For each simulated data set, compute the IV estimator (8.41). Then draw the empirical distribution of the realizations of the estimator on the same plot as the CDF of the normal distribution with mean zero and variance $\sigma_u^2/n\pi_0^2$. Explain why this is an appropriate way to compare the finite-sample and asymptotic distributions of the estimator.

In addition, for each simulated data set, compute the OLS estimator, and plot the EDF of the realizations of this estimator on the same axes as the EDF of the realizations of the IV estimator.

8.11 Redo Exercise 8.10 for a sample size of $n = 100$. If you have enough computer time available, redo it yet again for $n = 1000$, in order to see how quickly or slowly the finite-sample distribution tends to the asymptotic distribution.

8.12 Redo the simulations of Exercise 8.10, for $n = 10$, generating the exogenous instrument $\boldsymbol{w}$ as follows. For the first experiment, use independent drawings from the uniform distribution on $[-1, 1]$. For the second, use drawings from the AR(1) process $w_t = \alpha w_{t-1} + \varepsilon_t$, where $w_0 = 0$, $\alpha = 0.8$, and the ε_t are independent drawings from $\mathrm{N}(0, 1)$. In all cases, rescale $\boldsymbol{w}$ so that $\boldsymbol{w}^\top \boldsymbol{w} = n$. To what extent does the empirical distribution of $\hat{\beta}_{\mathrm{IV}}$ appear to depend on the properties of $\boldsymbol{w}$? What theoretical explanation can you think of for your results?

8.13 Include one more instrument in the simulations of Exercise 8.10. Continue to use the same DGP for $\boldsymbol{y}$ and $\boldsymbol{x}$, but replace the simple IV estimator by the generalized one, based on two instruments $\boldsymbol{w}$ and $\boldsymbol{z}$, where $\boldsymbol{z}$ is generated independently of everything else in the simulation. See if you can verify the theoretical prediction that the overidentified estimator computed with two instruments is more biased, but has thinner tails, than the just identified estimator.

Repeat the simulations twice more, first with two additional instruments and then with four. What happens to the distribution of the estimator as the number of instruments increases?

8.14 Verify that $\hat{\boldsymbol{\beta}}_{\mathrm{IV}}$ is the OLS estimator for model (8.10) when the regressor matrix is $\boldsymbol{X} = [\boldsymbol{Z}\;\; \boldsymbol{Y}] = \boldsymbol{W\Pi}$, with the matrix $\boldsymbol{V}$ in (8.44) equal to $\mathbf{O}$. Is this estimator consistent? Explain.

⋆8.15 Verify, by use of the assumption that the instruments in the matrix $\boldsymbol{W}$ are exogenous or predetermined, and by use of a suitable law of large numbers, that all the terms in (8.45) that involve $\boldsymbol{V}$ do not contribute to the probability limit of (8.45) as the sample size tends to infinity.

8.16 Show that the vector of residuals obtained by running the IVGNR (8.49) with $\boldsymbol{\beta} = \acute{\boldsymbol{\beta}}$ is equal to $\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}}_{\mathrm{IV}} + \boldsymbol{M_W X}(\hat{\boldsymbol{\beta}}_{\mathrm{IV}} - \acute{\boldsymbol{\beta}})$. Use this result to show that $\acute{s}^2$, the estimate of the error variance given by the IVGNR, is consistent for the error variance of the underlying model (8.10) if $\acute{\boldsymbol{\beta}}$ is root-n consistent.

8.17 Prove that expression (8.56) is equal to expression (8.57). **Hint:** Use the facts that $\boldsymbol{P}_{\boldsymbol{P_W X}_1}\boldsymbol{X}_1 = \boldsymbol{P_W X}_1$ and $\boldsymbol{P}_{\boldsymbol{P_W X}}\boldsymbol{P}_{\boldsymbol{P_W X}_1} = \boldsymbol{P}_{\boldsymbol{P_W X}_1}$.

⋆8.18 Show that k_2 times the artificial F statistic from the pair of IVGNRs (8.53) and (8.54) is asymptotically equal to the Wald statistic (8.48), using reasoning similar to that employed in Section 6.7. Why are these two statistics not numerically identical? Show that the asymptotic equality does not hold if different matrices of instruments are used in the two IVGNRs.