model, equation (13.93), adds p lagged values of g_2 additional variables to this regression. We can then perform an asymptotic F test of the hypothesis that the pg_2 coefficients of the lags of all the additional variables are jointly equal to zero. For this test to be asymptotically valid, the error terms must be homoskedastic. If this assumption does not seem to be correct, we should instead perform a heteroskedasticity-robust test, as discussed in Section 6.8.

Our discussion of Granger causality has been quite brief. Hamilton (1994, Chapter 11) provides a much more detailed discussion of this topic. That book also discusses a number of other aspects of VAR models in more detail than we have done here.

13.8 Final Remarks

The analysis of time-series data has engaged the interest of a great many statisticians and econometricians and generated a massive literature. This chapter has provided only a superficial introduction to the subject. In particular, we have said nothing at all about frequency domain methods, because they are a bit too specialized for this book. See Brockwell and Davis (1991), Box, Jenkins, and Reinsel (1994, Chapter 2), Hamilton (1994, Chapter 6), and Fuller (1995), among many others.

This chapter has dealt only with stationary time series. A great many economic time series are, or at least appear to be, nonstationary. Therefore, in the next chapter, we turn our attention to methods for dealing with nonstationary time series. Such methods have been a subject of an enormous amount of research in econometrics during the past two decades.

13.9 Exercises

13.1 Show that the solution to the Yule-Walker equations (13.07) for the AR(2) process is given by equations (13.08).

13.2 Demonstrate that the first $p+1$ Yule-Walker equations for the AR(p) process $u_t = \sum_{i=1}^{p} \rho_i u_{t-i} + \varepsilon_t$ are

$$\begin{aligned} v_0 - \sum_{i=1}^{p} \rho_i v_i &= \sigma_\varepsilon^2, \quad \text{and} \\ \rho_i v_0 - v_i + \sum_{j=1,\, j\neq i}^{p} \rho_j v_{|i-j|} &= 0, \quad i = 1, \ldots, p. \end{aligned} \tag{13.95}$$

Then rewrite these as a set of simultaneous equations for v_0, v_1, and v_2 using matrix notation.

13.3 Consider the AR(2) process

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t,$$