12.26 In the model (12.124), what is the identification status of each of the two equations? How would your answer change if an additional regressor, x_{t6}, were added to the first equation only, to the second equation only, or to both equations?

⋆12.27 Consider the linear simultaneous system of equations (12.90) and (12.91). Write down the estimating equations for the 3SLS estimator for the system, and show that they define the same estimator of the parameters of (12.90) as the IV estimator applied to that equation alone with instruments $\boldsymbol{W}$.

State and prove the analogous result for an SUR system in which only one equation is overidentified.

⋆12.28 In the just-identified case of LIML estimation, for which, in the notation of (12.91), the number of excluded instruments in the matrix $\boldsymbol{W}_1$ is equal to the number of included endogenous variables in the matrix $\boldsymbol{Y}$, show that the minimized value of the ratio κ given by (12.92) is equal to the global minimum of 1. Show further that the vector of estimates $\hat{\boldsymbol{\beta}}_2$ that attains this minimum is the IV, or 2SLS, estimator of $\boldsymbol{\beta}_2$ for equation (12.90) with instruments $\boldsymbol{W}$.

In the overidentified case of LIML estimation, explicitly formulate a model containing the model consisting of (12.90) and (12.91) as a special case, with the overidentifying restrictions relaxed. Show that the maximized loglikelihood for this unconstrained model is the same function of the data as for the constrained model, but with $\hat{\kappa}$ replaced by 1.

12.29 Consider the demand-supply model

$$\begin{aligned} q_t &= \beta_{11} + \beta_{21}x_{t2} + \beta_{31}x_{t3} + \gamma_{21}p_t + u_{t1} \\ q_t &= \beta_{12} + \beta_{42}x_{t4} + \beta_{52}x_{t5} + \gamma_{22}p_t + u_{t2}, \end{aligned} \tag{12.126}$$

where q_t is the log of quantity, p_t is the log of price, x_{t2} is the log of income, x_{t3} is a dummy variable that accounts for regular demand shifts, and x_{t4} and x_{t5} are the prices of inputs. Thus the first equation of (12.126) is a demand function and the second equation is a supply function.

For this model, precisely what is the vector $\boldsymbol{\beta}_\bullet$ that was introduced in equation (12.55)? What are the matrices $\boldsymbol{B}$ and $\boldsymbol{\Gamma}$ that were introduced in equation (12.68)? How many overidentifying restrictions are there?

12.30 The file **demand-supply.data** contains 120 observations generated by the model (12.126). Estimate this model by 2SLS, LIML, 3SLS, and FIML. In each case, test the overidentifying restrictions, either for each equation individually or for the whole system, as appropriate.

12.31 The second equation of (12.126) can be rewritten as

$$p_t = \beta'_{12} + \beta'_{42}x_{t4} + \beta'_{52}x_{t5} + \gamma'_{12}q_t + u'_{t2}. \tag{12.127}$$

Estimate the system that consists of the first equation of (12.126) and equation (12.127) by 3SLS and FIML. What is the relationship between the FIML estimates of this system and the FIML estimates of (12.126)? What is the relationship between the two sets of 3SLS estimates?