose that the Minister of Industry for some government meets the President of General Motors, who tells him, "We'd like to build our new plant here, but there's always Tennessee..." A model that sets out non-linear tax schedules imagines that the government's response to this kind of initiative is pre-determined. The Minister, handcuffed by policy, can do no more than explain his government's tax policy to the company president. Our belief is that the interaction between the two would not be so mechanical, that the Minister would make every possible effort to influence the location of the new plant. The Minister would negotiate with the President until they concluded a deal or decided that no deal was possible.

The Minister would also find that General Motors is not the only company that comes calling. Our model imagines that he treats each new opportunity in the same way, with negotiations continuing until a deal is made or a deal becomes impossible. The offer made to each firm is based on that firm's economic potential, rather than being part of an overarching strategy. That potential is evaluated using a given wage rate, because any one firm's location has no impact on the regional wage rates.

5.2 The Terms of Trade Effect

A fundamental question in tax competition is whether the profits tax will be positive or negative. In any fixed-rate model the answer to this question hinges upon Hamada's (1966) terms of trade effect. Consider an economy with two regions, and let J be the net transfer of after-tax profits from region 2 to region 1:

$$J \equiv \alpha \left(\int_{L_2} \gamma_1 (1 - t_2) y_2 dF - \int_{L_1} \gamma_2 (1 - t_1) y_1 dF \right)$$

Hamada showed that, if capital is homogeneous, the sign of each region's tax rate is entirely determined by the sign of J:

$$\text{sign}(J) = -\text{sign}(t_1) = \text{sign}(t_2)$$

If J is positive, so that region 1 is a net recipient of after-tax profits, region 1 will choose to subsidize capital. Its subsidization of capital will drive up the economy-wide after-tax return to capital, and since region 1 is a net recipient of profits, the increased return to capital will be beneficial to it. Region 2 will tax capital to drive down capital's economy-wide after-tax return, as a lower return to capital is beneficial to a region that pays out more after-tax profits than it receives.

The BCL model differs from the standard tax competition model in that it assumes that each unit of capital is embodied in a firm, and that the firms can earn locational rents. The existence of these locational rents pushes up the rates at which profits are taxed. Each government can capture rents that would otherwise accrue to foreigners by levying a positive profits tax, and the government's desire to capture these rents might ultimately determine the signs of the tax rates. Indeed, BCL show that the equilibrium tax rate is positive in any symmetric economy in which there is a positive measure of firms for which θ_1 and θ_2 are not equal. The terms of trade effect is still present in this model, but in a weaker form:

$$\mathrm{sign}(J) = \mathrm{sign}(t_2 - t_1)$$

That is, the terms of trade effect determines the relative sizes of the tax rates but not their signs.

By contrast, nothing resembling the terms of trade effect arises in the bargaining model. The government's offer to any given firm is not dependent upon the success or failure of the offers made to other firms, and hence cannot depend upon the equilibrium net transfer of profits.

The importance of the terms of trade effect in fixed-rate models implies that the distribution of ownership plays a critical role in determining the equilibrium tax rates. In the symmetric equilibrium in the BCL model, for example, increasing each region's ownership of the firms that are relatively productive in that region always leads to a decline in the equilibrium tax rate. (The part of after-tax profits that accrues to foreigners is smaller, so there is less incentive to tax profits.) Since the terms of trade effect has no role in the bargaining model, it is not surprising that the distribution of ownership has no impact on the equilibrium tax rates in that model.

If the terms of trade effect does not determine the signs of the tax rates in the bargaining model, what does? Lemma 3 shows that each firm's equilibrium after-tax profits could be as high as its maximal output or as low as its second highest output. If the firm's after-tax profits are equal to its maximal output, the firm is receiving a subsidy equal to the wage income that it generates. However, if its after-tax profits are equal to its second highest output, the firm is subsidized if

$$\widetilde{y}_h > \alpha \widetilde{y}_i$$

where $\widetilde{y}_i$ and $\widetilde{y}_h$ are its highest and second highest outputs, and it is taxed if

$$\widetilde{y}_h < \alpha \widetilde{y}_i$$

The equilibrium tax rate is certainly negative if the first inequality holds, but the equilibrium tax rate can be either positive or negative if the second inequality holds.

5.3 The Role of Locational Rents

Locational rents play a decisive role in both the bargaining model and the BCL model. BCL show that, in a symmetric equilibrium, the tax rate is positive whenever locational rents exist. Eliminating the locational rents, by squeezing the distribution F so that θ_1 and θ_2 converge for every firm, drives the tax rate to zero.

Eliminating the locational rents in the bargaining model can have even more extreme implications for the tax rate, but to derive these implications, the manner in which the distribution is altered must be exactly specified. Suppose that there are I regions, and for any given firm, let i and h be the regions in which the firm's output would be highest and second highest. Imagine that each firm's θ is altered by replacing θ_h with

$$\theta'_h \equiv \lambda\theta_h + (1-\lambda)\theta_i k^*_i / k^*_h$$

where λ is a constant between 0 and 1. That is, there is no change to any firm's productivity in its best location, but there is an improvement in every firm's productivity in its second best location. (Recall that $\widehat{y}_i > \widehat{y}_h$ implies $\theta_i k^*_i > \theta_h k^*_h$.) Altering the distribution in this fashion will be called a *compression of the locational rents*.

Compressing the locational rents does not induce any firm to switch regions, so there is no change in any region's aggregate productivity Z_j or its wage rate w_j. However, every firm's second highest output rises toward its highest output, partly because θ_h rises and partly because $\widetilde{n}_h(\theta_h, w_h)$ rises.

Successive compressions of the locational rents drive $\widetilde{y}_h$ toward $\widetilde{y}_i$ for every firm. Since $\widetilde{y}_h$ and $\widetilde{y}_i$ are the firm's minimum and maximum after-tax profits, successive compressions raise the firm's minimum after-tax profits toward its maximum after-tax profits. In the limit, when each firm's highest and second highest outputs are arbitrarily close, the governments have no bargaining power and must offer each firm a subsidy equal to labour's share of the firm's output.

In the BCL model, squeezing the locational rents from the economy reduces the tax rate to zero. In the bargaining model, squeezing the locational rents causes every firm to be subsidized. In the limit, every firm's after-tax profits are equal to its output.

6 The Lump-Sum Tax

It has been assumed that the government is able to levy lump-sum taxes on wages and on the domestic residents' share of the after-tax profits of all firms. The finding that the bargaining equilibrium gives rise to a Pareto optimal allocation relies on this assumption. A bargaining equilibrium in which every firm receives a subsidy equal to its wage bill is possible, and in that equilibrium, the government must appropriate all of the residents' wage income to pay the required subsidies. Once this transfer has been made, the residents' income consists only of their share of after-tax profits. The government is only able to provide a positive quantity of the public good if it is able to tax these profits. In light of this observation, what are the welfare implications of reducing the scope of the lump-sum tax?

One might also speculate that the scope of the lump-sum tax influences the nature of the bargains between governments and firms. If a government were unable to impose a lump-sum tax on the domestic residents' share of after-tax profits, no public goods would be provided in a region in which every firm received a subsidy equal to its wage bill. Would an equilibrium with such large subsidies continue to exist?

These questions are examined here by eliminating the lump-sum tax in a stepwise fashion. It is assumed first that only wages can be taxed in a lump-sum fashion, and then that lump-sum taxation is impossible. These changes have quite different (and perhaps surprising) effects on the equilibrium.

If the scope of the lump-sum tax is limited, some of the region's resources cannot be appropriated by the government and therefore cannot be allocated to the provision of the public good. Let R_i^g be the part of region i's resources that can be allocated to the public good, and let R_i^c be the part that cannot be allocated to it. The bargaining equilibrium examined below involves two stages. The first stage determines each firm's location, use of labour, and tax rate. The vectors $R^c \equiv (R_1^c, ..., R_I^c)$ and $R^g \equiv (R_1^g, ..., R_I^g)$ are implied by these values. The second stage determines c and g.

The government's optimization problem in the second stage is

$$\begin{aligned} &\max_{c_i, g_i} S_i = s_i(c_i, g_i) \\ \text{s.t. } &c_i + g_i = R_i^c + R_i^g \\ &g_i \leq R_i^g \end{aligned}$$

Let the solution to this problem be the functions $c_i(R_i^c, R_i^g)$ and $g_i(R_i^c, R_i^g)$, and let the associated maximum value function be $S_i(R_i^c, R_i^g)$. By the envelope theorem,

$$\frac{\partial S_i}{\partial R_i^c} = \frac{\partial s_i(c_i(R_i^c, R_i^g), g_i(R_i^c, R_i^g))}{\partial c_i} \tag{11}$$

$$\frac{\partial S_i}{\partial R_i^g} = \frac{\partial s_i(c_i(R_i^c, R_i^g), g_i(R_i^c, R_i^g))}{\partial g_i} \tag{12}$$

These partial derivatives are equal if the inequality constraint (in the optimization problem above) is not binding, and are unequal only if the inequality constraint is binding. Let the marginal rate of substitution MRS_i be the value of a unit of public goods, measured in units of private good, *evaluated at the optimum*:

$$MRS_i \equiv \frac{\partial s_i(c_i(R_i^c, R_i^g), g_i(R_i^c, R_i^g))}{\partial g_i} \Bigg/ \frac{\partial s_i(c_i(R_i^c, R_i^g), g_i(R_i^c, R_i^g))}{\partial c_i}$$

Then, (11) and (12) imply that MRS_i is also the value of another unit of R_i^g measured in units of R_i^c:

$$MRS_i = \frac{\partial S_i}{\partial R_i^g} \Bigg/ \frac{\partial S_i}{\partial R_i^c}$$

Since both partial derivatives are continuous functions of R_i^c and R_i^g, MRS_i is a continuous function of the same variables:

$$MRS_i = \psi_i(R_i^c, R_i^g)$$

By construction, each element of the vector $MRS \equiv (MRS_1, ..., MRS_I)$ is bounded below by 1. It is assumed henceforth that each element of MRS is bounded above by a positive finite number b.

Now consider the first stage of the equilibrium. The government recognizes that any one firm's location decision will have no impact upon the region's wage rate, and likewise, it recognizes that that decision will have no impact upon the relative values of public and private goods in its region. Thus, the first-stage equilibrium consists of the locations L, the labour allocation n, the taxes t, the wages w, and the marginal rates of substitution MRS such that:

1. Given L and w, no firm could raise its (pre-tax) profits by deviating from n.

2. Given w and t, no firm could raise its after-tax profits by deviating from L.

3. Let ρ_i be the increase in region i's resources, measured in units of R_i^c, generated by a firm of type (θ, γ). Given MRS, for each (θ, γ), no government j can increase its resources ρ_j by deviating from $t(\theta, \gamma)$.

4. The labour allocation n clears the labour market in every region.

5. If R^c and R^g are the resource vectors implied by L, n and t, then MRS satisfies the condition

$$MRS_i = \psi_i(R_i^c, R_i^g)$$

   for all i.

The remainder of this section examines the stage 1 equilibrium under alternative assumptions about lump-sum taxation.

6.1 Only Wages are Subject to Lump-Sum Taxation

Under this assumption, the government can finance the public good from either the wage tax or the profits tax, implying

$$R_i^g = (1-\alpha)Y_i + \alpha \int_{L_i(k)} t_i y_i dF = Y_i - \int_{L_i(k)} \pi_i dF$$

$$R_i^c = \sum_{j=1}^{I} \left(\int_{L_j(k)} \gamma_i \pi_j dF \right)$$

The contribution of a firm of type (θ, γ) to region i's resources, measured in units of R_i^c, is

$$\rho_i = \begin{cases} (y_i - \pi_i)\, MRS_i + \gamma_i \pi_i & \text{if the firm locates in } i \\ \gamma_i \pi_j & \text{if the firm locates in } j \ (j \neq i) \end{cases}$$

This equation replaces (7) in the determination of the equilibrium offers, but otherwise, the characterization of equilibrium proceeds in much the same manner.

Lemma 3 *Assume that the wages of domestic residents can be taxed in a lump-sum fashion. Assume that a firm of type (θ, γ) believes that it can hire as much labour as it wants at the wages contained in w. Suppose that the firm would produce its highest output if it located in region i and that it would produce its next highest output if it located in region h. Let $t^*(\theta, \gamma)$ be the vector of equilibrium tax rates. Then results 1–3 of Lemma 2 hold in any equilibrium.*

The range of possible after-tax profits for each firm is bounded by the firm's highest and second highest outputs, exactly as it was when all of income was subject to the lump-sum tax. The government of region i is willing to offer the same subsides when MRS_i is high—when subsidizing firms has a high social opportunity cost—as when it is low. The problem that faces the government when MRS_i is high is that R_i^g is low relative to R_i^c. Since the resources R_i^g are derived entirely from firms that locate within the region, and since any reduction in the after-tax profits offered to the firm will (in equilibrium) induce the firm to locate elsewhere, the government is unwilling to moderate its offer to the firm.

Theorem 3 *Assume that the wages of domestic residents can be taxed in a lump-sum fashion. Then an equilibrium exists. The market-clearing wages are the same in every equilibrium, and L and n are measurably identical across equilibria. Also,*

1. *(L, n) is measurably identical to $(L(k^*), \widehat{n}(k^*))$.*
2. *There exists an equilibrium in which no units of the public good are provided by any government.*

3. *An equilibrium that gives rise to a Pareto optimal allocation exists under some specifications of the model.*

Since the range of offers that can be made to each firm does not change when the scope of the lump-sum tax is restricted, neither does the actual location of the firm. The collection L is measurably identical to $L(k^*)$. With a measurably identical set of firms in each region, the same wage rate clears the labour market in each region, and the distribution of labour across firms is measurably identical to $\widehat{n}(k^*)$. Consequently, (L, n) maximizes total output even under the more restrictive tax assumption.

Every equilibrium satisfies the Pareto optimality conditions P1 and P2, but there is no guarantee that P3 will be satisfied. Indeed, an equilibrium always exists in which any or all of the governments provide no public goods at all. If a government makes the highest possible offer to every firm, all of the wages must be appropriated to pay the subsidies—that is,

$$\alpha \int_{L_i(k)} t_i y_i dF = -(1-\alpha)Y_i$$

so that R_i^g is equal to zero. Nevertheless, there are some specifications of the model under which P3 is also satisfied, so that the equilibrium allocation is Pareto optimal. Suppose that each government makes the smallest possible offer. If each firm's second highest output is small relative to its highest output, the government will be able to collect taxes from every firm, so that R_i^g exceeds $(1-\alpha)Y_i$. There are specifications of the social welfare function under which a government equipped with these resources will be able to provide the optimal quantity of public goods.

These results are the reverse of a common representation of Hamada's (1966) tax competition model. In that model, the existence of a lump-sum wage tax is commonly assumed to ensure the optimal provision of public goods, while the "terms of trade" effect causes capital to be misallocated across regions. Our findings are that firms (which embody the available capital) are correctly allocated across regions, but that the optimal provision of public goods is not assured.[5]

[5] Arguably, this characterization of the Hamada model does not take seriously the limit on lump-sum taxation. In that model, one region subsidizes firms if the other region taxes firms. The resources that the subsidizing region can devote to the provision of the public good are no greater than (in our terminology) $(1-\alpha)Y_i$. If the subsidies are

6.2 Eliminating the Lump-Sum Tax

Wilson (1999) studies a fixed-rate tax competition model in which the profits tax is the only available tax. He finds that, if the regions are not identical, the equilibrium tax rates distort both the division of capital between regions and the division of a region's resources between the private and public goods. If the regions are identical, each region will underprovide the public good.[6]

If the same assumption is imposed in the bargaining model,

$$R_i^g = \alpha \int_{L_i(k)} t_i y_i dF = \alpha Y_i - \int_{L_i(k)} \pi_i dF$$

$$R_i^c = (1-\alpha)Y_i + \sum_{j=1}^{I} \left(\int_{L_j(k)} \gamma_i \pi_j dF \right)$$

The value of the resources gained by attracting a single firm to the region is

$$\rho_i = \begin{cases} (\alpha y_i - \pi_i)\, MRS_i + (1-\alpha) y_i + \gamma_i \pi_i & \text{if the firm locates in } i \\ \gamma_i \pi_j & \text{if the firm locates in } j \ (j \neq i) \end{cases}$$

The locations of the firms varies with the vector MRS under this specification of ρ_i.

Lemma 4 *Assume that the profits tax is the only available tax. Consider a firm of type (θ, γ), and assume that the firm believes that it can hire as much labour as it wants at the wages w. Let $t^*(\theta, \gamma)$ be the vector of equilibrium tax rates. Define the variables*

$$x_j \equiv \widetilde{y}_j \left(\alpha + \frac{1-\alpha}{MRS_j} \right) \qquad \text{for all } j \in \mathcal{I}$$

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sufficiently large or the desire for the public good is sufficiently strong, the government will be unable to raise the revenue needed to drive MRS_i down to one. Hence, a more accurate characterization of the Hamada model would be that capital is generally misallocated (it is not misallocated in knife-edge circumstances, notably when the regions are identical), and that the optimal provision of public goods is not assured.

[6] Optimal provision of the public good requires equality between MRS, the value of another unit of public goods measured in private goods, and MRT, the amount of private goods that must be given up to produce another unit of public good. The latter amount is also the quantity of private good taken away as taxes when the government provides another unit of public good. In equilibrium, the government recognizes two costs to providing another unit of public goods: MRT and the private consumption lost by the region (but not the economy as a whole) when higher tax rates cause capital to abandon the region. The government chooses the quantity of public goods that equates the sum of these costs to MRS, leading to underprovision.

Then, in equilibrium, the firm locates in region i only if

$$x_i = \max [x_1, ..., x_I]$$

Furthermore, the firm's after-tax profits satisfy the condition

$$x_h \leq \widetilde{\pi}(\theta_i, w_i, t_i^*) \leq x_i \tag{13}$$

where

$$x_h = \max [x_1, ..., x_{i-1}, x_{i+1}, .., x_I]$$

The variable x_j is the *maximal* value of the firm's output, measured in public goods, conditional on the firm locating in region j. There would be αy_j units of profits, and each unit of profits—whether or not it is actually taken as taxes by the government—is valued at one unit of public goods. There would also be $(1-\alpha)y_j$ units of wages. Since wages are not taxable, each unit of wages is valued at one unit of private goods, which is worth only $1/MRS_j$ units of public goods.

Total output is maximized when no firm can increase its own output by moving to another region. When the profits tax is the only tax, this condition is satisfied if and only if every region has the same marginal rate of substitution. The only robust equilibrium satisfying this condition would be one in which each region raises enough revenue to provide the optimal quantity of public goods. Such an equilibrium might exist, but does not necessarily exist.

The taxes paid by each firm are (at last) influenced by the government's need for revenue. The *maximum* value of a firm's after-tax profits $\widetilde{\pi}_i$ is x_i, which is bounded below by $\alpha\widetilde{y}_i$. Since the firm is neither taxed nor subsidized when $\widetilde{\pi}_i$ is equal to $\alpha\widetilde{y}_i$, a firm that is earning the maximum after-tax profits is necessarily being subsidized. The subsidy shrinks as the region's marginal rate of substitution rises, but remains positive. A firm that is earning the *minimum* after-tax profits could be either paying taxes or receiving a subsidy. Specifically, the firm is paying taxes if

$$\widetilde{y}_h \left(1 + \left(\frac{1-\alpha}{\alpha}\right) \frac{1}{MRS_h}\right) < \widetilde{y}_i$$

The firm's tax rate rises as its second best output falls, and it also rises as the region in which it attains its second best output becomes more desparate for revenue. The firm is receiving a subsidy if the inequality is reversed. The factors that lead to higher tax rates also lead to lower subsidy rates.

Proving existence is more difficult in this model than in the earlier ones, so only a restricted set of equilibria—credible equilibria—will be considered.

Definition 3 *Let $\pi^*(\theta,\gamma)$ be the equilibrium after-tax profits of a firm of type (θ,γ). Let $\mathcal{J}(\theta,\gamma) \subset \mathcal{I}$ be the set containing the identities of the regions that offer after-tax profits of $\pi^*(\theta,\gamma)$ to firms of type (θ,γ). The equilibrium is credible if, for every (θ,γ) and every $j \in \mathcal{J}(\theta,\gamma)$, the resources that region j would obtain from a firm of type (θ,γ) are at least as great when the firm accepts region j's offer as when it rejects region j's offer.*

To understand the impact of the credibility assumption, suppose that firms of a particular type (θ,γ) locate in region i. Region i offered after-tax profits of $\pi^*(\theta,\gamma)$ to these firms; but at least one other region must have made the same offer, since otherwise region i's equilibrium offer would have been lower. Credibility requires that every region j that did make the same offer does not prefer its offer to be rejected. This condition is satisfied if and only if

$$\pi^*(\theta,\gamma) \leq x_j$$

Combining this inequality with (13) leads to the following conclusions: the second best offer is made by region h, and $\pi^*(\theta,\gamma)$ is equal to x_h. Credibility implies that each firm's after-tax profits are uniquely determined.

Theorem 4 *Let a_i be a lower bound of $\psi_i(R_i^c, 0)$, and define the lower bound*

$$a^\circ \equiv \min[a_1, ..., a_I]$$

If the profits tax is the only tax, a credible equilibrium exists for all sufficiently large values of a°. Also,

1. *In any two credible equilibria with the same MRS, the only differences are in L and n, and these differences are not measurable. However, MRS might not be the same in every credible equilibrium.*

2. *Total output is maximized if and only if every region has the same marginal rate of substitution.*

3. *A credible equilibrium that gives rise to a Pareto optimal allocation exists under some specifications of the model.*

A credible equilibrium exists for sufficiently large values of a°, but whether there exist multiple equilibria that are distinctly different remains an open question. The central (and unresolved) issue is whether all credible equilibria have the same vector MRS; if they do, the differences across equilibria are not measurable.

The efficiency properties of a credible equilibrium are also uncertain. The first property of Pareto optimal allocations, P1, is always satisfied. Almost all equilibria fall into one of two categories: both P2 and P3 are satisfied, or neither is satisfied. The only exceptions are knife-edge equilibria in which every region underprovides the public good but has the same marginal rate of substitution, so that P3 is violated but P2 is satisfied. Failure to maximize output follows not from the inability of regions to provide the optimal quantity of public goods, but from disparity in their provision of public goods.

The fixed-rate tax competition model does not generate a Pareto optimal allocation, but the bargaining model will sometimes to do so. This difference arises in part because the firms in the bargaining model are less mobile than the firms in the fixed-rate model. Capital in the fixed-rate model is truly mobile, in the sense that it can move to any region and obtain the same after-tax return in every region. Each firm in the bargaining model is mobile in the sense that it can locate in any region, but it is not necessarily mobile in the sense that it can move between regions without a significant loss of productivity. The extent of these productivity losses largely explains the difference in results. If each firm's second best option is almost as good as its best option, the governments will be forced to subsidize some firms and collect only small amounts of revenue from others. The net revenue collected by a government is likely to be so small that public goods will be underprovided. However, if each firm's second best option is much worse than its best option, each government will be able to collect taxes from almost every firm. If each region's preferences for the public good are not very strong, the net revenues might be large enough to allow each region to provide the optimal quantity of public goods. Thus, relatively high mobility (in the sense of movement without significant loss) gives rise to allocations that are not Pareto optimal, as in Wilson (1999) while relatively low mobility gives rise to Pareto optimal allocations.

7 Conclusions

The standard model of tax competition assumes that each government taxes every firm's profits at the same rate. Regional differences in productivity or in endowments lead to an equilibrium in which there is a range of tax rates. Resources are misallocated, with the low tax regions using too much capital and the high tax regions using too little. By contrast, the model set out above assumes that the governments negotiate separately with every firm. The predictions of the model depend upon the nature of the supplementary taxes in the economy. If all of income is subject to a lump-sum tax, a Pareto optimal allocation is reached; if only wages are subject to a lump-sum tax, there can be underprovision of the public good but capital is optimally allocated.

There is the potential for very large subsidies to be paid to the firm under either lump-sum tax. In the most extreme equilibrium, all of the wages paid to the workers in a region are appropriated by the government to subsidize the firms, and all of the after-tax income is earned as profits. There are only two assumptions under which such an extreme outcome makes sense. One is that the residents of the region are identical, with each resident supplying the same amount of labour and owning equal shares in each of the firms. The appropriated wages are then simply returned to the workers that earned them in a different form, namely after-tax profits. The other assumption is that the residents are not identical—some earn mostly wages and others earn mostly profits—but that there is a hidden system of lump-sum transfers that returns the economy to some preferred distribution. While either assumption make sense of the outcome, both assumptions are very strong. We suspect that a complete understanding of subsidies requires the abandonment of a social welfare function in which only aggregates matter.

A Appendix

Let k° be the solution to the equation system

$$k_i = \eta_i \left(\frac{1}{Z_i(k)} \right)^{1-\alpha} \qquad \text{for all } i \in \mathcal{I} \tag{14}$$

where each η_i is a positive constant. Then k° has these properties.

FP1. The vector k° exists and is unique, and k_i° and $Z_i(k^\circ)$ are strictly positive for each $i \in \mathcal{I}$.

FP2. An increase in η_j causes k_j° and each ratio k_j° / k_i° $(i \in \mathcal{I}, i \neq j)$ to rise.

Proof of FP1: Let k° be a fixed point of (14), and let s° be the vector such that

$$s_i^\circ = \frac{k_i^\circ}{\sum_{l=1}^{I} k_l^\circ} \qquad \text{for all } i \in \mathcal{I}$$

Then

$$s_i^\circ = \frac{\eta_i Z_i(k^\circ)^{\alpha-1}}{\sum_{l=1}^{I} \eta_l Z_l(k^\circ)^{\alpha-1}} = \frac{\eta_i Z_i(s^\circ)^{\alpha-1}}{\sum_{l=1}^{I} \eta_l Z_l(s^\circ)^{\alpha-1}}$$

(The second equality follows from the observation that each firm's location depends only upon the relative sizes of the elements of k°, and hence that $Z_i(k^\circ) = Z_i(\lambda k^\circ)$ for all positive λ.) Alternatively, s° is the fixed point of the equation system

$$s_i = \frac{\eta_i Z_i(s)^{\alpha-1}}{\sum_{l=1}^{I} \eta_l Z_l(s)^{\alpha-1}} \qquad \text{for all } i \in \mathcal{I} \tag{15}$$

Every fixed point of (14) gives rise to a unique fixed point of (15). Likewise, each fixed point of (15) is associated with a unique fixed point of (14):

$$k_i^\circ = \eta_i Z_i(k^\circ)^{\alpha-1} = \eta_i Z_i(s^\circ)^{\alpha-1} \qquad \text{for all } i \in \mathcal{I}$$

Thus, k° exists and is unique if s° exists and is unique. The existence and uniqueness of s° are proved in turn:

1. The difficulty of proving the existence of the fixed point of (15) is that the right-hand side of (15) is not bounded or not defined if some k_i is zero. This problem is circumvented by considering the mapping

$$q_i^\epsilon(s) = \frac{\eta_i \left(Z_i(s) + \epsilon\right)^{\alpha-1}}{\sum_{l=1}^{I} \eta_l \left(Z_l(s) + \epsilon\right)^{\alpha-1}} \qquad \text{for all } i \in \mathcal{I}$$

Define the set

$$\mathbb{S} \equiv \left\{ s \in \mathbb{R}_+^I : \sum_{l=1}^{I} s_l = 1 \right\}$$

This set is non-empty, compact and convex. For each $\epsilon > 0$, the mapping $q^\epsilon : \mathbb{S} \to \mathbb{S}$ is well-defined even when some elements of s are zero. The assumptions on F ensure that q^ϵ is a continuous mapping from $\mathbb{S}$ into $\mathbb{S}$. Taken together, these conditions ensure the existence of a fixed point $s^\epsilon = (s_1^\epsilon, \cdots, s_I^\epsilon)$. Furthermore, the construction of the mapping ensures that s_i^ϵ is strictly positive for all i and all $\epsilon > 0$. Since s^ϵ is a fixed-point of q^ϵ, we have

$$s_i^\epsilon = \frac{\eta_i \left(Z_i(s^\epsilon) + \epsilon\right)^{\alpha-1}}{\sum_{l=1}^{I} \eta_l \left(Z_l(s^\epsilon) + \epsilon\right)^{\alpha-1}} \qquad \text{for all } i \in \mathcal{I}$$

Since $0 < s_i^\epsilon < 1$ for all i at each ϵ, the positive sequence $\{s^\epsilon\}$ is bounded. Therefore, there exists a subsequence of $\{s^\epsilon\}$ that must converge as $\epsilon \to 0$. For simplicity, assume that we choose this convergent subsequence right from the start so that $\{s^\epsilon\}$ itself converges to s° as $\epsilon \to 0$.

Inspection shows that no element of s° is negative, and since the elements of s° sum to 1, no element of s° can be greater than 1. To show that $0 < s_i^\circ < 1$ for all $i \in \mathcal{I}$, suppose that this condition is not satisfied. Then there exists a non-empty subset $\mathcal{D} \subset \mathcal{I}$ such that $s_i^\circ = 0$ for all $i \in \mathcal{D}$. Then we have $Z_i(s^\circ) = 0$ for all $i \in \mathcal{D}$ because every firm will be located in some region i' such that $s_{i'}^\circ > 0$. Furthermore, $Z_i(s^\epsilon) \to Z_i(s^\circ) = 0$ as $\epsilon \to 0$. $\sum_{l \in \mathcal{D}} s_l^\epsilon$ is expressed by

$$\sum_{l \in \mathcal{D}} s_l^\epsilon = \frac{\sum_{l \in \mathcal{D}} \eta_l \left(Z_l(s^\epsilon) + \epsilon\right)^{\alpha-1}}{\sum_{l \in \mathcal{D}} \eta_l \left(Z_l(s^\epsilon) + \epsilon\right)^{\alpha-1} + \sum_{l \notin \mathcal{D}} \eta_l \left(Z_l(s^\epsilon) + \epsilon\right)^{\alpha-1}}$$

Note that s^ϵ is a fixed point of q^ϵ, so $Z_l(s^\epsilon) + \epsilon$ is bounded and positive for all $l \notin \mathcal{D}$ at any ϵ. This implies that $\left\{\sum_{l \notin \mathcal{D}} \left(N_l/(Z_l(s^\epsilon) + \epsilon)\right)^{1-\alpha}\right\}$ is a bounded and positive sequence. Furthermore, $\sum_{l \notin \mathcal{D}} \eta_l \left(Z_l(s^\epsilon) + \epsilon\right)^{\alpha-1} \to \infty$ as $\epsilon \to 0$ because $Z_l(s^\epsilon) + \epsilon \to 0$ for all $l \in \mathcal{D}$ as $\epsilon \to 0$. Therefore, we have $\sum_{l \in \mathcal{D}} s_l^\epsilon \to 1$ as $\epsilon \to 0$. This contradicts $\sum_{l \in \mathcal{D}} s_l^\circ = 0$. It follows that $0 < s_i^\circ < 1$ for all i. Since s° is a fixed point of (15) and $0 < s_i^\circ < 1$ for all i, we have $Z_i(s^\circ) > 0$ for all i. Therefore, a fixed point k° (with $k_i^\circ > 0$ for all i) of (14) exists and $Z_i(k^\circ) = Z_i(s^\circ) > 0$ for all i

2. Assume that s° is not a unique fixed point, and let s^0 and s^1 be two of the fixed points. Let region a be the region in which the ratio s_i^0/s_i^1 is lowest. This ratio must be smaller than 1. (If it were not, every element of s^0 would be greater than the corresponding element of s^1. Since every fixed point has the property that $\sum_i s_i = 1$, at least one of the two vectors could not be a fixed point.) Then, for all $\theta \in \Theta$ and all $j \neq a$,

$$\theta_j \frac{s_j^1}{s_a^1} \leq \theta_j \frac{s_j^0}{s_a^0}, \tag{16}$$

Furthermore, the inequality must be strict for some j. Since a firm locates in region a if and only if $\theta_a > \theta_j(s_j/s_a)$ for all $j \neq a$, (16) implies that $Z_a(s^0) < Z_a(s^1)$. Then, using (15),

$$\frac{s_a^0}{s_a^1} = \left(\frac{Z_a(s^1)}{Z_a(s^0)}\right)^{1-\alpha} > 1$$

which contradicts the initial assumption that s_a^0/s_a^1 is smaller than 1. Thus, the fixed point s° must be unique. ■

Proof of FP2: This property is proved by demonstrating that any other outcome leads to a contradiction. If $Z_j(k^\circ)$ does not rise when η_j rises, k_j° must rise to satisfy the j^{th} equation in the system. However, if $Z_j(k^\circ)$ does not rise, there must be at least one element k_i° that rises by a greater proportion than k_j°. Let k_h° be the element that experiences the greatest proportionate increase. Since k_h° and $Z_h(k^\circ)$ both rise, the h^{th} equation in the system is not satisfied. Thus, $Z_j(k^\circ)$ must rise in response to the increase in η_j. If k_j° does not rise while $Z_j(k^\circ)$ rises, there must be at least one element k_i° that falls by a greater proportion than k_j°. Let k_h° be the element that experiences the greatest proportionate decline. Since both k_h° and $Z_h(k^\circ)$ fall, the h^{th} equation in the system is not satisfied. Thus, the rise in $Z_j(k^\circ)$ must be accompanied by an increase in k_j°. Let k_h° be the element of k° that experiences the greatest proportionate increase. If its proportionate increase is at least as great as that of k_j°, $Z_h(k^\circ)$ also rises, so that the h^{th} equation in the system cannot be satisfied. It follows that the proportionate increase in k_j° must be greater than the proportionate increase in any other element of k°. ■

Proof of Lemma 1: Since (2) is (14) with η_i set equal to $(N_i)^{1-\alpha}$, the first part of Lemma 1 follows immediately from FP1. To prove the second part of Lemma 1, suppose that L is not measurably identical to $L(k^*)$. Then, under L, there exists a compact set M_i of firms in some region i such that $F(M_i) > 0$ and such that, for some j, $k_j\theta_j > k_i\theta_i$ for all firms in M_i. It will be shown that moving a subset of these firms from region i to region j raises total output. Let $\mathcal{M}_i$ be the set of all subsets of M_i, and identify some (θ, γ) in the interior of M_i. Define a mapping $m : \mathbb{R}_+ \to \mathcal{M}_i$ such that (i) $m(0) = (\theta, \gamma)$, (ii) $m(x') \subset m(x)$ for all x' and x such that $x' < x$, and (iii) $C = F \circ m$ is continuous and differentiable at all $x \in \mathbb{R}_+$. The firms in the set $m(x)$ will be moved from region i to region j; $C(x)$ is their measure. For each $m(x)$, let $g(x)$ be the decline in region i's aggregate productivity when the firms are moved out of region i, and let $h(x)$ be the increase in region j's aggregate productivity when the firms are moved into that region. Since C is continuous and differentiable, g and h are continuous and differentiable. Assuming that labour is reallocated in accordance with (1), the movement of the firms causes total output to rise by

$$D(x;k^*) = \frac{1}{1-\alpha}\left[\left(Z_j(k^*)+h(x)\right)^{\alpha} N_j^{1-\alpha} - \left(Z_i(k^*)-g(x)\right)^{\alpha} N_i^{1-\alpha}\right]$$

Taking the first-order derivative of D with respect to x and evaluating it at $x=0$ gives

$$\begin{aligned} D'(0;k^*) &= \frac{1}{1-\alpha}\left[h'(0)Z_j^{\alpha-1}N_j^{1-\alpha} - g'(0)Z_i^{\alpha-1}N_i^{1-\alpha}\right] \\ &= \frac{1}{1-\alpha}\left[h'(0)k_j^* - g'(0)k_i^*\right] \end{aligned}$$

Since

$$\theta_j k_j^* > \theta_i k_i^*$$

for every firm that is moved between regions, this derivative is positive. That is, moving a small but positive measure of firms between the regions raises total output. Thus, L does not maximize total output if L is not measurably identical to $L(k^*)$. Since there is a well-defined maximum total output, and since it is not attained under any L that is not measurably identical to $L(k^*)$, it must be attained under each L that is measurably identical to $L(k^*)$.■

Proof of Theorem 1: P1 follows from Lemma 1. P2 follows from the facts that the marginal product of labour is equalized across firms within each region and that L is measurably identical to $L(k^*)$. Since s_i is strictly concave, increasing, and twice differentiable, P3 is the necessary and sufficient condition for (c_i, g_i) to maximize s_i subject to the constraint (4). Let $(c_i^*(R_i), g_i^*(R_i))$ be the solution to this maximization problem. Since s_i is strictly concave, strictly increasing, and twice differentiable, $s_i(c_i^*(R_i), g_i^*(R_i))$ is strictly increasing in R_i. Consequently, shifting resources from one region to another raises one region's welfare at the expense of the other region. Then any allocation of output that satisfies (3) can be part of a Pareto optimal allocation. ■

Proof of Lemma 2: Assume that, in equilibrium, a firm with characteristics (θ, γ) chooses to locate in region i, and assume that this firm's next best offer came from region m. The resources that region i extracts from the firm rise as the firm's after-tax profits fall, so region i will offer the smallest after-tax profits that induce the firm to locate there.

$$\widetilde{\pi}_i(\theta_i, w_i, t_i^*) = \widetilde{\pi}_m(\theta_m, w_m, t_m^*) \tag{17}$$

Also, since region i chooses to induce the firm to locate within its boundaries, the firm must contribute more to region i's resources by it locating there than it would by locating in region m:

$$\widetilde{y}_i(\theta_i, w_i) - (1-\gamma_i)\widetilde{\pi}_i(\theta_i, w_i, t_i^*) \geq \gamma_i \widetilde{\pi}_m(\theta_m, w_m, t_m^*) \tag{18}$$

The resources that region m gains from the firm must be at least as great when the firm locates in region i as when it locates in region m:

$$\gamma_m \widetilde{\pi}_i(\theta_i, w_i, t_i^*) \geq \widetilde{y}_m(\theta_m, w_m) - (1-\gamma_m)\widetilde{\pi}_m(\theta_m, w_m, t_m^*) \tag{19}$$

(If this condition did not hold, region m would be able to make an offer to the firm that induces the firm to locate in region m and increases that region's resources.) Conditions (17) and (18) imply

$$\widetilde{y}_i \geq \widetilde{\pi}_m(\theta_m, w_m, t_m^*) \tag{20}$$

while (17) and (19) imply

$$\widetilde{\pi}_i(\theta_i, w_i, t_i^*) \geq \widetilde{y}_m \tag{21}$$

Combining (20) and (21) shows that $\widetilde{y}_i$ is at least as large as $\widetilde{y}_m$. Regions i and m have no profitable deviation if (17), (20) and (21) are satisfied. Now consider some other region j. Assume that the region j could attract the firm by offering a tax rate t_j' such that

$$\widetilde{\pi}_j(\theta_j, w_j, t_j') = \widetilde{\pi}_i(\theta_i, w_i, t_i^*) \tag{22}$$

Then, there exist no profitable deviations for region j ($j \neq i$) if

$$\widetilde{y}_j(\theta_j, w_j) - (1-\gamma_j)\widetilde{\pi}_j(\theta_j, w_j, t_j') \leq \gamma_j \widetilde{\pi}_i(\theta_i, w_i, t_i^*) \tag{23}$$

Using (22), (20) and (23) implies that if the following condition holds, for all ($j \neq i$)

$$y_i \geq \widetilde{\pi}_i(\theta_i, w_i, t_i^*) \geq \widetilde{y}_j \tag{24}$$

then no region has incentive to deviate from its offer. Consequently, no other region j ($j \neq i$) has a profitable deviation if

$$\widetilde{\pi}_i(\theta_i, w_i, t_i^*) \geq \widetilde{y}_h \tag{25}$$

where $\widetilde{y}_h$ is the firm's second highest output. There is no requirement that regions h and m are the same region. Conditions (21) and (25) are the same if they are, and (25) is the tighter constraint if they are not. Thus,

$$\widetilde{y}_h \leq \widetilde{\pi}_i(\theta_i, w_i, t_i^*(\theta, \gamma)) \leq \widetilde{y}_i \tag{26}$$

implying

$$1 - \frac{1}{\alpha} \leq t_i^*(\theta, \gamma) \leq 1 - \frac{\widetilde{y}_h}{\alpha \widetilde{y}_i}$$

The firm's after-tax profits are equal to $\widetilde{y}_i$ at the lower tax rate and equal to $\widetilde{y}_h$ at the higher tax rate. By (26), the firm has located in a region in which it has the greatest possible output. ■

Proof of Theorem 2: The first step is to show that a vector of market-clearing wages exists, and that this vector is unique. Define the vector $\widetilde{k}(w) \equiv (\widetilde{k}_1(w), ..., \widetilde{k}_I(w))$, where

$$\widetilde{k}_i(w) = \left(\frac{1}{w_i}\right)^{(1-\alpha)/\alpha} \qquad \text{for all } i \in \mathcal{I}$$

Then, by (8), the firm's output in region i is

$$\widetilde{y}_i = \left(\frac{1}{1-\alpha}\right) \theta_i \widetilde{k}_i(w)$$

Lemma 2 shows that, under any wage vector w, a firm locates in the region in which its output is highest, so the firms' locations are given by $L(\widetilde{k}(w))$. The labour demand of a firm that locates in region i is

$$\widetilde{n}_i(\theta_i, w_i) = \theta_i \left(1/w_i\right)^{1/\alpha} = \theta_i \widetilde{k}_i(w)^{1/(1-\alpha)}$$

The aggregate demand for labour is found by integrating over the demands of the individual firms in the region:

$$N_i^D = \left(\int_{L_i(\widetilde{k}(w))} \theta_i dF\right) \widetilde{k}_i(w)^{1/(1-\alpha)} = Z_i(\widetilde{k}(w)) \widetilde{k}_i(w)^{1/(1-\alpha)}$$

The labour market clearing condition equates this demand to the supply of labour. This condition can be written as

$$\widetilde{k}_i(w) = \left[\frac{N_i}{Z_i(\widetilde{k}(w))}\right]^{1-\alpha} \tag{27}$$

Thus, a vector of market-clearing wages exists if and only if there exists a vector $\widetilde{k}(w)$ such that this condition is satisfied for all I markets. The required vector $\widetilde{k}(w)$ is simply a fixed point of (2), and by Lemma 1, this fixed point exists and is unique. Since $\widetilde{k}(w)$ is equal to k^* under the market-clearing vector w, $L(\widetilde{k}(w))$ and $L(k^*)$ are the same. Also, by (27), $\widetilde{n}(\theta_i, w_i)$ is the same as $\widehat{n}(k^*)$ under the market-clearing vector w. Thus, (L, n) is measurably identical to $(L(k^*), \widehat{n}(k^*))$ in any equilibrium. Now consider the issue of Pareto optimality. It has just been shown that P1 and P2 are satisfied. Since the governments use their lump-sum taxes to attain an optimal division of their resources between the public and private good, P3 is also satisfied. Therefore, any equilibrium allocation is Pareto optimal. ■

Proof of Lemma 3: The proof of Lemma 3 follows the same steps as the proof of Lemma 2, except that the government of any region i seeks to maximize ρ_i rather than r_i. Here, we present only the proof of the range of the equilibrium after-tax profits and the equilibrium location of firms. (18) and (19) are replaced with

$$[\widetilde{y}_i(\theta_i, w_i) - \widetilde{\pi}_i(\theta_i, w_i, t_i^*)]\, MRS_i + \gamma_i \widetilde{\pi}_i(\theta_i, w_i, t_i^*) \geq \gamma_i \widetilde{\pi}_m(\theta_m, w_m, t_m^*)$$

and

$$\gamma_m \widetilde{\pi}_i(\theta_i, w_i, t_i^*) \geq [\widetilde{y}_m(\theta_m, w_m) - \widetilde{\pi}_m(\theta_m, w_m, t_m^*)]\, MRS_m + \gamma_m \widetilde{\pi}_m(\theta_m, w_m, t_m^*)$$

where MRS_m is positive and bounded above by b.) Since $\widetilde{\pi}_i(\theta_i, w_i, t_i^*) = \widetilde{\pi}_m(\theta_m, w_m, t_m^*)$ in equilibrium, these inequalities yields

$$\widetilde{y}_i(\theta_i, w_i) \geq \widetilde{\pi}_i(\theta_i, w_i, t_i^*) \geq \widetilde{y}_m(\theta_m, w_m)$$

Likewise, (23) is replaced with

$$[\widetilde{y}_j(\theta_j, w_j) - \widetilde{\pi}_j(\theta_j, w_j, t_j)]\, MRS_j + \gamma_j \widetilde{\pi}_j(\theta_j, w_j, t_j) \leq \gamma_j \widetilde{\pi}_i(\theta_i, w_i, t_i^*) \tag{28}$$

for all j $(j \neq i)$. This inequality is simplified as

$$\widetilde{y}_j(\theta_j, w_j) \leq \widetilde{\pi}_j(\theta_j, w_j, t_j)$$

for all j $(j \neq i)$. if the following condition holds, for all $(j \neq i)$

$$\widetilde{y}_i \geq \widetilde{\pi}_i(\theta_i, w_i, t_i^*) \geq \widetilde{y}_j$$

then no region has incentive to deviate from its offer. Consequently, no other region j $(j \neq i)$ has a profitable deviation if

$$\widetilde{\pi}_i(\theta_i, w_i, t_i^*) \geq \widetilde{y}_h$$

where $\widetilde{y}_h$ is the firm's second highest output. ■

Proof of Theorem 3: Since every firm locates in a region where it can attain its highest output, the argument used in the proof of Theorem 2 can be used to show that (i) the wage vector w exists and is unique, and (ii) (L, n) is measurably identical to $(L(k^*), \widehat{n}(k^*))$ in any equilibrium. Item 1, the maximization of total output, follows immediately. Item 2 follows from the observation that there exists an equilibrium in which all wage income is used to subsidize the firms $(\widetilde{\pi}_i(\theta_i, w_i, t_i^*) = \widetilde{y}_i)$, so that the governments are unable to provide any public good. Item 3 follows from the observation that there can be an equilibrium in which no firms are subsidized, allowing each government to provide a quantity of public goods at least as great as the region's total wages. ■

Proof of Lemma 4: Lemma 4 is proved in the same way as Lemma 3, using the revised expression for ρ_i. ■

Proof of Theorem 4: Consider first the existence of a credible equilibrium. Define the variables

$$\mu_i \equiv \alpha + \frac{1-\alpha}{MRS_i} \qquad \text{for all } i \in \mathcal{I} \tag{29}$$

and let μ be the vector $(\mu_1, ...\mu_I)$. Suppose that, in equilibrium, a firm goes to region i only if

$$\theta_i k_i \geq \theta_j k_j \qquad \text{for all } j \in \mathcal{I}$$

Lemma 4 implies that, in equilibrium, a firm goes to region i only if

$$\theta_i \left(\frac{N_i}{Z_i(k)}\right) \mu_i \geq \theta_j \left(\frac{N_j}{Z_j(k)}\right) \mu_j \qquad \text{for all } j \in \mathcal{I}$$

Then an equilibrium distribution of firms across regions is described by $L(\widehat{k}(\mu))$, where $\widehat{k}(\mu)$ is the solution to the equation system

$$k_i = \left(\frac{N_i}{Z_i(k)}\right)^{1-\alpha} \mu_i \qquad \text{for all } i \in \mathcal{I} \tag{30}$$

By FP1, $\widehat{k}(\mu)$ exists and is unique for every strictly positive μ. The function $\widehat{k}(\mu)$ is continuous in its arguments. In any credible equilibrium, R_i^g and R_i^c are continuous functions of $\widehat{k}(\mu)$ and hence continuous functions of μ itself. Let these functions be $R_i^g(\mu)$ and $R_i^c(\mu)$. By FP1, $Z_i(\widehat{k}(\mu))$ is positive for every strictly positive vector μ, and hence $R_i^c(\mu)$ is positive for every strictly positive μ. $R_i^g(\mu)$ might be negative for some μ; but there is some $\overline{\epsilon}$ sufficiently small that $R_i^g(\mu)$ is positive whenever $\alpha < \mu_i < \alpha + \overline{\epsilon}$. To show this, let μ^ϵ be a vector in which μ_i^ϵ is equal to $\alpha + \epsilon$; and let μ^δ be identical to μ^ϵ except that μ_i^δ is set equal to $\alpha - \delta > 0$. By FP2,

$$\frac{\widehat{k}_i(\mu^\epsilon)}{\widehat{k}_j(\mu^\epsilon)} > \frac{\widehat{k}_i(\mu^\delta)}{\widehat{k}_j(\mu^\delta)}$$

for every j other than i, and hence

$$L_i(\widehat{k}_i(\mu^\delta)) \subset L_i(\widehat{k}_i(\mu^\epsilon))$$

Let the set $M(\epsilon)$ contain all of the elements of $L_i(\widehat{k}_i(\mu^\epsilon))$ that are not elements of $L_i(\widehat{k}_i(\mu^\delta))$. Then

$$R_i^g(\mu^\epsilon) = \int_{L_i(\widehat{k}_i(\mu^\delta))} (\alpha y_i - \pi_i) dF + \int_{M(\epsilon)} (\alpha y_i - \pi_i) dF$$

Consider a firm of some type (θ, γ) contained in $M(\epsilon)$, and suppose that this firm's next best offer came from region h. Credibility implies that π_i is equal to $y_h \mu_h$, and since the firm chose region i over region h,

$$y_h \mu_h \leq y_i(\alpha + \epsilon)$$

The taxes paid by the firm are

$$\alpha y_i - \pi_i \geq -\epsilon y_i$$

Now consider a firm of some type (θ, γ) contained in $L_i(\widehat{k}_i(\mu^\delta))$, and suppose again that the firm's next best offer came from region h. For this firm,

$$y_h \mu_h \leq y_i(\alpha - \delta)$$

and the taxes that it pays are

$$\alpha y_i - \pi_i \geq \delta y_i$$

Thus,

$$R_i^g(\mu^\epsilon) \geq \delta \int_{L_i(\widehat{k}_i(\mu^\delta))} y_i dF - \epsilon \int_{M(\epsilon)} y_i dF$$

By FP2, $M(\epsilon)$ shrinks to some non-empty set M^* as ϵ falls, so that the second term falls toward zero as ϵ falls. The first term is independent of ϵ. It follows that $R_i^g(\mu^\epsilon)$ is positive for all μ^ϵ in which ϵ smaller than some critical value $\bar{\epsilon}$.

Now consider the mapping

$$\phi_i(\mu) = \alpha + \frac{1-\alpha}{\psi_i\left(R_i^c(\mu), \max[R_i^g(\mu), 0]\right)} \qquad i \in \mathcal{I}$$

Define the set

$$\mathbb{D} \equiv \left[\alpha + \frac{1-\alpha}{b}, 1\right]^I$$

The mapping $\phi : \mathbb{D} \to \mathbb{D}$ is continuous and well-defined, so the mapping has a fixed point μ^*.

If $R_i^g(\mu^*)$ is non-negative for every i, all of the elements of a credible equilibrium can be inferred from μ^*. The equilibrium vector MRS is obtained from (29). The equilibrium L is $L(\widehat{k}(\mu))$ or is measurably identical to it. The equilibrium n is $n(\widehat{k}(\mu))$ or is measurably identical to it. Each region's wage is equal to that region's equilibrium marginal product. Credibility ensures that each firm's after-tax profits are well-defined, so its tax rate is uniquely determined by its output and after-tax profits. Government i's revenues are $R_i^c(\mu^*)$ and $R_i^g(\mu^*)$, and region i's consumption of private and public goods is $c_i(R_i^c(\mu^*), R_i^g(\mu^*))$ and $g_i(R_i^c(\mu^*), R_i^g(\mu^*))$.

On the other hand, no equilibrium can be inferred from μ^* if $R_i^g(\mu^*)$ is negative for some region i, so it is necessary to identify the conditions under which every government's revenues are non-negative. Assume that

$$\bar{\epsilon} > \frac{1-\alpha}{a^\circ} \tag{31}$$

and recall that, by definition,

$$\frac{1-\alpha}{a^\circ} \geq \frac{1-\alpha}{\psi_i(R_i^c, 0)}$$

for all $i \in \mathcal{I}$ and for all $R_i^c \geq 0$. Then $R_i^g(\mu^*)$ must be non-negative for every i. Suppose to the contrary that there were some region j for which $R_j^g(\mu^*)$

were negative. Then

$$\mu_j^* = \alpha + \frac{1-\alpha}{\psi_j(R_j^c(\mu^*),0)} < \alpha + \bar{\epsilon}$$

but by construction, $\mu_j < \alpha + \bar{\epsilon}$ implies $R_j^g(\mu) > 0$, contrary to assumption. Consequently, $R_i^g(\mu^*)$ is non-negative for all $i \in \mathcal{I}$ when (31) holds, and (31) holds if a° is sufficiently large.

Total output is maximized if and only if there is some positive λ such that $\widehat{k}_i(\mu^*) = \lambda k_i^*$ for all $i \in \mathcal{I}$. If every element of MRS is the same, every element of μ^* is equal to some λ, where $\alpha < \lambda \leq 1$. Since $Z_i(k)$ is linearly homogeneous, and since $\widehat{k}(\mu) = k^*$ when every element of μ is equal to 1, $\widehat{k}_i(\mu^*) = \lambda k^*$ so that total output is maximized. Now suppose that total output is maximized, implying $\widehat{k}(\mu^*) = \lambda k^*$. Evaluating (30) at λk^* determines a unique vector μ^*. Since it has already been shown that $\widehat{k}(\mu^*) = \lambda k^*$ when every element of μ^* is equal to λ, this is the unique solution for μ^*. Thus, total output maximization implies that every element of μ^* is the same and hence every element of MRS is the same.

The possibility that the equilibrium allocation is Pareto optimal still exists. If each firm's second best option is small relative to its best option, every firm will pay taxes in equilibrium. If each region's preferences for the public good are not too strong, each government's revenue will be sufficient to provide the optimal quantity of public goods. ■

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