10.31 Formulate a Box-Cox regression model which includes the first and second models of the previous exercise as special cases. Use the OPG regression to perform an LM test of the hypothesis that the Box-Cox parameter $\lambda = 0$, that is, that the loglinear model is correctly specified. Obtain both asymptotic and bootstrap P values.

10.32 The model (9.122) that was estimated in Exercise 10.30 can be written as

$$\Delta c_t = \beta_1 + \beta_2 \Delta y_t + \beta_3 \Delta y_{t-1} + \sigma \varepsilon_t,$$

where $\varepsilon_t \sim \text{NID}(0, 1)$. Suppose now that the ε_t, instead of being standard normal, follow the Cauchy distribution, with density $f(\varepsilon_t) = \big(\pi(1 + \varepsilon_t^2)\big)^{-1}$. Estimate the resulting model by maximum likelihood, and compare the maximized value of the loglikelihood with the one obtained in Exercise 10.30.

10.33 Suppose that the dependent variable y_t is a proportion, so that $0 < y_t < 1$, $t = 1, \ldots, n$. An appropriate model for such a dependent variable is

$$\log\left(\frac{y_t}{1 - y_t}\right) = \boldsymbol{X}_t \boldsymbol{\beta} + u_t,$$

where $\boldsymbol{X}_t$ is a $k \times 1$ vector of exogenous variables, and $\boldsymbol{\beta}$ is a k–vector. Write down the loglikelihood function for this model under the assumption that $u_t \sim \text{NID}(0, \sigma^2)$. How would you maximize this loglikelihood function?