willing to transform the dependent variable as well as the independent ones, the linearity assumption can be made even less restrictive. As an example, consider the nonlinear regression model

$$y_t = e^{\beta_1} x_{t2}^{\beta_2} x_{t3}^{\beta_3} + u_t, \tag{1.23}$$

in which there are two explanatory variables, x_{t2} and x_{t3}, and the regression function is multiplicative. If the notation seems odd, suppose that there is implicitly a third explanatory variable, x_{t1}, which is constant and always equal to e. Notice that the regression function in (1.23) can be evaluated only when x_{t2} and x_{t3} are positive for all t.[4] It is a genuinely nonlinear regression function, because it is clearly linear neither in parameters nor in variables. For reasons that will shortly become apparent, a nonlinear model like (1.23) is very rarely estimated in practice.

A model like (1.23) is not as outlandish as may appear at first glance. It could arise, for instance, if we wanted to estimate a Cobb-Douglas production function. In that case, y_t would be output for observation t, and x_{t2} and x_{t3} would be inputs, say labor and capital. Since e^{β_1} is just a positive constant, it plays the role of the scale factor that is present in every Cobb-Douglas production function.

As (1.23) is written, everything enters multiplicatively except the error term. But it is easy to modify (1.23) so that the error term also enters multiplicatively. One way to do this is to write

$$y_t = e^{\beta_1} x_{t2}^{\beta_2} x_{t3}^{\beta_3} + u_t \equiv \big(e^{\beta_1} x_{t2}^{\beta_2} x_{t3}^{\beta_3}\big)(1 + v_t), \tag{1.24}$$

where the error factor $1 + v_t$ multiplies the regression function. If we now assume that the underlying errors v_t are IID, it follows that the additive errors u_t are proportional to the regression function. This may well be a more plausible specification than that in which the u_t are supposed to be IID, as was implicitly assumed in (1.23). To see this, notice first that the additive error u_t has the same units of measurement as y_t. If (1.23) is interpreted as a production function, then u_t is measured in units of output. However, the multiplicative error v_t is dimensionless. In other words, it is a pure number, like 0.02, which could be expressed as 2 percent. If the u_t are assumed to be IID, then we are assuming that the error in output is of the same order of magnitude regardless of the scale of production. If, on the other hand, the v_t are assumed to be IID, then the error is proportional to total output. This second assumption is almost always more reasonable than the first.

If the model (1.24) is a good one, the v_t should be quite small, usually less than about 0.05. For small values of the argument w, a standard approximation to

[4] If x and a are real numbers, x^a is not usually a real number unless $x > 0$. Think of the square root of -1.