(e) $\boldsymbol{P_X y} = \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u}$;
(f) $\boldsymbol{M}_1\boldsymbol{y} = \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u}$;
(g) $\boldsymbol{M}_1\boldsymbol{y} = \boldsymbol{M}_1\boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u}$;
(h) $\boldsymbol{M}_1\boldsymbol{y} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{M}_1\boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u}$;
(i) $\boldsymbol{M}_1\boldsymbol{y} = \boldsymbol{M}_1\boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{M}_1\boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u}$;
(j) $\boldsymbol{P_X y} = \boldsymbol{M}_1\boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u}$.

Here $\boldsymbol{P}_1$ projects orthogonally on to the span of $\boldsymbol{X}_1$, and $\boldsymbol{M}_1 = \mathbf{I} - \boldsymbol{P}_1$. For which of the above regressions are the estimates of $\boldsymbol{\beta}_2$ the same as for the original regression? Why? For which are the residuals the same? Why?

2.17 Consider the linear regression

$$\boldsymbol{y} = \beta_1\boldsymbol{\iota} + \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u},$$

where $\boldsymbol{\iota}$ is an n–vector of 1s, and $\boldsymbol{X}_2$ is an $n \times (k-1)$ matrix of observations on the remaining regressors. Show, using the FWL Theorem, that the OLS estimators of β_1 and $\boldsymbol{\beta}_2$ can be written as

$$\begin{bmatrix} \hat{\beta}_1 \\ \hat{\boldsymbol{\beta}}_2 \end{bmatrix} = \begin{bmatrix} n & \boldsymbol{\iota}^\top\boldsymbol{X}_2 \\ \mathbf{0} & \boldsymbol{X}_2^\top\boldsymbol{M}_{\iota}\boldsymbol{X}_2 \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{\iota}^\top\boldsymbol{y} \\ \boldsymbol{X}_2^\top\boldsymbol{M}_{\iota}\boldsymbol{y} \end{bmatrix},$$

where, as usual, $\boldsymbol{M}_{\iota}$ is the matrix that takes deviations from the sample mean.

2.18 Using equations (2.35), show that $\boldsymbol{P_X} - \boldsymbol{P}_1$ is an orthogonal projection matrix. That is, show that $\boldsymbol{P_X} - \boldsymbol{P}_1$ is symmetric and idempotent.

★2.19 Show that $\boldsymbol{P_X} - \boldsymbol{P}_1 = \boldsymbol{P}_{\boldsymbol{M}_1\boldsymbol{X}_2}$, where $\boldsymbol{P}_{\boldsymbol{M}_1\boldsymbol{X}_2}$ is the projection on to the span of $\boldsymbol{M}_1\boldsymbol{X}_2$. This can be done most easily by showing that any vector in $\mathcal{S}(\boldsymbol{M}_1\boldsymbol{X}_2)$ is invariant under the action of $\boldsymbol{P_X} - \boldsymbol{P}_1$, and that any vector orthogonal to this span is annihilated by $\boldsymbol{P_X} - \boldsymbol{P}_1$.

2.20 Let $\boldsymbol{\iota}$ be a vector of 1s, and let $\boldsymbol{X}$ be an $n \times 3$ matrix, with full rank, of which the first column is $\boldsymbol{\iota}$. What can you say about the matrix $\boldsymbol{M}_{\iota}\boldsymbol{X}$? What can you say about the matrix $\boldsymbol{P}_{\iota}\boldsymbol{X}$? What is $\boldsymbol{M}_{\iota}\boldsymbol{M_X}$ equal to? What is $\boldsymbol{P}_{\iota}\boldsymbol{M_X}$ equal to?

2.21 Express the four seasonal variables, $\boldsymbol{s}_i$, $i = 1, 2, 3, 4$, defined in (2.46), as functions of the constant $\boldsymbol{\iota}$ and the three variables $\boldsymbol{s}'_i$, $i = 1, 2, 3$, defined in (2.49).

2.22 Show that the full n–dimensional space E^n is the span of the set of **unit basis vectors** $\boldsymbol{e}_t$, $t = 1, \ldots, n$, where all the components of $\boldsymbol{e}_t$ are zero except for the t^{th}, which is equal to 1.

2.23 The file **tbrate.data** contains data for 1950:1 to 1996:4 for three series: r_t, the interest rate on 90-day treasury bills, π_t, the rate of inflation, and y_t, the logarithm of real GDP. For the period 1950:4 to 1996:4, run the regression

$$\Delta r_t = \beta_1 + \beta_2\pi_{t-1} + \beta_3\Delta y_{t-1} + \beta_4\Delta r_{t-1} + \beta_5\Delta r_{t-2} + u_t, \qquad (2.70)$$

where Δ is the **first-difference operator**, defined so that $\Delta r_t = r_t - r_{t-1}$. Plot the residuals and fitted values against time. Then regress the residuals on the fitted values and on a constant. What do you learn from this second