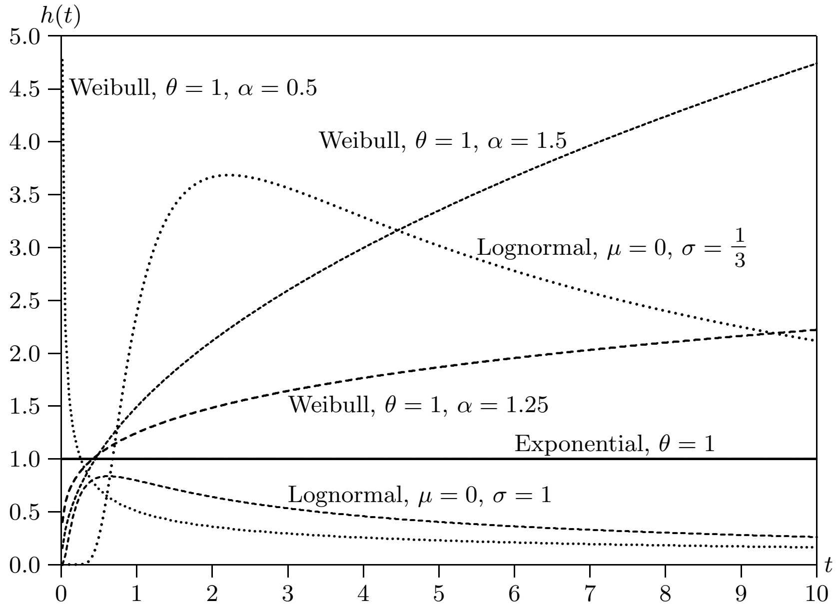

Figure 11.4 Various hazard functions

Maximum Likelihood Estimation

It is reasonably straightforward to estimate many duration models by maximum likelihood. In the simplest case, the data consist of n independent observations t_i on observed durations, each with an associated regressor vector $\boldsymbol{X}_i$. The loglikelihood function for $\boldsymbol{t}$, the vector of observations with typical element t_i, is just

$$\ell(\boldsymbol{t}, \boldsymbol{\theta}) = \sum_{i=1}^{n} \log f(t_i \,|\, \boldsymbol{X}_i, \boldsymbol{\theta}), \tag{11.84}$$

where $f(t_i \,|\, \boldsymbol{X}_i, \boldsymbol{\theta})$ denotes the density of t_i conditional on the data vector $\boldsymbol{X}_i$ for the parameter vector $\boldsymbol{\theta}$. In many cases, it may be easier to write the loglikelihood function as

$$\ell(\boldsymbol{t}, \boldsymbol{\theta}) = \sum_{i=1}^{n} \log h(t_i \,|\, \boldsymbol{X}_i, \boldsymbol{\theta}) + \sum_{i=1}^{n} \log S(t_i \,|\, \boldsymbol{X}_i, \boldsymbol{\theta}), \tag{11.85}$$

where $h(t_i \,|\, \boldsymbol{X}_i, \boldsymbol{\theta})$ is the hazard function and $S(t_i \,|\, \boldsymbol{X}_i, \boldsymbol{\theta})$ is the survivor function. The equivalence of (11.84) and (11.85) is ensured by (11.81), in which the hazard function was defined.

As with other models we have looked at in this chapter, it is convenient to let the loglikelihood depend on explanatory variables through an index function. As an example, suppose that duration follows a Weibull distribution, with