7.15 Use nonlinear least squares to estimate, over the period 1968:1 to 1998:4, the model that results if u_t in (7.98) follows an AR(1) process. Then test the common factor restrictions that are implicit in this model. Calculate an asymptotic P value for the test.

7.16 Test the common factor restrictions of Exercise 7.15 again using a GNR. Calculate both an asymptotic P value and a bootstrap P value based on at least $B = 99$ bootstrap samples. **Hint:** To obtain a consistent estimate of ρ for the GNR, use the fact that the coefficient of r_{t-1} in the unrestricted model (7.74) is equal to $-\rho$ times the coefficient of r_t.

7.17 Use nonlinear least squares to estimate, over the period 1968:1 to 1998:4, the model that results if u_t in (7.98) follows an AR(2) process. Is there any evidence that an AR(2) process is needed here?

⋆7.18 The algorithm called **iterated Cochrane-Orcutt**, alluded to in Section 7.8, is just iterated feasible GLS without the first observation. This algorithm is begun by running the regression $\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{u}$ by OLS, preferably omitting observation 1, in order to obtain the first estimate of $\boldsymbol{\beta}$. The residuals from this equation are then used to estimate ρ according to equation (7.70). What is the next step in this procedure? Complete the description of iterated Cochrane-Orcutt as iterated feasible GLS, showing how each step of the procedure can be carried out using an OLS regression.

Show that, when the algorithm converges, conditions (7.69) for NLS estimation are satisfied. Also show that, unlike iterated feasible GLS including observation 1, this algorithm *must* eventually converge, although perhaps only to a local, rather than the global, minimum of $\text{SSR}(\boldsymbol{\beta}, \rho)$.

7.19 Consider once more the model that you estimated in Exercise 7.15. Estimate this model using the iterated Cochrane-Orcutt algorithm, using a sequence of OLS regressions, and see how many iterations are needed to achieve the same estimates as those achieved by NLS. Compare this number with the number of iterations used by NLS itself.

Repeat the exercise with a starting value of 0.5 for ρ instead of the value of 0 that is conventionally used.

7.20 Test the hypothesis that the error terms of the linear regression model (7.98) are serially uncorrelated against the alternatives that they follow the simple AR(4) process $u_t = \rho_4 u_{t-4} + \varepsilon_t$ and that they follow a general AR(4) process.

Test the hypothesis that the error terms of the nonlinear regression model you estimated in Exercise 7.15 are serially uncorrelated against the same two alternative hypotheses. Use Gauss-Newton regressions.

7.21 Consider the linear regression model

$$\boldsymbol{y} = \boldsymbol{X}_0\boldsymbol{\beta}_0 + \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u}, \quad \boldsymbol{u} \sim \text{IID}(\boldsymbol{0}, \sigma^2\mathbf{I}), \tag{7.99}$$

where there are n observations, and k_0, k_1, and k_2 denote the numbers of parameters in $\boldsymbol{\beta}_0$, $\boldsymbol{\beta}_1$, and $\boldsymbol{\beta}_2$, respectively. Let H_0 denote the hypothesis that $\boldsymbol{\beta}_1 = \boldsymbol{0}$ and $\boldsymbol{\beta}_2 = \boldsymbol{0}$, H_1 denote the hypothesis that $\boldsymbol{\beta}_2 = \boldsymbol{0}$, and H_2 denote the model (7.99) with no restrictions.

Show that the F statistics for testing H_0 against H_1 and for testing H_1 against H_2 are asymptotically independent of each other.