The finite-sample bias of the ML estimator in binary response models can cause an important practical problem for the bootstrap. Since the probabilities associated with $\hat{\boldsymbol{\beta}}$ tend to be more extreme than the true ones, samples generated using $\hat{\boldsymbol{\beta}}$ are more prone to having a perfect classifier. Therefore, even though there is no perfect classifier for the original data, there may well be perfect classifiers for some of the bootstrap samples. The simplest way to deal with this problem is just to throw away any bootstrap samples for which a perfect classifier exists. However, if there is more than a handful of such samples, the bootstrap results must then be viewed with skepticism.

Specification Tests

Maximum likelihood estimation of binary response models almost always yields inconsistent estimates if the form of the transformation function, that is, $F(\boldsymbol{X}_t\boldsymbol{\beta})$, is misspecified. It is therefore very important to test whether this function has been specified correctly.

In Section 11.2, we derived the probit model by starting with the latent variable model (11.04), which has normally distributed, homoskedastic errors. A more general specification for a latent variable model, which allows for the error terms to be heteroskedastic, is

$$y_t^\circ = \boldsymbol{X}_t\boldsymbol{\beta} + u_t, \quad u_t \sim \mathrm{N}\big(0, \exp(2\boldsymbol{Z}_t\boldsymbol{\gamma})\big), \tag{11.24}$$

where $\boldsymbol{Z}_t$ is a row vector of dimension r of observations on variables that belong to the information set Ω_t, and $\boldsymbol{\gamma}$ is an r–vector of parameters to be estimated along with $\boldsymbol{\beta}$. To ensure that both $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are identifiable, $\boldsymbol{Z}_t$ must not include a constant term or the equivalent. With this precaution, the model (11.04) is obtained by setting $\boldsymbol{\gamma} = \mathbf{0}$. Combining (11.24) with (11.05) yields the model

$$P_t \equiv \mathrm{E}(y_t \,|\, \Omega_t) = \Phi\bigg(\frac{\boldsymbol{X}_t\boldsymbol{\beta}}{\exp(\boldsymbol{Z}_t\boldsymbol{\gamma})}\bigg),$$

in which P_t depends on both the regression function $\boldsymbol{X}_t\boldsymbol{\beta}$ and the skedastic function $\exp(2\boldsymbol{Z}_t\boldsymbol{\gamma})$. Thus it is clear that heteroskedasticity of the u_t in a latent variable model affects the form of the transformation function.

Even when the binary response model being used is not the probit model, it still seems quite reasonable to consider the alternative hypothesis

$$P_t = F\bigg(\frac{\boldsymbol{X}_t\boldsymbol{\beta}}{\exp(\boldsymbol{Z}_t\boldsymbol{\gamma})}\bigg). \tag{11.25}$$

We can test against this alternative by using a BRMR to test the hypothesis that $\boldsymbol{\gamma} = \mathbf{0}$. The appropriate BRMR is

$$\tilde{V}_t^{-1/2}(y_t - \tilde{F}_t) = \tilde{V}_t^{-1/2}\tilde{f}_t\boldsymbol{X}_t\boldsymbol{b} - \tilde{V}_t^{-1/2}\boldsymbol{X}_t\tilde{\boldsymbol{\beta}}\tilde{f}_t\boldsymbol{Z}_t\boldsymbol{c} + \text{residual}, \tag{11.26}$$