estimation (Chapter 9). The key idea of feasible GLS estimation, namely, that an unknown covariance matrix may in some circumstances be replaced by a consistent estimate of that matrix without changing the asymptotic properties of the resulting estimator, will also be encountered again in Chapter 9.

The remainder of the chapter dealt with the treatment of heteroskedasticity and serial correlation in linear regression models, and with error-components models for panel data. Although this material is of considerable practical importance, most of the techniques we discussed, although sometimes complicated in detail, are conceptually straightforward applications of feasible GLS estimation, NLS estimation, and methods for testing hypotheses that were introduced in Chapters 4 and 6.

7.12 Exercises

7.1 Using the fact that $\mathrm{E}(\boldsymbol{u}\boldsymbol{u}^\top \,|\, \boldsymbol{X}) = \boldsymbol{\Omega}$ for regression (7.01), show directly, without appeal to standard OLS results, that the covariance matrix of the GLS estimator $\hat{\boldsymbol{\beta}}_{\rm GLS}$ is given by the rightmost expression of (7.05).

7.2 Show that the matrix (7.11), reproduced here for easy reference,

$$\boldsymbol{X}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{X} - \boldsymbol{X}^\top\boldsymbol{W}(\boldsymbol{W}^\top\boldsymbol{\Omega}\boldsymbol{W})^{-1}\boldsymbol{W}^\top\boldsymbol{X},$$

is positive semidefinite. As in Section 6.2, this may be done by showing that this matrix can be expressed in the form $\boldsymbol{Z}^\top\boldsymbol{M}\boldsymbol{Z}$, for some $n \times k$ matrix $\boldsymbol{Z}$ and some $n \times n$ orthogonal projection matrix $\boldsymbol{M}$. It is helpful to express $\boldsymbol{\Omega}^{-1}$ as $\boldsymbol{\Psi}\boldsymbol{\Psi}^\top$, as in equation (7.02).

7.3 Using the data in the file **earnings.data**, run the regression

$$y_t = \beta_1 d_{1t} + \beta_2 d_{2t} + \beta_3 d_{3t} + u_t,$$

which was previously estimated in Exercise 5.3. Recall that the d_{it} are dummy variables. Then test the null hypothesis that $\mathrm{E}(u_t^2) = \sigma^2$ against the alternative that

$$\mathrm{E}(u_t^2) = \gamma_1 d_{1t} + \gamma_2 d_{2t} + \gamma_3 d_{3t}.$$

Report P values for F and nR^2 tests.

7.4 If u_t follows the stationary AR(1) process

$$u_t = \rho u_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathrm{IID}(0, \sigma_\varepsilon^2), \quad |\rho| < 1,$$

show that $\mathrm{Cov}(u_t, u_{t-j}) = \mathrm{Cov}(u_t, u_{t+j}) = \rho^j\sigma_\varepsilon^2/(1-\rho^2)$. Then use this result to show that the correlation between u_t and u_{t-j} is just ρ^j.

7.5 Consider the nonlinear regression model $y_t = x_t(\boldsymbol{\beta}) + u_t$. Derive the GNR for testing the null hypothesis that the u_t are serially uncorrelated against the alternative that they follow an AR(1) process.