At first glance, the RESET procedure may not seem to be based on an artificial regression. But it is easy to show (Exercise 15.2) that the t statistic for $\gamma = 0$ in regression (15.09) is identical to the t statistic for $c = 0$ in the GNR

$$\hat{u}_t = \boldsymbol{X}_t \boldsymbol{b} + c(\boldsymbol{X}_t \hat{\boldsymbol{\beta}})^2 + \text{residual}, \tag{15.10}$$

where $\hat{u}_t$ is the t^{th} residual from regression (15.08). The test regression (15.10) is clearly a special case of the artificial regression (15.05), with $\hat{\boldsymbol{\beta}}$ playing the role of $\acute{\boldsymbol{\theta}}$ and $(\boldsymbol{X}_t \hat{\boldsymbol{\beta}})^2$ playing the role of $\acute{\boldsymbol{Z}}$. It is not hard to check that the three conditions for a valid specification test regression are satisfied. First, the predeterminedness of $\boldsymbol{X}_t$ implies that $\mathrm{E}\big((\boldsymbol{X}_t\boldsymbol{\beta}_0)^2(y_t - \boldsymbol{X}_t\boldsymbol{\beta}_0)\big) = 0$, where $\boldsymbol{\beta}_0$ is the true parameter vector, so that condition R1 holds. Condition R2 is equally easy to check. For condition R3, let $\boldsymbol{z}(\boldsymbol{\beta})$ be the n–vector with typical element $(\boldsymbol{X}_t\boldsymbol{\beta})^2$. Then the derivative of $n^{-1}\boldsymbol{z}^\top(\boldsymbol{\beta})(y_t - \boldsymbol{X}_t\boldsymbol{\beta})$ with respect to β_i, for $i = 1, \ldots, k$, evaluated at $\boldsymbol{\beta}_0$, is

$$\frac{2}{n}\sum_{t=1}^{n} \boldsymbol{X}_t\boldsymbol{\beta}_0\, x_{ti} u_t - \frac{1}{n}\sum_{t=1}^{n} (\boldsymbol{X}_t\boldsymbol{\beta}_0)^2 x_{ti}.$$

The first term above is $n^{-1/2}$ times an expression which, by a central limit theorem, is asymptotically normal with mean zero and finite variance. It is therefore $O_p(n^{-1/2})$. The second term is an element of the vector $-n^{-1}\boldsymbol{z}^\top(\boldsymbol{\beta}_0)\boldsymbol{X}$. Thus condition R3 holds, and the RESET test, implemented by either of the regressions (15.09) or (15.10), is seen to be asymptotically valid.

Actually, the RESET test is not merely valid asymptotically. It is exact in finite samples whenever the model that is being tested satisfies the strong assumptions needed for t statistics to have their namesake distribution; see Section 4.4 for a statement of those assumptions. To see why, note that the vector of fitted values $\boldsymbol{X}\hat{\boldsymbol{\beta}}$ is orthogonal to the residual vector $\hat{\boldsymbol{u}}$, so that $\mathrm{E}(\hat{\boldsymbol{\beta}}^\top\boldsymbol{X}^\top\hat{\boldsymbol{u}}) = 0$. Under the assumption of normal errors, it follows that $\boldsymbol{X}\hat{\boldsymbol{\beta}}$ is independent of $\hat{\boldsymbol{u}}$. As Milliken and Graybill (1970) first showed, and as readers are invited to show in Exercise 15.3, this implies that the t statistic for $c = 0$ yields an exact test under classical assumptions.

Like most specification tests, the RESET procedure is designed to have power against a variety of alternatives. However, it can also be derived as a test against a specific alternative. Suppose that

$$y_t = \frac{\tau(\delta\boldsymbol{X}_t\boldsymbol{\beta})}{\delta} + u_t, \tag{15.11}$$

where δ is a scalar parameter, and $\tau(x)$ may be any scalar function that is monotonically increasing in its argument x and satisfies the conditions

$$\tau(0) = 0,\ \ \tau'(0) = 1,\ \ \text{and}\ \ \tau''(0) \neq 0,$$