13.17 The file **hstarts.data** contains the housing starts data graphed in Figure 13.2. For the period 1966:1 to 2001:4, regress the unadjusted series h_t on a constant, h_{t-1}, the three seasonal dummies defined in (2.49), those dummies interacted with the elements of the trend vector $\boldsymbol{t}_q$ defined in (13.68), and those dummies interacted with the squares of the elements of $\boldsymbol{t}_q$. Then test the null hypothesis that the error terms for this regression are serially independent against the alternative that they follow the simple AR(4) process (13.63).

For the period 1966:1 to 1999:4, regress the adjusted series h'_t on the unadjusted series h_t, a constant, and the nine seasonal dummy variables used in the previous regression.

For the period 1966:1 to 1999:4, run the regression

$$h'_t = \beta_0 + \sum_{j=-8}^{8} \delta_j h_{t+j} + u_t.$$

Compare the performance of this regression with that of the dummy variable regression you just estimated. Which of them provides a better approximation to the way in which the seasonally adjusted data were actually generated?

⋆13.18 Consider the GARCH(1, 1) model with conditional variance given by equation (13.78). Calculate the unconditional fourth moment of the stationary distribution of the series u_t generated as $u_t = \sigma_t \varepsilon_t$ with $\varepsilon_t \sim \text{NID}(0, 1)$. It may be advisable to begin by calculating the unconditional fourth moment of the stationary distribution of σ_t. What is the necessary condition for the existence of these fourth moments? Show that, when the parameter δ_1 is zero, this condition becomes $3\alpha_1^2 < 1$, as for an ARCH(1) process.

13.19 This exercise is an extension of Exercise 4.2. By considering the derivative of the function $z^{2r+1}\phi(z)$, where $\phi(\cdot)$ is the standard normal density, and using an inductive argument, show that the $(2r)^{\text{th}}$ moment of the N(0, 1) distribution is equal to $\prod_{j=1}^{r}(2j-1)$.

⋆13.20 Use the result of the previous exercise to show that a necessary condition for the existence of the $2r^{\text{th}}$ moment of the ARCH(1) process

$$u_t = \sigma_t \varepsilon_t; \quad \sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2; \quad \varepsilon_t \sim \text{NID}(0, 1)$$

is that $\alpha_1^r \prod_{j=1}^{r}(2j-1) < 1$.

⋆13.21 Consider the regression model $\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{u}$, where $\boldsymbol{X}$ is an $n \times k$ matrix, in which the errors follow a GARCH(1, 1) process with conditional variance given by equation (13.78). Show that the skedastic function $\sigma_t^2(\boldsymbol{\beta}, \boldsymbol{\theta})$ used in the loglikelihood contribution $\ell_t(\boldsymbol{\beta}, \boldsymbol{\theta})$ given in (13.86) can be written explicitly as

$$\sigma_t^2(\boldsymbol{\beta}, \boldsymbol{\theta}) = \frac{\alpha_0(1-\delta_1^{t-1})}{1-\delta_1} + \alpha_1 \sum_{s=1}^{t-1} \delta_1^{s-1} u_{t-s}^2 + \frac{\alpha_0 \delta_1^{t-1}}{1-\alpha_1-\delta_1}, \tag{13.96}$$

where u_t stands for the residual $y_t - \boldsymbol{X}_t\boldsymbol{\beta}$, and all unavailable instances of both u_t^2 and σ_t^2 are replaced by the unconditional expectation $\alpha_0/(1-\alpha_1-\delta_1)$.